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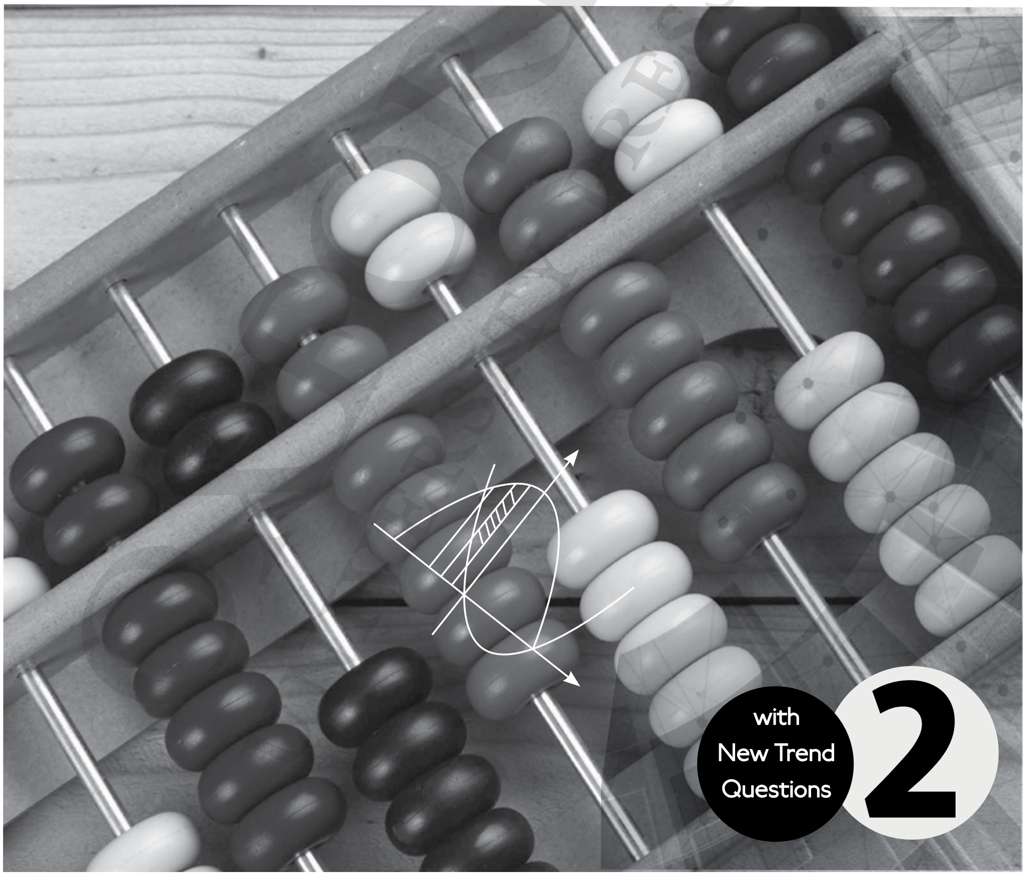
7th EDITION

NEW SYLLABUS MATHEMATICS

WORKBOOK FULL SOLUTIONS



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ANSWERS

Chapter 1 Direct and Inverse Proportions

Basic

1. Cost of 15 l of petrol $= \frac{\$14.70}{7} \times 15$

$= \$31.50$

2. **(i)** $y = kx$

When $x = 200$, $y = 40$,

$40 = k(200)$

$k = \frac{40}{200}$

$= \frac{1}{5}$

$\therefore y = \frac{1}{5}x$

(ii) When $x = 15$,

$y = \frac{1}{5}(15)$

$= 3$

(iii) When $y = 8$,

$8 = \frac{1}{5}x$

$x = 40$

3. **(i)** $s = kt^2$

When $t = 4$, $s = 8$,

$8 = k(4)^2$

$k = \frac{8}{16}$

$= \frac{1}{2}$

$\therefore s = \frac{1}{2}t^2$

(ii) When $t = 3$,

$s = \frac{1}{2}(3)^2$

$= 4\frac{1}{2}$

(iii) When $s = 32$,

$32 = \frac{1}{2}t^2$

$t^2 = 64$

$t = \pm 8$

4. **(i)** $y = k(4x + 1)$

When $x = 2$, $y = 3$,

$3 = k(8 + 1)$

$k = \frac{3}{9}$

$= \frac{1}{3}$

$\therefore y = \frac{1}{3}(4x + 1)$

(ii) When $x = 5$,

$y = \frac{1}{3}(20 + 1)$

$= 7$

(iii) When $y = 11$,

$11 = \frac{1}{3}(4x + 1)$

$33 = 4x + 1$

$4x = 32$

$x = 8$

5. **(i)** $D^3 = kL$

When $L = 6$, $D = 2$,

$2^3 = k(6)$

$k = \frac{8}{6}$

$= \frac{4}{3}$

$\therefore D^3 = \frac{4}{3}L$

(ii) When $L = 48$,

$D^3 = \frac{4}{3}(48)$

$= 64$

$D = 4$

(iii) When $D = \frac{2}{3}$,

$\left(\frac{2}{3}\right)^3 = \frac{4}{3}L$

$\frac{8}{27} = \frac{4}{3}L$

$L = \frac{8}{27} \div \frac{4}{3}$

$= \frac{2}{9}$

6. Time taken for 1 tap to fill the bath tub = 15×2

$= 30$ minutes

Time taken for 3 taps to fill the bath tub = $\frac{30}{3}$

$= 10$ minutes

7. **(i)** When $x = 5$,

$y = 100 \times 2$

$= 200$

(ii) $y = \frac{k}{x}$

When $x = 10$, $y = 100$,

$100 = \frac{k}{10}$

$k = 1000$

$\therefore y = \frac{1000}{x}$

(iii) When $y = 80$,

$80 = \frac{1000}{x}$

$x = \frac{1000}{80}$

$= 12.5$

8. **(i)** $y = \frac{k}{\sqrt{x}}$

When $x = 16$, $y = 5$,

$5 = \frac{k}{\sqrt{16}}$

$= \frac{k}{4}$

$k = 20$

$\therefore y = \frac{20}{\sqrt{x}}$

(ii) When $x = 100$,

$y = \frac{20}{\sqrt{100}}$

$= \frac{20}{10}$

$= 2$

(iii) When $y = 4$,

$4 = \frac{20}{\sqrt{x}}$

$\sqrt{x} = 5$

$x = 5^2$

$= 25$

Intermediate

9. **(i)** $a = kb$

When $b = 15$, $a = 75$,

$75 = k(15)$

$k = \frac{75}{15}$

$= 5$

$\therefore a = 5b$

When $b = 37.5$,

$a = 5(37.5)$

$= 187.5$

(ii) When $a = 195$,

$195 = 5b$

$b = \frac{195}{5}$

$= 39$

10. $h = kl$

When $l = 36$, $h = 30$,

$30 = k(36)$

$k = \frac{30}{36}$

$= \frac{5}{6}$

$\therefore h = \frac{5}{6}l$

When $h = 15$,

$15 = \frac{5}{6}l$

$l = \frac{6}{5} \times 15$

$= 18$

When $l = 72$,

$h = \frac{5}{6}(72)$

$= 60$

When $h = 75$,

$75 = \frac{5}{6}l$

$l = \frac{6}{5} \times 75$

$= 90$

h	15	30	60	75
l	18	36	72	90

11. **(i)** $w = kt$

When $t = 0.3$, $w = 1.8$,

$1.8 = k(0.3)$

$k = \frac{1.8}{0.3}$

$= 6$

$\therefore w = 6t$

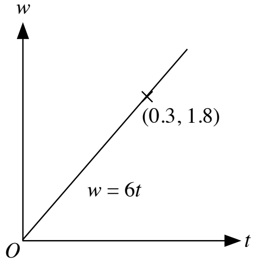
(ii) When $t = 2.5$,

$w = 6(2.5)$

$= 15$

$\therefore$ 15 g of silver will be deposited.

(iii)



12. (i) $F = km$

When $m = 250$, $F = 60$,

$60 = k(250)$

$k = \frac{60}{250}$

$= \frac{6}{25}$

$\therefore F = \frac{6}{25}m$

(ii) When $m = 300$,

$F = \frac{6}{25}(300)$

$= 72$

$\therefore$ The net force required is 72 newtons.

(iii) When $F = 102$,

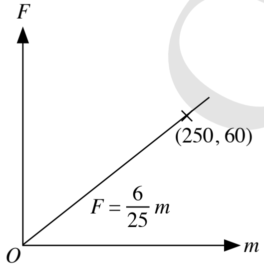
$102 = \frac{6}{25}m$

$m = \frac{25}{6} \times 102$

$= 425$

$\therefore$ The mass of the box is 425 kg.

(iv)



13. (i) $C = an + b$

When $n = 200$, $C = 55\ 000$,

$55\ 000 = 200a + b$ —(1)

When $n = 500$, $C = 62\ 500$,

$62\ 500 = 500a + b$ —(2)

(2) – (1): $300a = 7500$

$a = \frac{7500}{300}$

$= 25$

Substitute $a = 25$ into (1):

$200(25) + b = 55\ 000$

$5000 + b = 55\ 000$

$b = 55\ 000 - 5000$

$= 50\ 000$

$\therefore a = 25, b = 50\ 000$

(ii) $C = 25n + 50\ 000$

When $n = 420$,

$C = 25(420) + 50\ 000$

$= 60\ 500$

$\therefore$ The total cost is \$60 500.

(iii) When $C = 70\ 000$,

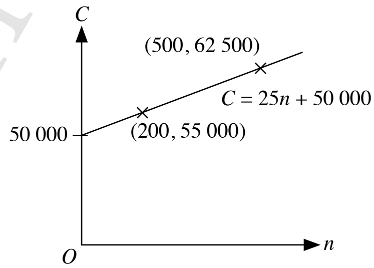
$70\ 000 = 25n + 50\ 000$

$25n = 20\ 000$

$n = \frac{20\ 000}{25}$

$= 800$

(iv)



No, C is not directly proportional to n since the graph of C against n does not pass through the origin.

14. (i) Annual premium payable $= \$25 + \frac{\$20\ 000}{\$1000} \times \$2$

$= \$65$

(ii) Face value $= (\$155 - \$25) \times \frac{\$1000}{\$2}$

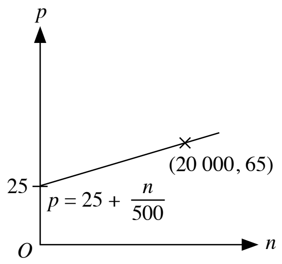
$= \$65\ 000$

(iii) $p = 25 + \frac{n}{1000} \times 2$

$= 25 + \frac{2n}{1000}$

$= 25 + \frac{n}{500}$

(iv)



No, p is not directly proportional to n since the graph of p against n does not pass through the origin.

15. **(a)** y and x^5

(b) y^3 and $\sqrt{x}$

(c) $(y-2)^2$ and x

16. **(i)** $n = km^3$

When $m = 1\frac{1}{2}$, $n = 27$,

$27 = k\left(\frac{3}{2}\right)^3$

$= \frac{27}{8}k$

$k = 8$

$\therefore n = 8m^3$

When $m = 2$,

$n = 8(2)^3$

$= 64$

(ii) When $n = 125$,

$125 = 8m^3$

$m^3 = \frac{125}{8}$

$m = \frac{5}{2}$

$= 2\frac{1}{2}$

17. $y = k(x+2)(x+7)$

When $x = 1$, $y = 4$,

$4 = k(3)(8)$

$k = \frac{4}{24}$

$= \frac{1}{6}$

$\therefore y = \frac{1}{6}(x+2)(x+7)$

When $x = 5$,

$y = \frac{1}{6}(7)(12)$

$= 14$

18. **(i)** $h^2 = kl$

When $l = \frac{1}{8}$, $h = \frac{1}{2}$,

$\left(\frac{1}{2}\right)^2 = k\left(\frac{1}{8}\right)$

$\frac{1}{4} = \frac{1}{8}k$

$k = 2$

$\therefore h^2 = 2l$

When $l = 8$,

$h^2 = 2(8)$

$= 16$

$h = \pm 4$

(ii) When $h = 6$,

$6^2 = 2l$

$36 = 2l$

$l = \frac{36}{2}$

$= 18$

19. $y = k\sqrt{x+1}$

When $x = 224$, $y = 5$,

$5 = k\sqrt{225}$

$= 15k$

$k = \frac{5}{15}$

$= \frac{1}{3}$

$\therefore y = \frac{1}{3}\sqrt{x+1}$

When $x = -1$, $y = p$,

$p = \frac{1}{3}\sqrt{0}$

$= 0$

When $x = q$, $y = 3\frac{1}{3}$,

$3\frac{1}{3} = \frac{1}{3}\sqrt{q+1}$

$10 = \sqrt{q+1}$

$q + 1 = 100$

$q = 99$

$\therefore p = 0, q = 99$

20. (i) $m = kr^3$

When $r = 3$, $m = 54$,

$54 = k(3)^3$

$= 27k$

$k = \frac{54}{27}$

$= 2$

$\therefore m = 2r^3$

(ii) When $r = 4$,

$m = 2(4)^3$

$= 128$

$\therefore$ The mass of the sphere is 128 g.

21. (i) $v = k\sqrt{r}$

When $r = 121$, $v = 22$,

$22 = k\sqrt{121}$

$= 11k$

$k = \frac{22}{11}$

$= 2$

$\therefore v = 2\sqrt{r}$

When $r = 81$,

$v = 2\sqrt{81}$

$= 18$

$\therefore$ The safe speed is 18 m/s.

(ii) When $v = 11$,

$11 = 2\sqrt{r}$

$\sqrt{r} = \frac{11}{2}$

$r = \left(\frac{11}{2}\right)^2$

$= 30.25$

$\therefore$ The radius is 30.25 m.

22. $H = kd^3$

When $d = 6$, $H = 120$,

$120 = k(6)^3$

$= 216k$

$k = \frac{120}{216}$

$= \frac{5}{9}$

$\therefore H = \frac{5}{9}d^3$

When $d = 9$,

$H = \frac{5}{9}(9)^3$

$= 405$

$\therefore$ The shaft can transmit 405 horsepower.

23. Number of workers to complete in 1 day $= 6 \times 8$

$= 48$

Number of workers to complete in 12 days $= \frac{48}{12}$

$= 4$

24.

Number of girls		Number of paper cranes		Number of minutes	
8		5	$\times 24$	6	$\times 24$
8	$\div 8$	120		144	$\times 8$
1	$\times 36$	120		1152	$\div 36$
36		120		32	

$\therefore$ 36 girls take 32 minutes to fold 120 paper cranes.
Assume that all the girls have the same rate of folding paper cranes.

25. (i) $f = \frac{k}{w}$

When $w = 1.5 \times 10^3$, $f = 2.0 \times 10^5$,

$2.0 \times 10^5 = \frac{k}{1.5 \times 10^3}$

$k = (2.0 \times 10^5) \times (1.5 \times 10^3)$

$= 3.0 \times 10^8$

$\therefore f = \frac{3.0 \times 10^8}{w}$

When $w = 480$,

$f = \frac{3.0 \times 10^8}{480}$

$= 625\,000$

$\therefore$ The frequency is 625 000 Hz.

(ii) When $f = 9.6 \times 10^5$,

$9.6 \times 10^5 = \frac{3.0 \times 10^8}{w}$

$w = \frac{3.0 \times 10^8}{9.6 \times 10^5}$

$= 312.5$

$\therefore$ The wavelength is 312.5 m.

26. $P = \frac{k}{V}$

When $V = 2$, $P = 500$,

$500 = \frac{k}{2}$

$k = 500 \times 2$

$= 1000$

$\therefore P = \frac{1000}{V}$

When $V = 5$,

$P = \frac{1000}{5}$

$= 200$

$\therefore$ The pressure of the gas is 200 pascals.

27. **(a)** y and x^5

(b) y^2 and $\sqrt{x}$

(c) $y - 1$ and x

28. $y = \frac{k}{x^2}$

When $x = 4$, $y = 5$,

$5 = \frac{k}{4^2}$

$k = 5 \times 16$

$= 80$

$\therefore y = \frac{80}{x^2}$

When $x = 2$,

$y = \frac{80}{2^2}$

$= 20$

29. $y = \frac{k}{r^2 + 1}$

When $r = 1$, $y = 32$,

$32 = \frac{k}{1 + 1}$

$= \frac{k}{2}$

$k = 64$

$\therefore y = \frac{64}{r^2 + 1}$

When $r = 7$,

$y = \frac{64}{7^2 + 1}$

$= 1\frac{7}{25}$

30. $u = \frac{k}{\sqrt{v}}$

When $v = 9$, $u = 10$,

$10 = \frac{k}{\sqrt{9}}$

$k = 10 \times 3$

$= 30$

$\therefore u = \frac{30}{\sqrt{v}}$

When $v = 25$,

$u = \frac{30}{\sqrt{25}}$

$= 6$

31. $y = \frac{k}{x^2}$

When $x = 10$, $y = 2$,

$2 = \frac{k}{10^2}$

$k = 2 \times 100$

$= 200$

$\therefore y = \frac{200}{x^2}$

When $x = 5$,

$y = \frac{200}{5^2}$

$= 8$

When $y = \frac{8}{9}$,

$\frac{8}{9} = \frac{200}{x^2}$

$x^2 = 225$

$x = \pm 15$

x	5	10	15
y	8	2	$\frac{8}{9}$

32. **(i)** $F = \frac{k}{R^2}$

(ii) When $R = 32$, $F = 50$,

$50 = \frac{k}{32^2}$

$k = 50 \times 32^2$

$= 51\ 200$

(iii) $F = \frac{51\ 200}{R^2}$

When $F = 512$,

$512 = \frac{51\ 200}{R^2}$

$R^2 = 100$

$R = \pm 10$

$\therefore R = 10$

Advanced

33. $y = kx^3$

When $x = a, y = p$,

$p = ka^3$

When $x = \frac{1}{2}a$,

$y = k\left(\frac{1}{2}a\right)^3$

$= \frac{1}{8}ka^3$

$= \frac{1}{8}p$

34. (i) $W = \frac{k}{d^2}$

When $d = 6500, W = 800$,

$800 = \frac{k}{6500^2}$

$k = 800 \times 6500^2$

$= 3.38 \times 10^{10}$

$\therefore W = \frac{3.38 \times 10^{10}}{d^2}$

When $d = 2.5 \times 10^4 + 6500 = 31\,500$,

$W = \frac{3.38 \times 10^{10}}{31\,500^2}$

$= 34.1$ (to 3 s.f.)

$\therefore$ The weight of the astronaut is 34.1 N.

(ii) When $W = 400$,

$400 = \frac{3.38 \times 10^{10}}{d^2}$

$d^2 = 8.45 \times 10^7$

$d = \pm 9190$ (to 3 s.f.)

$\therefore$ The astronaut is 9190 km above the centre of the earth.

New Trend

35. Number of pails Tap A can fill in 1 minute $= \frac{8}{5}$

Number of pails Tap B can fill in 1 minute $= \frac{7}{4}$

Number of pails Tap A and B can fill in 1 minute

$= \frac{8}{5} + \frac{7}{4}$

$= 3\frac{7}{20}$

Time taken to fill 16 pails $= \frac{16}{3\frac{7}{20}}$

$= 4\frac{52}{67}$

$= 4$ min 47 s (to nearest second)

36. (a) $f = k\sqrt[3]{T}$

When $f = 320, T = 64$,

$320 = k\sqrt[3]{64}$

$k = \frac{320}{4}$

$= 80$

$\therefore f = 80\sqrt[3]{T}$

(b) When $f = 450$,

$450 = 80\sqrt[3]{T}$

$\sqrt[3]{T} = \frac{450}{80}$

$T = \frac{91125}{512}$

$= 178$ N (to 3 s.f.)

(c) $\frac{f_1}{f_2} = \frac{80\sqrt[3]{T_1}}{80\sqrt[3]{T_2}}$

$\frac{1}{2} = \frac{\sqrt[3]{T_1}}{\sqrt[3]{T_2}}$

$\frac{T_1}{T_2} = \frac{1}{8}$

$\therefore$ The ratio of the tensions in the string is 1 : 8.

37. (a) (i) $P = kR^2$

When $P = 200, R = 5$,

$200 = k(5)^2$

$k = \frac{200}{25}$

$= 8$

$\therefore P = 8R^2$

(ii) When $R = 20$,

$P = 8(20)^2$

$= 3200$ kPa

(b) $\frac{P_A}{P_B} = \frac{(1.8)^2}{(1)^2}$

$= \left(1\frac{4}{5}\right)^2$

$= \left(\frac{9}{5}\right)^2$

$= \frac{81}{25}$

$\therefore$ The ratio of the pressure acting on disc A to the pressure acting on disc B is 81 : 25.

38. (i) $s = kt^2$

When $t = 3$, $s = 45$,

$45 = k(3)^2$

$= 9k$

$k = \frac{45}{9}$

$= 5$

$\therefore s = 5t^2$

When $t = 7$,

$s = 5(7)^2$

$= 245$

$\therefore$ The distance is 245 m.

(ii) When $s = 20$,

$20 = 5t^2$

$t^2 = \frac{20}{5}$

$= 4$

$t = \pm 2$

$\therefore$ The time taken is 2 s.

Chapter 2 Linear Graphs and Simultaneous Linear Equations

Basic

1. **(a)** Take two points (0, 2) and (7, 2).
Vertical change (or rise) = 2 – 2 = 0
Horizontal change (or run) = 7 – 0 = 7

$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{0}{7} = 0$

(b) Take two points (7, 0) and (7, 7).
Vertical change (or rise) = 7 – 0 = 7
Horizontal change (or run) = 7 – 7 = 0

$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{7}{0}$ = undefined

(c) Take two points (0, 2) and (4, 6).
Vertical change (or rise) = 6 – 2 = 4
Horizontal change (or run) = 4 – 0 = 4
Since the line slopes upwards from the left to the right, its gradient is positive.

$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{4}{4} = 1$

(d) Take two points (4, 6) and (7, 0).
Vertical change (or rise) = 6 – 0 = 6
Horizontal change (or run) = 7 – 4 = 3
Since the line slopes downwards from the left to the right, its gradient is negative.

$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= -\frac{6}{3} = -2$

2. **(a)** Take two points (–3, 4) and (4, 4).
Vertical change (or rise) = 4 – 4 = 0
Horizontal change (or run) = 4 – (–3) = 7

$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{0}{7} = 0$

(b) Take two points (–3, –3) and (4, –3).
Vertical change (or rise) = –3 – (–3) = 0
Horizontal change (or run) = 4 – (–3) = 7

$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{0}{7} = 0$

(c) Take two points (–3, 4) and (–3, –3).
Vertical change (or rise) = 4 – (–3) = 7
Horizontal change (or run) = –3 – (–3) = 0

$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{7}{0}$ = undefined

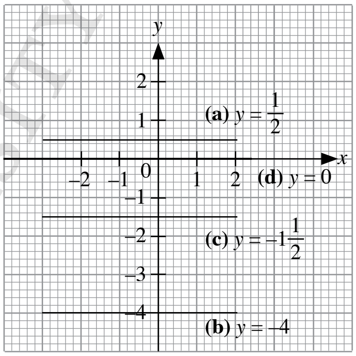
(d) Take two points (–4, 4) and (0, –3).
Vertical change (or rise) = 4 – (–3) = 7
Horizontal change (or run) = 0 – (–4) = 4
Since the line slopes downwards from the left to the right, its gradient is negative.

$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= -\frac{7}{4}$

(e) Take two points (0, –3) and (4, 4).
Vertical change (or rise) = 4 – (–3) = 7
Horizontal change (or run) = 4 – 0 = 4
Since the line slopes upwards from the left to the right, its gradient is positive.

$\therefore$ Gradient $= \frac{\text{rise}}{\text{run}}$
$= \frac{7}{4}$

3.



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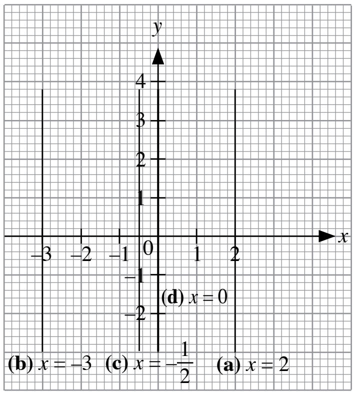


5. **(i)** Line 1: $x = 1$

Line 2: $x = -1.2$

Line 3: $y = 2$

Line 4: $y = -2.6$

(ii) Area enclosed $= (2.2)(4.6)$

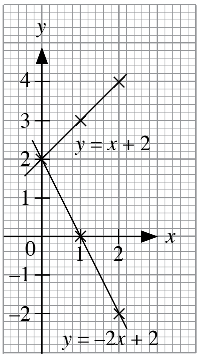
$= 10.12$ units^2

6. **(a)** $y = x + 2$

x	0	1	2
y	2	3	4

$y = -2x + 2$

x	0	1	2
y	2	0	–2



From the graph,

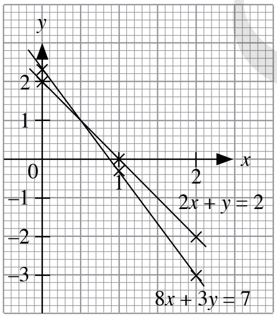
$x = 0$ and $y = 2$

(b) $8x + 3y = 7$

x	0	1	2
y	2.3	–0.3	–3

$2x + y = 2$

x	0	1	2
y	2	0	–2



From the graph,

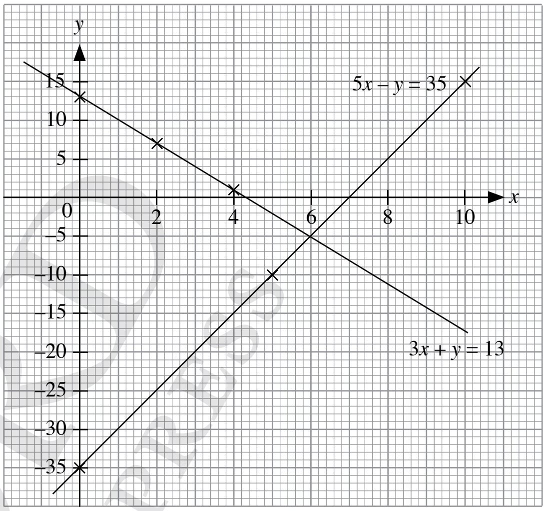
$x = \frac{1}{2}$ and $y = 1$.

(c) $3x + y = 13$

x	0	2	4
y	13	7	1

$5x - y = 35$

x	0	5	10
y	–35	–10	15



From the graph,

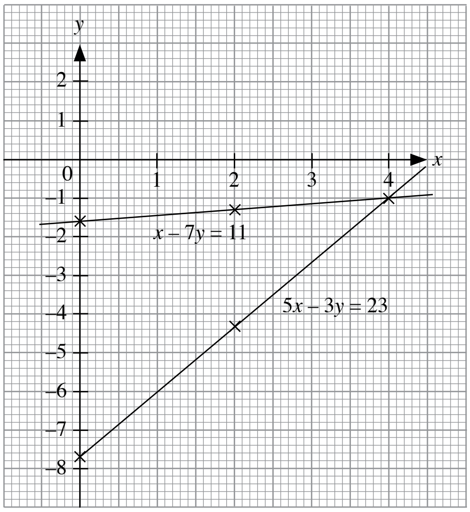
$x = 6$ and $y = -5$.

(d) $5x - 3y = 23$

x	0	2	4
y	–7.7	–4.3	–1

$x - 7y = 11$

x	0	2	4
y	–1.6	–1.3	–1



From the graph,

$x = 4$ and $y = -1$.

7. **(a)** $x + y = 7$ —(1)
$x - y = 3$ —(2)
(1) + (2): $2x = 10$
$x = 5$
Substitute $x = 5$ into (1):
$5 + y = 7$
$y = 2$
$\therefore x = 5, y = 2$

(b) $5x - 4y = 18$ —(1)
$3x + 2y = 13$ —(2)
(2) × 2: $6x + 4y = 26$ —(3)
(1) + (3): $11x = 44$
$x = 4$
Substitute $x = 4$ into (1):
$5(4) - 4y = 18$
$20 - 4y = 18$
$4y = 2$
$y = \frac{1}{2}$
$\therefore x = 4, y = \frac{1}{2}$

(c) $x + 3y = 7$ —(1)
$x + y = 3$ —(2)
(1) – (2): $2y = 4$
$y = 2$
Substitute $y = 2$ into (2):
$x + 2 = 3$
$x = 1$
$\therefore x = 1, y = 2$

(d) $3x - 5y = 19$ —(1)
$5x + 2y = 11$ —(2)
(1) × 2: $6x - 10y = 38$ —(3)
(2) × 5: $25x + 10y = 55$ —(4)
(3) + (4): $31x = 93$
$x = 3$
Substitute $x = 3$ into (2):
$5(3) + 2y = 11$
$15 + 2y = 11$
$2y = -4$
$y = -2$
$\therefore x = 3, y = -2$

(e) $3x - 4y = 30$ —(1)
$2x - 7y = 33$ —(2)
(1) × 2: $6x - 8y = 60$ —(3)
(2) × 3: $6x - 21y = 99$ —(4)
(3) – (4): $13y = -39$
$y = -3$
Substitute $y = -3$ into (2):
$2x - 7(-3) = 33$
$2x + 21 = 33$
$2x = 12$
$x = 6$
$\therefore x = 6, y = -3$

8. **(a)** $3x + y = 17$ —(1)
$3x - y = 19$ —(2)
From (1),
$y = 17 - 3x$ —(3)
Substitute (3) into (2):
$3x - (17 - 3x) = 19$
$3x - 17 + 3x = 19$
$6x = 36$
$x = 6$
Substitute $x = 6$ into (3):
$y = 17 - 3(6)$
$= -1$
$\therefore x = 6, y = -1$

(b) $2x - y = 3$ —(1)
$x + y = 0$ —(2)
From (1),
$y = 2x - 3$ —(3)
Substitute (3) into (2):
$x + (2x - 3) = 0$
$x + 2x - 3 = 0$
$3x = 3$
$x = 1$
Substitute $x = 1$ into (3):
$y = 2(1) - 3$
$= 2 - 3$
$= -1$
$\therefore x = 1, y = -1$

(c) $3x + 3 = 6y$ —(1)
$x - y = 1$ —(2)
From (2),
$y = x - 1$ —(3)
Substitute (3) into (1):
$3x + 3 = 6(x - 1)$
$= 6x - 6$
$3x = 9$
$x = 3$
Substitute $x = 3$ into (3):
$y = 3 - 1$
$= 2$
$\therefore x = 3, y = 2$

(d) $6x + 2y = -3$ —(1)
$4x - 7y = 23$ —(2)
From (1),
$y = \left(\dfrac{-3 - 6x}{2}\right)$ —(3)
Substitute (3) into (2):
$4x - 7\left(\dfrac{-3 - 6x}{2}\right) = 23$
$8x + 21 + 42x = 46$
$50x = 25$
$x = \dfrac{1}{2}$
Substitute $x = \dfrac{1}{2}$ into (3):
$y = \dfrac{-3 - 6\left(\frac{1}{2}\right)}{2}$
$= -3$
$\therefore x = \dfrac{1}{2}, y = -3$

(e) $5x + y = 7$ —(1)
$3x - 5y = 13$ —(2)
From (1),
$y = 7 - 5x$ —(3)
Substitute (3) into (2):
$3x - 5(7 - 5x) = 13$
$3x - 35 + 25x = 13$
$28x = 48$
$x = 1\dfrac{5}{7}$
Substitute $x = 1\dfrac{5}{7}$ into (3):
$y = 7 - 5\left(1\dfrac{5}{7}\right)$
$= -1\dfrac{4}{7}$
$\therefore x = 1\dfrac{5}{7}, y = -1\dfrac{4}{7}$

9. **(a)** $3x - y = -1$ —(1)
$x + y = -3$ —(2)
(1) + (2): $4x = -4$
$x = -1$
Substitute $x = -1$ into (2):
$-1 + y = -3$
$y = -2$
$\therefore x = -1, y = -2$

(b) $2x - 3y = 13$ —(1)
$3x - 12y = 42$ —(2)
From (2),
$x - 4y = 14$
$x = 4y + 14$ —(3)
Substitute (3) into (1):
$2(4y + 14) - 3y = 13$
$8y + 28 - 3y = 13$
$5y = -15$
$y = -3$
Substitute $y = -3$ into (3):
$x = 4(-3) + 14$
$= -12 + 14$
$= 2$
$\therefore x = 2, y = -3$

(c) $14x + 6y = 9$ —(1)
$6x - 15y = -2$ —(2)
(1) × 5: $70x + 30y = 45$ —(3)
(2) × 2: $12x - 30y = -4$ —(4)
(3) + (4): $82x = 41$
$x = \dfrac{1}{2}$
Substitute $x = \dfrac{1}{2}$ into (2):
$6\left(\dfrac{1}{2}\right) - 15y = -2$
$3 - 15y = -2$
$15y = 5$
$y = \dfrac{1}{3}$
$\therefore x = \dfrac{1}{2}, y = \dfrac{1}{3}$

(d) $8x + y = 24$ —(1)
$4x - y = 6$ —(2)
(1) + (2): $12x = 30$
$x = 2\dfrac{1}{2}$
Substitute $x = 2\dfrac{1}{2}$ into (2):
$4\left(2\dfrac{1}{2}\right) - y = 6$
$10 - y = 6$
$y = 4$
$\therefore x = 2\dfrac{1}{2}, y = 4$

(e) $3x + 7y = 17$ —(1)

$3x - 6y = 4$ —(2)

(1) – (2): $13y = 13$

$y = 1$

Substitute $y = 1$ into (1):

$3x + 7(1) = 17$

$3x + 7 = 17$

$3x = 10$

$x = 3\frac{1}{3}$

$\therefore x = 3\frac{1}{3}, y = 1$

(f) $7x - 3y = 6$ —(1)

$7x - 4y = 8$ —(2)

(1) – (2): $y = -2$

Substitute $y = -2$ into (1):

$7x - 3(-2) = 6$

$7x + 6 = 6$

$7x = 0$

$x = 0$

$\therefore x = 0, y = -2$

Intermediate

10. For L_1:

Vertical change (or rise) = 6 – 2 = 4

Horizontal change (or run) = 4 – 0 = 4

Since the line slopes upwards from the left to the right, its gradient is positive.

m = gradient of line

$= \frac{4}{4}$

$= 1$

c = y-intercept

$= 2$

For L_2:

Vertical change (or rise) = 6 – (–2) = 8

Horizontal change (or run) = 4 – 0 = 4

Since the line slopes upwards from the left to the right, its gradient is positive.

m = gradient of line

$= \frac{8}{4}$

$= 2$

c = y-intercept

$= -2$

For L_3:

Vertical change (or rise) = 4 – 0 = 4

Horizontal change (or run) = 4 – 0 = 4

Since the line slopes downwards from the left to the right, its gradient is negative.

m = gradient of line

$= -\frac{4}{4}$

$= -1$

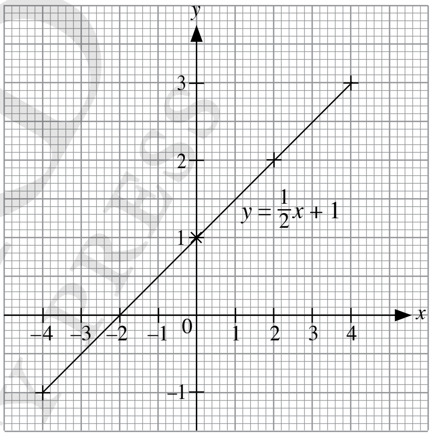
c = y-intercept

$= 4$

11. (i)

x	–4	0	2	4
$y = \frac{1}{2}x + 1$	$y = \frac{1}{2}(-4) + 1$ $= -1$	$y = \frac{1}{2}(0) + 1$ $= 1$	$y = \frac{1}{2}(2) + 1$ $= 2$	$y = \frac{1}{2}(4) + 1$ $= 3$

(ii)



(iii) From the graph, the point (3, 2.5) lies on the line but the point $\left(-1, -\frac{1}{2}\right)$ does not lie on the line.

(iv) From the graph, the line cuts the x-axis at $x = -2$. The coordinates are (–2, 0).

(v) Vertical change (or rise) = 3 – (–1) = 4

Horizontal change (or run) = 4 – (–4) = 8

Since the line slopes upwards from the left to the right, its gradient is positive.

m = gradient of line

$= \frac{4}{8}$

$= \frac{1}{2}$

12. The equation of a straight line is in the form of $y = mx + c$, where m is the gradient. So, to find the gradient of the lines, express the equation of the given lines to be in the form of the equation of a straight line.

(a) $y + x = 5$

$y = -x + 5$

From the equation, the value of the gradient m is –1.

(b) $3y + x = 6$

$3y = -x + 6$

$\frac{3y}{3} = \frac{-x + 6}{3}$

$y = \frac{-x}{3} + 2$

$= -\frac{1}{3}x + 2$

From the equation, the value of m is $-\frac{1}{3}$.

(c) $2y + 3x = 7$

$2y = -3x + 7$

$\frac{2y}{2} = \frac{-3x + 7}{2}$

$y = \frac{-3x}{2} + \frac{7}{2}$

From the equation, the value of m is $-\frac{3}{2}$.

(d) $2x - 5y = 9$

$2x = 9 + 5y$

$2x - 9 = 5y$

$5y = 2x - 9$

$\frac{5y}{5} = \frac{2x - 9}{5}$

$y = \frac{2x}{5} - \frac{9}{5}$

From the equation, the value of m is $\frac{2}{5}$.

(e) $4x - 6y + 1 = 0$

$4x + 1 = 6y$

$6y = 4x + 1$

$\frac{6y}{6} = \frac{4x + 1}{6}$

$y = \frac{4x}{6} - \frac{1}{6}$

$y = \frac{2x}{3} - \frac{1}{6}$

From the equation, the value of m is $\frac{2}{3}$.

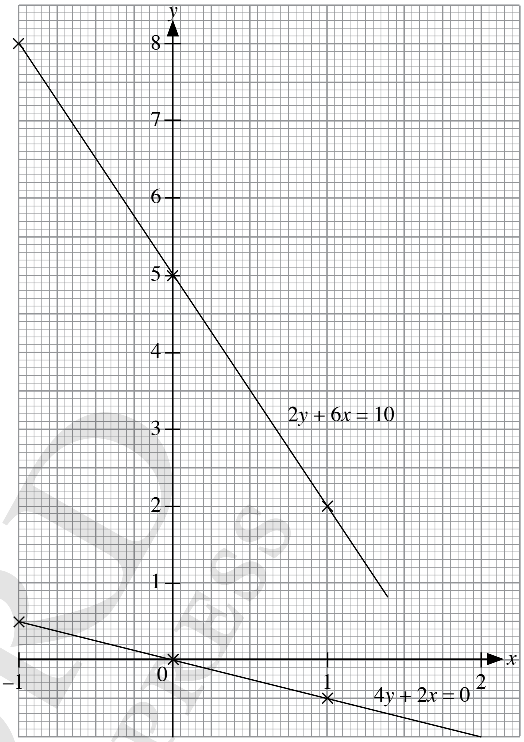
(f) $\frac{1}{2}x - \frac{2}{3}y - 5 = 0$

$\frac{2}{3}y = \frac{1}{2}x - 5$

$y = \frac{3}{4}x - 7\frac{1}{2}$

From the equation, the value of m is $\frac{3}{4}$.

13. (a)



For $4y + 2x = 0$,

Vertical change (or rise) $= \frac{1}{2} - 0$

$= \frac{1}{2}$

Horizontal change (or run) $= 0 - (-1)$

$= 1$

Since the line slopes downwards from the left to the right, its gradient is negative.

m = gradient of line

$= \frac{\frac{1}{2}}{1}$

$= -\frac{1}{2}$

For $2y + 6x = 10$,

Vertical change (or rise) $= 5 - 2$

$= 3$

Horizontal change (or run) $= 1 - 0$

$= 1$

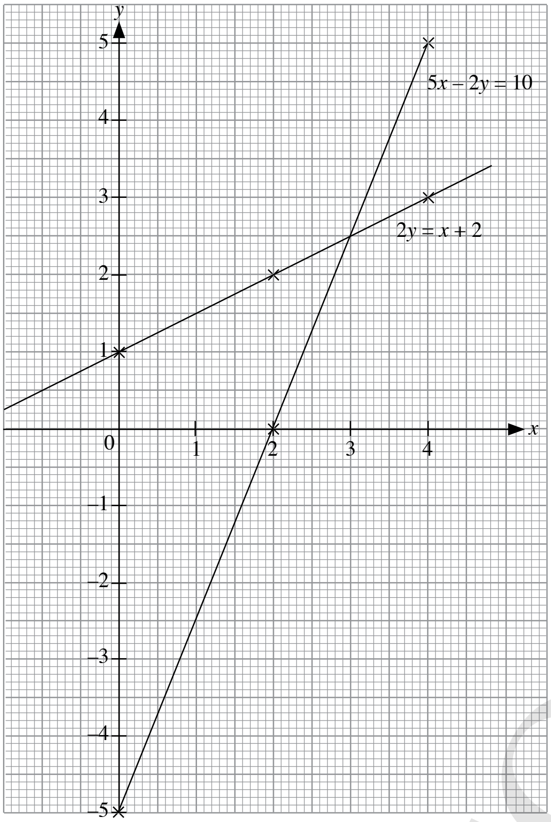
Since the line slopes downwards from the left to the right, its gradient is negative.

m = gradient of line

$= -\frac{3}{1}$

$= -3$

(b)



For $2y = x + 2$,

Vertical change (or rise) $= 2\frac{1}{2} - 1\frac{1}{2}$

$= -1$

Horizontal change (or run) $= 3 - 1$

$= -2$

Since the line slopes upwards from the left to the right, its gradient is positive.

m = gradient of line

$= \frac{1}{2}$

For $5x - 2y = 10$,

Vertical change (or rise) $= 2\frac{1}{2} - \left(-2\frac{1}{2}\right)$

$= 5$

Horizontal change (or run) $= 3 - 1$

$= 2$

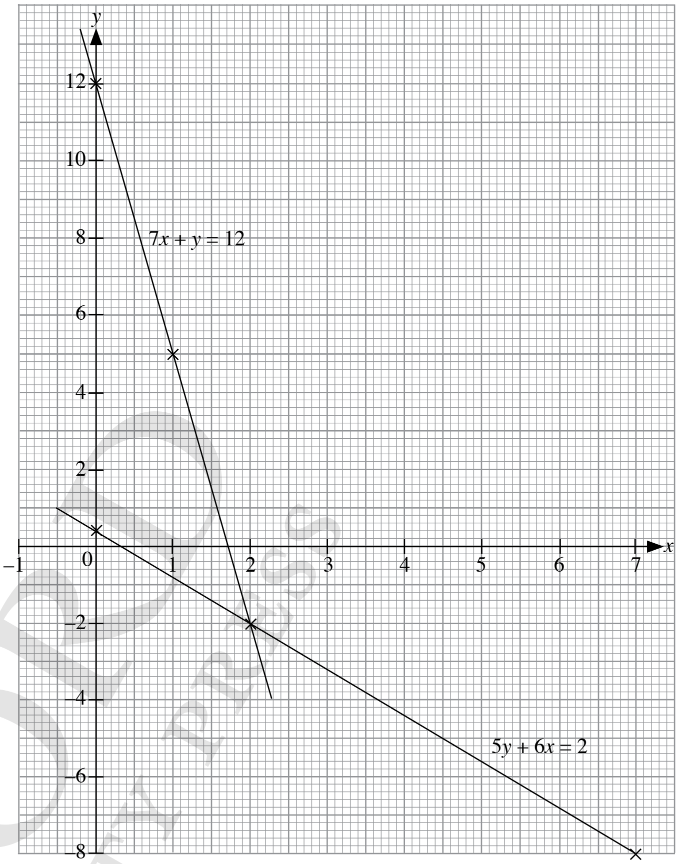
Since the line slopes upwards from the left to the right, its gradient is positive.

m = gradient of line

$= \frac{5}{2}$

$= 2\frac{1}{2}$

(c)



For $7x + y = 12$,

Vertical change (or rise) $= 12 - 5$

$= 7$

Horizontal change (or run) $= 1 - 0$

$= 1$

Since the line slopes downwards from the left to the right, its gradient is negative.

m = gradient of line

$= \frac{7}{1}$

$= -7$

For $5y + 6x = 2$,

Vertical change (or rise) $= -2 - (-8)$

$= 6$

Horizontal change (or run) $= 7 - 2$

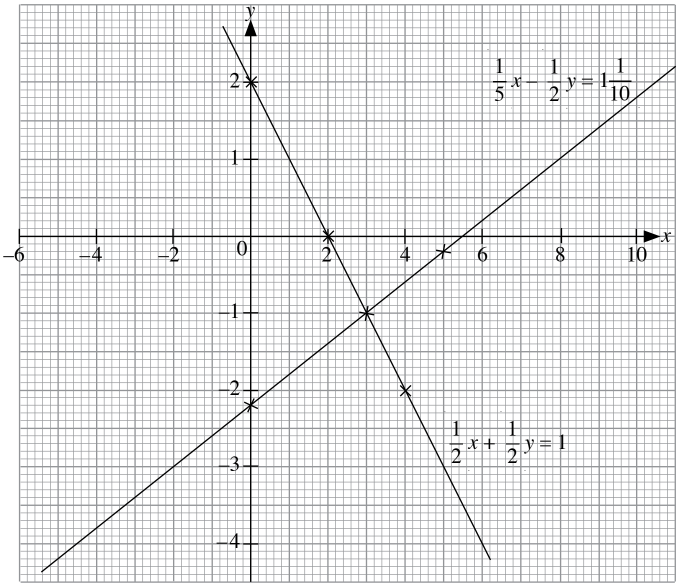
$= 5$

Since the line slopes downwards from the left to the right, its gradient is negative.

m = gradient of line

$= -\frac{6}{5}$

(d)



For $\frac{1}{2}x + \frac{1}{2}y = 1$,

Vertical change (or rise) $= 0 - (-4)$
$= 4$

Horizontal change (or run) $= 6 - 2$
$= 4$

Since the line slopes downwards from the left to the right, its gradient is negative.

m = gradient of line
$= -\frac{4}{4}$
$= -1$

For $\frac{1}{5}x - \frac{1}{2}y = 1\frac{1}{10}$,

Vertical change (or rise) $= 1 - (-1)$
$= 2$

Horizontal change (or run) $= 8 - 3$
$= 5$

Since the line slopes upwards from the left to the right, its gradient is positive.

m = gradient of line
$= \frac{2}{5}$

14. (i) From the graph, the value of x can be obtained by taking the value of the y-intercept, i.e. when the number of units used is zero.

$\therefore x = 14$

The value of y can be obtained by find the gradient of the line since the gradient, in this case, represents the cost for every unit of electricity used.

Vertical change (or rise) $= 54 - 14 = 40$

Horizontal change (or run) $= 400 - 0 = 400$

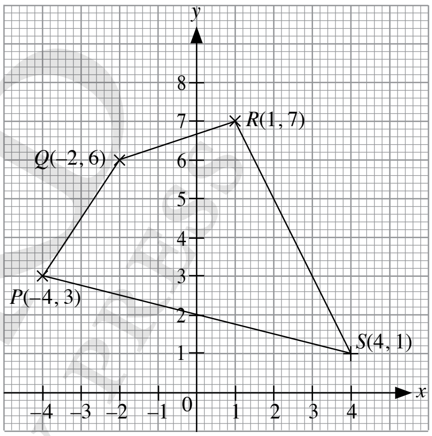
Since the line slopes upwards from the left to the right, its gradient is positive.

$y = m$ = gradient of line
$= \frac{40}{400}$
$= \frac{1}{10}$

(ii) From the graph, the cost of using 300 units of electricity is \$44.

(iii) From the graph, the number of units of electricity used if the cost is \$32 is 180.

15. (a)



(b) (i) Vertical change (or rise) $= 6 - 3 = 3$

Horizontal change (or run) $= -2 - (-4) = 2$

Since the line slopes upwards from the left to the right, its gradient is positive.

Gradient of line $= \frac{3}{2}$

(ii) Vertical change (or rise) $= 7 - 6 = 1$

Horizontal change (or run) $= 1 - (-2) = 3$

Since the line slopes upwards from the left to the right, its gradient is positive.

Gradient of line $= \frac{1}{3}$

(iii) Vertical change (or rise) $= 7 - 1 = 6$

Horizontal change (or run) $= 4 - 1 = 3$

Since the line slopes downwards from the left to the right, its gradient is negative.

Gradient of line $= -\frac{6}{3} = -2$

(iv) Vertical change (or rise) $= 3 - 1 = 2$

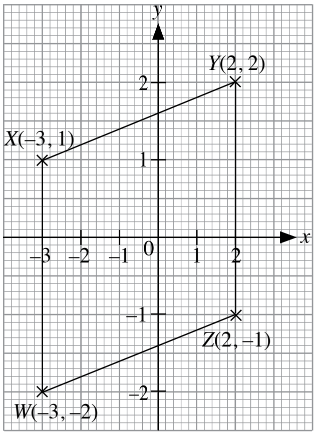
Horizontal change (or run) $= 4 - (-4) = 8$

Since the line slopes downwards from the left to the right, its gradient is negative.

Gradient of line $= -\frac{2}{8} = -\frac{1}{4}$

(c) From the graph, the coordinates of the point is (0, 2).

16. (a)



(b) (i) Vertical change (or rise) = 1 – (–2) = 3

Horizontal change (or run) = –3 – (–3) = 0

Gradient of line = $\frac{3}{0}$ = undefined

(ii) Vertical change (or rise) = 2 – 1 = 1

Horizontal change (or run) = 2 – (–3) = 5

Since the line slopes upwards from the left to the right, its gradient is positive.

Gradient of line = $\frac{1}{5}$

(iii) Vertical change (or rise) = 2 – (–1) = 3

Horizontal change (or run) = 2 – 2 = 0

Gradient of line = $\frac{3}{0}$ = undefined

(iv) Vertical change (or rise) = –1 – (–2)

Horizontal change (or run) = 2 – (–3) = 5

Since the line slopes upwards from the left to the right, its gradient is positive.

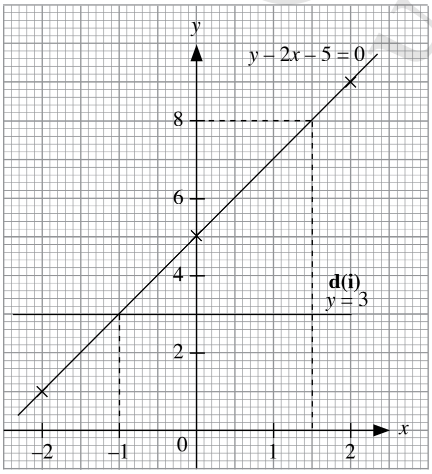
Gradient of line = $\frac{1}{5}$

(c) The quadrilateral *WXYZ* is a parallelogram.

17. (a)

x	–2	0	2
y	1	5	9

(b)



(c) From the graph, $k = 8$.

(d) (ii) $x = -1$

18. (a) $y - 2x = 4$

$y = 2x + 4$

When $x = -6$,

$y = -8$

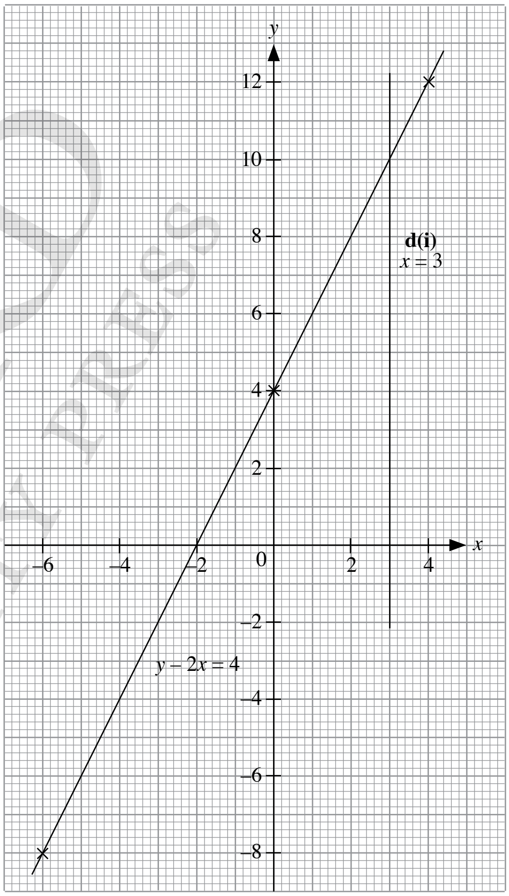
$\therefore a = -8$

When $x = 4$,

$y = 12$

$\therefore b = 12$

(b)



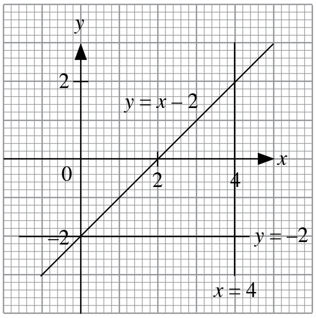
(c) $h = -4$

(d) (ii) Area of triangle

$= \frac{1}{2} \times 5 \times 10$

$= 25 \text{ units}^2$

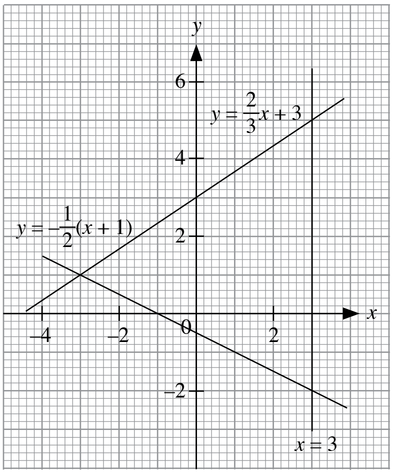
19. (i)



(ii) Area of triangle $= \frac{1}{2} \times 4 \times 4$

$= 8 \text{ units}^2$

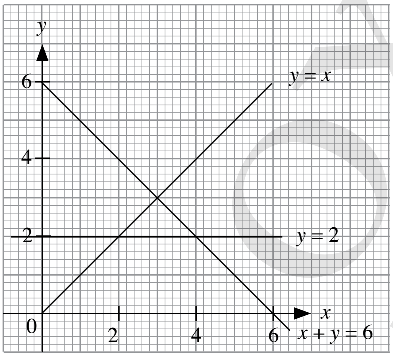
20. (i)



(ii) Area of triangle $= \frac{1}{2} \times 7 \times 6$

$= 21 \text{ units}^2$

21. (i)



(ii) Area of trapezium $= \frac{1}{2} \times (2 + 6) \times 2$

$= 8 \text{ units}^2$

22. (a) $y = -5x$ —(1)

$y = 5$ —(2)

Substitute (2) into (1):

$5 = -5x$

$x = -1$

$\therefore$ Coordinates of vertices of triangle are (0, 0), (–1, 5) and (5, 5).

Area of shaded region $= \frac{1}{2}[5 - (-1)](5)$

$= 15 \text{ units}^2$

(b) Coordinates of vertices of triangle are (0, 0), (4, 2) and (4, 4).

Area of shaded region $= \frac{1}{2}(4 - 2)(4)$

$= 4 \text{ units}^2$

(c) $y = -x - 3$

When $y = 0$,

$-x - 3 = 0$

$x = -3$

$2y = x - 6$

When $y = 0$,

$x - 6 = 0$

$x = 6$

$\therefore$ Coordinates of vertices of triangle are (–3, 0), (6, 0) and (0, –3).

Area of shaded region $= \frac{1}{2}[6 - (-3)](3)$

$= 13.5 \text{ units}^2$

(d) $2y = x - 2$

When $x = 0$,

$2y = -2$

$y = -1$

$y = -\frac{1}{8}x + 4$ —(1)

$2y = x - 2$ —(2)

Substitute (1) into (2):

$2\left(-\frac{1}{8}x + 4\right) = x - 2$

$-\frac{1}{4}x + 8 = x - 2$

$\frac{5}{4}x = 10$

$x = 8$

$y = 3$

$\therefore$ Coordinates of vertices of triangle are (0, –1), (0, 4) and (8, 3).

Area of shaded region $= \frac{1}{2}[4 - (-1)](8)$

$= 20 \text{ units}^2$

23. **(a)** $4x - 6y = 12$ —(1)

$2x + 4y = -4.5$ —(2)

(1) ÷ 2: $2x - 3y = 6$ —(3)

(2) – (3): $7y = -10.5$

$y = -1.5$

Substitute $y = -1.5$ into (3):

$2x - 3(-1.5) = 6$

$2x + 4.5 = 6$

$2x = 1.5$

$x = 0.75$

$\therefore x = 0.75, y = -1.5$

(b) $3x - 5y = 2$ —(1)

$x - 2y = \frac{4}{15}$ —(2)

(2) × 3: $3x - 6y = \frac{4}{5}$ —(3)

(1) – (3): $y = \frac{6}{5}$

$= 1\frac{1}{5}$

Substitute $y = 1\frac{1}{5}$ into (2):

$x - 2\left(1\frac{1}{5}\right) = \frac{4}{15}$

$x = \frac{8}{3}$

$= 2\frac{2}{3}$

$\therefore x = 2\frac{2}{3}, y = 1\frac{1}{5}$

(c) $5x - 8y = 23\frac{1}{2}$ —(1)

$4x + y = 22\frac{1}{2}$ —(2)

(2) × 8: $32x + 8y = 180$ —(3)

(1) + (3): $37x = 203\frac{1}{2}$

$x = 5\frac{1}{2}$

Substitute $x = 5\frac{1}{2}$ into (2):

$4\left(5\frac{1}{2}\right) + y = 22\frac{1}{2}$

$22 + y = 22\frac{1}{2}$

$y = \frac{1}{2}$

$\therefore x = 5\frac{1}{2}, y = \frac{1}{2}$

(d) $5x - 3y = 1.4$ —(1)

$2x + 5y = 14.2$ —(2)

(1) × 2: $10x - 6y = 2.8$ —(3)

(2) × 5: $10x + 25y = 71$ —(4)

(4) – (3): $31y = 68.2$

$y = 2.2$

Substitute $y = 2.2$ into (2):

$2x + 5(2.2) = 14.2$

$2x + 11 = 14.2$

$2x = 3.2$

$x = 1.6$

$\therefore x = 1.6, y = 2.2$

24. **(a)** $15x - 7y = 14\frac{1}{4}$ —(1)

$5x - y = 3\frac{3}{4}$ —(2)

From (2),

$y = 5x - 3\frac{3}{4}$ —(3)

Substitute (3) into (1):

$15x - 7\left(5x - 3\frac{3}{4}\right) = 14\frac{1}{4}$

$15x - 35x + \frac{105}{4} = \frac{57}{4}$

$20x = 12$

$x = \frac{3}{5}$

Substitute $x = \frac{3}{5}$ into (3):

$y = 5\left(\frac{3}{5}\right) - 3\frac{3}{4}$

$= 3 - 3\frac{3}{4}$

$= -\frac{3}{4}$

$\therefore x = \frac{3}{5}, y = -\frac{3}{4}$

(b) $3x + 1.4y = 0.1$ —(1)

$x - 3.6y = 10.2$ —(2)

From (2),

$x = 3.6y + 10.2$ —(3)

Substitute (3) into (1):

$3(3.6y + 10.2) + 1.4y = 0.1$

$10.8y + 30.6 + 1.4y = 0.1$

$12.2y = -30.5$

$y = -2.5$

Substitute $y = -2.5$ into (3):

$x = 3.6(-2.5) + 10.2$

$= 1.2$

$\therefore x = 1.2, y = -2.5$

(c) $\frac{1}{2}x - \frac{1}{3}y - 1 = 0$ —(1)

$x + 6y + 8 = 0$ —(2)

From (2),

$x = -6y - 8$ —(3)

Substitute (3) into (1):

$\frac{1}{2}(-6y - 8) - \frac{1}{3}y - 1 = 0$

$-3y - 4 - \frac{1}{3}y - 1 = 0$

$-\frac{10}{3}y = 5$

$y = -\frac{3}{2}$

$= -1\frac{1}{2}$

Substitute $y = -1\frac{1}{2}$ into (3):

$x = -6\left(-1\frac{1}{2}\right) - 8$

$= 9 - 8$

$= 1$

$\therefore x = 1, y = -1\frac{1}{2}$

(d) $3x - 2y = 8$ —(1)

$\frac{1}{8}x + \frac{1}{2}y = 1.25$ —(2)

From (2),

$\frac{1}{2}y = 1.25 - \frac{1}{8}x$

$y = 2.5 - \frac{1}{4}x$ —(3)

Substitute (3) into (1):

$3x - 2\left(2.5 - \frac{1}{4}x\right) = 8$

$3x - 5 + \frac{1}{2}x = 8$

$\frac{7}{2}x = 13$

$x = \frac{26}{7}$

$= 3\frac{5}{7}$

Substitute $x = 3\frac{5}{7}$ into (3):

$y = 2\frac{1}{2} - \frac{1}{4}\left(3\frac{5}{7}\right)$

$= 1\frac{4}{7}$

$\therefore x = 3\frac{5}{7}, y = 1\frac{4}{7}$

25. (a) $3x + 2y + 7 = 0$ —(1)

$5x - 2y + 1 = 0$ —(2)

(1) + (2): $8x + 8 = 0$

$8x = -8$

$x = -1$

Substitute $x = -1$ into (1):

$3(-1) + 2y + 7 = 0$

$-3 + 2y + 7 = 0$

$2y = -4$

$y = -2$

$\therefore x = -1, y = -2$

(b) $2y - 7x + 69 = 0$ —(1)

$4x - 3y - 45 = 0$ —(2)

(1) × 3: $6y - 21x + 207 = 0$ —(3)

(2) × 2: $8x - 6y - 90 = 0$ —(4)

(3) + (4): $-13x + 117 = 0$

$13x = 117$

$x = 9$

Substitute $x = 9$ into (1):

$2y - 7(9) + 69 = 0$

$2y - 63 + 69 = 0$

$2y = -6$

$y = -3$

$\therefore x = 9, y = -3$

(c) $0.5x - 0.2y = 2$ —(1)

$2.5x + 0.6y = 2$ —(2)

(1) × 3: $1.5x - 0.6y = 6$ —(3)

(2) + (3): $4x = 8$

$x = 2$

Substitute $x = 2$ into (1):

$0.5(2) - 0.2y = 2$

$1 - 0.2y = 2$

$0.2y = -1$

$y = -5$

$\therefore x = 2, y = -5$

(d) $x + \frac{1}{2}y = 9$ —(1)

$3x - 2y = 13$ —(2)

(1) × 4: $4x + 2y = 36$ —(3)

(2) + (3): $7x = 49$

$x = 7$

Substitute $x = 7$ into (1):

$7 + \frac{1}{2}y = 9$

$\frac{1}{2}y = 2$

$y = 4$

$\therefore x = 7, y = 4$

(e) $\frac{1}{3}(x + 1) + y - 8 = 0$ —(1)

$x + 4 = \frac{y + 1}{3}$ —(2)

From (1),

$x + 1 + 3y - 24 = 0$

$x = 23 - 3y$ —(3)

Substitute (3) into (2):

$23 - 3y + 4 = \frac{y + 1}{3}$

$27 - 3y = \frac{y + 1}{3}$

$81 - 9y = y + 1$

$10y = 80$

$y = 8$

Substitute $y = 8$ into (3):

$x = 23 - 3(8)$

$= 23 - 24$

$= -1$

$\therefore x = -1, y = 8$

(f) $\frac{1}{5}x + \frac{3}{4}y = -1\frac{1}{2}$ —(1)

$\frac{5}{6}x - \frac{1}{8}y = 13\frac{1}{4}$ —(2)

(1) × 20: $4x + 15y = -30$ —(3)

(2) × 24: $20x - 3y = 318$ —(4)

From (4),

$3y = 20x - 318$ —(5)

Substitute (5) into (3):

$4x + 5(20x - 318) = -30$

$4x + 100x - 1590 = -30$

$104x = 1560$

$x = 15$

Substitute $x = 15$ into (5):

$3y = 20(15) - 318$

$= -18$

$y = -6$

$\therefore x = 15, y = -6$

(g) $\frac{1}{3}x - \frac{2}{3}y + 5 = 0$ —(1)

$\frac{1}{2}x + \frac{1}{3}y - \frac{1}{2} = 0$ —(2)

(1) × 3: $x - 2y + 15 = 0$ —(3)

(2) × 6: $3x + 2y - 3 = 0$ —(4)

(3) + (4): $4x + 12 = 0$

$4x = -12$

$x = -3$

Substitute $x = -3$ into (3):

$-3 - 2y + 15 = 0$

$2y = 12$

$y = 6$

$\therefore x = -3, y = 6$

(h) $\frac{x + y}{13 - 7y} = \frac{1}{3}$ —(1)

$\frac{4x - 4y - 3}{6y - 3x + 2} = \frac{4}{3}$ —(2)

From (1),

$3x + 3y = 13 - 7y$

$3x + 10y = 13$ —(3)

From (2),

$12x - 12y - 9 = 24y - 12x + 8$

$24x - 36y = 17$ —(4)

From (3),

$3x = 13 - 10y$ —(5)

Substitute (5) into (4):

$8(13 - 10y) - 36y = 17$

$104 - 80y - 36y = 17$

$116y = 87$

$y = \frac{3}{4}$

Substitute $y = \frac{3}{4}$ into (5):

$3x = 13 - 10\left(\frac{3}{4}\right)$

$= \frac{11}{2}$

$x = \frac{11}{6}$

$= 1\frac{5}{6}$

$\therefore x = 1\frac{5}{6}, y = \frac{3}{4}$

26. (a) $4x + 4 = 5x = 60y - 100$

$4x + 4 = 5x$ —(1)

$5x = 60y - 100$ —(2)

From (1),

$x = 4$

Substitute $x = 4$ into (2):

$5(4) = 60y - 100$

$20 = 60y - 100$

$60y = 120$

$y = 2$

$\therefore x = 4, y = 2$

(b) $2x - 2 + 12y = 9 = 4x - 2y$

$2x - 2 + 12y = 9$ —(1)

$4x - 2y = 9$ —(2)

From (1),

$2x + 12y = 11$

$x = \dfrac{11 - 12y}{2}$ —(3)

Substitute (3) into (2):

$4\left(\dfrac{11 - 12y}{2}\right) - 2y = 9$

$22 - 24y - 2y = 9$

$26y = 13$

$y = \dfrac{1}{2}$

Substitute $y = \dfrac{1}{2}$ into (3):

$x = \dfrac{11 - 12\left(\frac{1}{2}\right)}{2}$

$= \dfrac{5}{2}$

$= 2\dfrac{1}{2}$

$\therefore x = 2\dfrac{1}{2}, y = \dfrac{1}{2}$

(c) $5x + 3y = 2x + 7y = 29$

$5x + 3y = 29$ —(1)

$2x + 7y = 29$ —(2)

(1) × 2: $10x + 6y = 58$ —(3)

(2) × 5: $10x + 35y = 145$ —(4)

(4) – (3): $29y = 87$

$y = 3$

Substitute $y = 3$ into (2):

$2x + 7(3) = 29$

$2x + 21 = 29$

$2x = 8$

$x = 4$

$\therefore x = 4, y = 3$

(d) $10x - 15y = 12x - 8y = 150$

$10x - 15y = 150$ —(1)

$12x - 8y = 150$ —(2)

(1) ÷ 5: $2x - 3y = 30$ —(3)

(2) ÷ 2: $6x - 4y = 75$ —(4)

From (3),

$2x = 3y + 30$ —(5)

Substitute (5) into (4):

$3(3y + 30) - 4y = 75$

$9y + 90 - 4y = 75$

$5y = -15$

$y = -3$

Substitute $y = -3$ into (5):

$2x = 3(-3) + 30$

$= -9 + 30$

$= 21$

$x = \dfrac{21}{2}$

$= 10\dfrac{1}{2}$

$\therefore x = 10\dfrac{1}{2}, y = -3$

(e) $x + y + 3 = 3y - 2 = 2x + y$

$x + y + 3 = 3y - 2$ —(1)

$x + y + 3 = 2x + y$ —(2)

From (2),

$x = 3$

Substitute $x = 3$ into (1):

$3 + y + 3 = 3y - 2$

$2y = 8$

$y = 4$

$\therefore x = 3, y = 4$

(f) $5x - 8y = 3y - x + 8 = 2x - y + 1$

$5x - 8y = 3y - x + 8$ —(1)

$5x - 8y = 2x - y + 1$ —(2)

From (1),

$6x - 11y = 8$ —(3)

From (2),

$3x - 7y = 1$

$3x = 7y + 1$ —(4)

Substitute (4) into (3):

$2(7y + 1) - 11y = 8$

$14y + 2 - 11y = 8$

$3y = 6$

$y = 2$

Substitute $y = 2$ into (4):

$3x = 7(2) + 1$

$= 15$

$x = 5$

$\therefore x = 5, y = 2$

(g) $4x + 2y = x - 3y + 1 = 2x + y + 3$

$4x + 2y = x - 3y + 1$ —(1)

$4x + 2y = 2x + y + 3$ —(2)

From (1),

$3x + 5y = 1$ —(3)

From (2),

$2x + y = 3$

$y = 3 - 2x$ —(4)

Substitute (4) into (3):

$3x + 5(3 - 2x) = 1$

$3x + 15 - 10x = 1$

$7x = 14$

$x = 2$

Substitute $x = 2$ into (4):

$y = 3 - 2(2)$

$= 3 - 4$

$= -1$

$\therefore x = 2, y = -1$

(h) $3x - 4y - 7 = y + 10x - 10 = 4x - 7y$

$3x - 4y - 7 = y + 10x - 10$ —(1)

$3x - 4y - 7 = 4x - 7y$ —(2)

From (1),

$7x + 5y = 3$ —(3)

From (2),

$x - 3y = -7$

$x = 3y - 7$ —(4)

Substitute (4) into (3):

$7(3y - 7) + 5y = 3$

$21y - 49 + 5y = 3$

$26y = 52$

$y = 2$

Substitute $y = 2$ into (4):

$x = 3(2) - 7$

$= 6 - 7$

$= -1$

$\therefore x = -1, y = 2$

27. $6x - 3y = 4$ —(1)

$y = 2x + 5$ —(2)

Substitute (2) into (1):

$6x - 3(2x + 5) = 4$

$6x - 6x - 15 = 4$

$-15 = 4$ (N.A.)

From (1),

$3y = 6x - 4$

$y = 2x - \frac{4}{3}$

Since the gradients of the lines are equal, the lines are parallel and have no solution.

28. $6y + 3x = 15$ —(1)

$y = -\frac{1}{2}x + \frac{5}{2}$ —(2)

From (1),

$6y = -3x + 15$

$y = -\frac{1}{2}x + \frac{5}{2}$

Since the lines are identical, they overlap each other and have an infinite number of solutions.

29. (a) $x + y + 2 = 3y + 1 = 2x$

$x + y + 2 = 3y + 1$ —(1)

$3y + 1 = 2x$ —(2)

From (1),

$x = 2y - 1$ —(3)

Substitute (3) into (2):

$3y + 1 = 2(y - 1)$

$= 4y - 2$

$y = 3$

$\therefore$ Perimeter $= 3[3(3) + 1]$

$= 30$ cm

(b) $x + 5y + 9 = 2x + 3y - 3 = x + y + 1$

$x + 5y + 9 = 2x + 3y - 3$ —(1)

$x + 5y + 9 = x + y + 1$ —(2)

From (2),

$4y = -8$

$y = -2$

Substitute $y = -2$ into (1):

$x + 5(-2) + 9 = 2x + 3(-2) - 3$

$x - 1 = 2x - 9$

$x = 8$

$\therefore$ Perimeter $= 3[8 + (-2) + 1]$

$= 21$ cm

30. (a) $2x + y + 1 = 12$ —(1)

$4x + y + 2 = 3x + 3y$ —(2)

From (1),

$y = 11 - 2x$ —(3)

Substitute (3) into (2):

$4x + 11 - 2x + 2 = 3x + 3(11 - 2x)$

$2x + 13 = 3x + 33 - 6x$

$= 33 - 3x$

$5x = 20$

$x = 4$

Substitute $x = 4$ into (3):

$y = 11 - 2(4)$

$= 11 - 8$

$= 3$

$\therefore$ Perimeter $= 2[3(4) + 3(3) + 12]$

$= 66$ cm

Area $= 12[3(4) + 3(3)]$

$= 252\ \text{cm}^2$

(b) $3x + y + 6 = 4x - y$ —(1)

$5x - 2y + 1 = 6x + y$ —(2)

From (1),

$x = 2y + 6$ —(3)

Substitute (3) into (2):

$5(2y + 6) - 2y + 1 = 6(2y + 6) + y$

$10y + 30 - 2y + 1 = 12y + 36 + y$

$8y + 31 = 13y + 36$

$5y = -5$

$y = -1$

Substitute $y = -1$ into (3):

$x = 2(-1) + 6$

$= -2 + 6$

$= 4$

$\therefore$ Perimeter $= 2[6(4) + (-1) + 4(4) - (-1)]$

$= 80$ cm

Area $= [6(4) + (-1)][4(4) - (-1)]$

$= 391\text{ cm}^2$

31. $y - 1 = x + 5$ —(1)

$2x + y + 1 = 3x - y + 18$ —(2)

From (1),

$y = x + 6$ —(3)

Substitute (3) into (2):

$2x + x + 6 + 1 = 3x - (x + 6) + 18$

$3x + 7 = 3x - x - 6 + 18$

$= 2x + 12$

$x = 5$

Substitute $x = 5$ into (3):

$y = 5 + 6$

$= 11$

$\therefore$ Perimeter $= 2[2(5) + 11 + 1 + 11 - 1]$

$= 64$ cm

32. $y + 2 = x - 1$ —(1)

$2x + y = 3x - y + 12$ —(2)

From (1),

$y = x - 3$ —(3)

Substitute (3) into (2):

$2x + x - 3 = 3x - (x - 3) + 12$

$3x - 3 = 3x - x + 3 + 12$

$= 2x + 15$

$x = 18$

Substitute $x = 18$ into (3):

$y = 18 - 3$

$= 15$

$y + 2 = 15 + 2$

$= 17$

$2x + y = 2(18) + 15$

$= 51$

Since the lengths of the sides are not equal, the quadrilateral is not a rhombus.

33. $0.3x + 0.4y = 7$ —(1)

$1.1x - 0.3y = 8$ —(2)

(1) × 30: $9x + 12y = 210$ —(3)

(2) × 40: $44x - 12y = 320$ —(4)

(3) + (4): $53x = 530$

$x = 10$

Substitute $x = 10$ into (1):

$0.3(10) + 0.4y = 7$

$3 + 0.4y = 7$

$0.4y = 4$

$y = 10$

$\therefore p = 10, q = 10$

34. $3x - y = 7$ —(1)

$2x + 5y = -1$ —(2)

From (1),

$y = 3x - 7$ —(3)

Substitute (3) into (2):

$2x + 5(3x - 7) = -1$

$2x + 15x - 35 = -1$

$17x = 34$

$x = 2$

Substitute $x = 2$ into (3):

$y = 3(2) - 7$

$= 6 - 7$

$= -1$

$\therefore$ Coordinates of point of intersection are $(2, -1)$.

35. $x^2 + ax + b = 0$ —(1)

Substitute $x = 3$ into (1):

$3^2 + a(3) + b = 0$

$3a + b = -9$ —(2)

Substitute $x = -4$ into (1):

$(-4)^2 + a(-4) + b = 0$

$4a - b = 16$ —(3)

(2) + (3): $7a = 7$

$a = 1$

Substitute $a = 1$ into (2):

$3(1) + b = -9$

$b = -9 - 3$

$= -12$

$\therefore a = 1, b = -12$

36. $ax - by = 1$ —(1)

$ay + bx = -7$ —(2)

Substitute $x = -1, y = 2$ into (1):

$a(-1) - b(2) = 1$

$-a - 2b = 1$ —(3)

Substitute $x = -1, y = 2$ into (2):

$a(2) + b(-1) = -7$

$2a - b = -7$

$b = 2a + 7$ —(4)

Substitute (4) into (3):

$-a - 2(2a + 7) = 1$

$-a - 4a - 14 = 1$

$5a = -15$

$a = -3$

Substitute $a = -3$ into (4):

$b = 2(-3) + 7$

$= 1$

$\therefore a = -3, b = 1$

37. Using the same method,

$4x - 3y = 48x + 8y$

$44x = -11y$

$4x = -y$

$\therefore$ This method cannot be used as we have one equation with two unknowns at the end.

38. Let Khairul's age be x years and his aunt's age be y years.

$y = 4x$ —(1)

$y + 8 = \frac{5}{2}(x + 8)$ —(2)

Substitute (1) into (2):

$4x + 8 = \frac{5}{2}(x + 8)$

$8x + 16 = 5x + 40$

$3x = 24$

$x = 8$

Substitute $x = 8$ into (1):

$y = 4(8)$

$= 32$

$\therefore$ His aunt's present age is 32 years.

39. (i) Let Jun Wei's age be x years and his mother's age be y years.

$x + y = 61$ —(1)

$y - x = 29$ —(2)

(1) – (2): $2x = 32$

$x = 16$

$\therefore$ Jun Wei's present age is 16 years.

(ii) Substitute $x = 16$ into (2):

$y - 16 = 29$

$y = 45$

$y + 5 = 45 + 5$

$= 50$

$\therefore$ Jun Wei's mother will be 50 years old.

40. Let the numbers be x and y.

$y + 7 = 4x$ —(1)

$x + 28 = 2y$ —(2)

From (1),

$y = 4x - 7$ —(3)

Substitute (3) into (2):

$x + 28 = 2(4x - 7)$

$= 8x - 14$

$7x = 42$

$x = 6$

Substitute $x = 6$ into (3):

$y = 4(6) - 7$

$= 17$

$\therefore$ The numbers are 17 and 6.

41. Let the original fraction be $\frac{x}{y}$.

$\frac{x - 1}{y - 1} = \frac{3}{4}$ —(1)

$\frac{x + 1}{y + 1} = \frac{4}{5}$ —(2)

From (1),

$4x - 4 = 3y - 3$

$4x - 3y = 1$ —(3)

From (2),

$5x + 5 = 4y + 4$

$4y = 5x + 1$

$y = \frac{1}{4}(5x + 1)$ —(4)

Substitute (4) into (3):

$4x - \frac{3}{4}(5x + 1) = 1$

$16x - 15x - 3 = 4$

$x = 7$

Substitute $x = 7$ into (4):

$y = \frac{1}{4}(35 + 1)$

$= 9$

$\therefore$ The fraction is $\frac{7}{9}$.

42. Let the fractions be represented by x and y.

$x + y = 3(y - x)$ —(1)

$6x - y = \frac{3}{2}$ —(2)

From (2),

$y = 6x - \frac{3}{2}$ —(3)

Substitute (3) into (1):

$x + 6x - \frac{3}{2} = 3\left(6x - \frac{3}{2} - x\right)$

$7x - \frac{3}{2} = 15x - \frac{9}{2}$

$8x = 3$

$x = \frac{3}{8}$

Substitute $x = \frac{3}{8}$ into (3):

$y = 6\left(\frac{3}{8}\right) - \frac{3}{2}$

$= \frac{3}{4}$

$\therefore$ The fractions are $\frac{3}{4}$ and $\frac{3}{8}$.

43. Let the price of a chicken be \$$x$ and that of a duck be \$$y$.

$5x + 5y = 100$ —(1)

$10x + 17y = 287.5$ —(2)

From (1),

$x + y = 20$

$y = 20 - x$ —(3)

Substitute (3) into (2):

$10x + 17(20 - x) = 287.5$

$10x + 340 - 17x = 287.5$

$7x = 52.5$

$x = 7.5$

Substitute $x = 7.5$ into (3):

$y = 20 - 7.5$

$= 12.5$

$3x + 2y = 3(7.5) + 2(12.5)$

$= 47.5$

$\therefore$ He will receive \$47.50.

44. Let the number of chickens and goats be x and y respectively.

$x + y = 45$ —(1)

$2x + 4y = 150$ —(2)

From (2),

$x + 2y = 75$ —(3)

(2) – (1): $y = 30$

Substitute $y = 30$ into (1):

$x + 30 = 45$

$x = 15$

$y - x = 30 - 15$

$= 15$

$\therefore$ There are 15 more goats than chickens.

45. Let the cost of 1 can of condensed milk and 1 jar of instant coffee be \$$x$ and \$$y$ respectively.

$5x + 3y = 27$ —(1)

$12x + 5y = 49.4$ —(2)

From (1),

$3y = 27 - 5x$

$y = 9 - \frac{5}{3}x$ —(3)

Substitute (3) into (2):

$12x + 5\left(9 - \frac{5}{3}x\right) = 49.4$

$12x + 45 - \frac{25}{3}x = 49.4$

$\frac{11}{3}x = 4.4$

$x = 1.2$

Substitute $x = 1.2$ into (3):

$y = 9 - \frac{5}{3}(1.2)$

$= 7$

$7x + 2y = 7(1.2) + 2(7)$

$= 22.4$

$\therefore$ The total cost is \$22.40.

46. Let the cost of 1 kiwi fruit and 1 pear be \$$x$ and \$$y$ respectively.

$8x + 7y = 4.1$ —(1)

$4x + 9y = 3.7$ —(2)

(2) × 2: $8x + 18y = 7.4$ —(3)

(3) – (1): $11y = 3.3$

$y = 0.3$

Substitute $y = 0.3$ into (1):

$8x + 7(0.3) = 4.1$

$8x = 2.0$

$x = 0.25$

$2x + 2y = 2(0.25) + 2(0.3)$

$= 1.1$

$\therefore$ The cost is \$1.10.

47. Let the number of research staff and laboratory assistants be x and y respectively.

$x + y = 540$ —(1)

$240x + 200y = 120\,000$ —(2)

From (2),

$6x + 5y = 3000$ —(3)

(1) × 5: $5x + 5y = 2700$ —(4)

(3) – (4): $x = 300$

Substitute $x = 300$ into (1):

$300 + y = 540$

$y = 240$

∴ The facility employs 300 research staff and 240 laboratory assistants.

48. Let the time taken to travel at 90 km/h and 80 km/h be x hours and y hours respectively.

$x + y = 8$ —(1)

$90x + 80y = 690$ —(2)

From (2),

$9x + 8y = 69$ —(3)

(1) × 9: $9x + 9y = 72$ —(4)

(4) – (3): $y = 3$

$80y = 80(3)$

$= 240$

∴ The distance he covered was 240 km.

Advanced

49. For the line AC,

Vertical change (or rise) = $2 - n$

Horizontal change (or run) = $3 - (-2) = 5$

Since the line slopes upwards from the left to the right, its gradient is positive.

Gradient of line = $\dfrac{2-n}{5} = \dfrac{4}{5}$

$2 - n = 4$

$n = 2 - 4$

$= -2$

For the line AB,

Vertical change (or rise) = $m - 2$

Horizontal change (or run) = $3 - (-2) = 5$

Since the line slopes downwards from the left to the right, its gradient is negative.

Gradient of line = $-\dfrac{m-2}{5} = -\dfrac{1}{5}$

$m - 2 = 1$

$m = 1 + 2$

$= 3$

50. **(a)** From the graph, Jun Wei left home at 1100.

(b) He took 45 minutes to have lunch.

(c) The distance between Jun Wei's house and his friend's house is 60 km.

(d) **(i)** Vertical change (or rise) = $40 - 0 = 40$

Horizontal change (or run)

= 1145 – 1100

= 45 min

$= 1\frac{3}{4}$ h

Since the line slopes upwards from the left to the right, its gradient is positive.

Gradient of line = $\dfrac{40}{\frac{3}{4}}$

$= 53\frac{1}{3}$ (to 3 s.f.)

The gradient represents the speed at which Jun Wei travels to his lunch venue.

(ii) Vertical change (or rise) = $40 - 40 = 0$

Horizontal change (or run)

= 1230 – 1145

= 45 min

$= \frac{3}{4}$ h

Gradient of line = $\dfrac{0}{\frac{3}{4}} = 0$

The gradient represents the speed. In this case, Jun Wei is taking his lunch and so his speed is zero.

(iii) Vertical change (or rise) = $60 - 40 = 20$

Horizontal change (or run)

= 1300 – 1230

= 30 min

$= \frac{1}{2}$ h

Since the line slopes upwards from the left to the right, its gradient is positive.

Gradient of line = $\dfrac{20}{\frac{1}{2}} = 40$

The gradient represents the speed at which Jun Wei travels from the lunch venue to Jurong West.

51. **(a)** $\frac{2}{3}x - \frac{3}{5}y - 4 = \frac{1}{20}x - y + \frac{17}{30} = 2x - y - 18\frac{14}{15}$

$\frac{2}{3}x - \frac{3}{5}y - 4 = \frac{1}{20}x - y + \frac{17}{30}$ —(1)

$\frac{1}{20}x - y + \frac{17}{30} = 2x - y - 18\frac{14}{15}$ —(2)

From (1),

$40x - 36y - 240 = 3x - 60y + 34$

$37x + 24y = 274$ —(3)

From (2),

$3x - 60y + 34 = 120x - 60y - 1136$

$117x = 1170$

$x = 10$

Substitute $x = 10$ into (3):

$37(10) + 24y = 274$

$24y = -96$

$y = -4$

$\therefore x = 10, y = -4$

(b) $\frac{2}{7}x + \frac{3}{4}y - 4 = \frac{3}{5}x - \frac{2}{7}y - 44 = \frac{7}{15}x + y - 3\frac{1}{3}$

$\frac{2}{7}x + \frac{3}{4}y - 4 = \frac{3}{5}x - \frac{2}{7}y - 44$ —(1)

$\frac{3}{5}x - \frac{2}{7}y - 44 = \frac{7}{15}x + y - 3\frac{1}{3}$ —(2)

From (1),

$40x + 105y - 560 = 84x - 40y - 6160$

$44x - 145y = 5600$ —(3)

From (2),

$63x - 30y - 4620 = 49x + 105y - 350$

$14x = 135y + 4270$

$x = \frac{135}{14}y + 305$ —(4)

Substitute (4) into (3):

$44\left(\frac{135}{14}y + 305\right) - 145y = 5600$

$\frac{2970}{7}y + 13\,420 - 145y = 5600$

$\frac{1955}{7}y = -7820$

$y = -28$

Substitute $y = -28$ into (4):

$x = \frac{135}{14}(-28) + 305$

$= 35$

$\therefore x = 35, y = -28$

52. Let the number be represented by $10x + y$.

$10x + y = 4(x + y)$ —(1)

$(10y + x) - (10x + y) = 27$ —(2)

From (1),

$10x + y = 4x + 4y$

$6x = 3y$

$y = 2x$ —(3)

From (2),

$9y - 9x = 27$

$y - x = 3$ —(4)

Substitute (3) into (4):

$2x - x = 3$

$x = 3$

Substitute $x = 3$ into (3):

$y = 2(3)$

$= 6$

$\therefore$ The original number is 36.

53. Let the digit in the tens place be x and the digit in the ones place be y.

$x = \frac{1}{2}y$ —(1)

$(10y + x) - (10x + y) = 36$ —(2)

From (2),

$9y - 9x = 36$

$y - x = 4$ —(3)

Substitute (1) into (3):

$y - \frac{1}{2}y = 4$

$\frac{1}{2}y = 4$

$y = 8$

Substitute $y = 8$ into (1):

$x = \frac{1}{2}(8)$

$= 4$

$\therefore$ The original number is 48.

54. Let the larger number be x and the smaller number be y.

$x + y = 55$ —(1)

$x = 2y + 7$ —(2)

Substitute (2) into (1):

$2y + 7 + y = 55$

$3y = 48$

$y = 16$

Substitute $y = 16$ into (2):

$x = 2(16) + 7$

$= 39$

Difference in the reciprocals $= \frac{1}{16} - \frac{1}{39}$

$= \frac{23}{624}$

55. Let the walking speed of Ethan and Michael be x m/s and y m/s respectively.

$8x + 8y = 64$ —(1)

$32x - 64 = 32y$ —(2)

From (1),

$x + y = 8$ —(3)

From (2),

$32x - 32y = 64$

$x - y = 2$ —(4)

(3) + (4): $2x = 10$

$x = 5$

Substitute $x = 5$ into (4):

$5 - y = 2$

$y = 3$

$\therefore$ Ethan's walking speed is 5 m/s and Michael's walking speed is 3 m/s.

The assumption is that when they are walking in the same direction, Ethan starts off 64 m behind Michael.

New Trend

56. $3x = y + 1$ —(1)

$y - x = 3$ —(2)

From (1),

$y = 3x - 1$ —(3)

Substitute (3) into (2):

$3x - 1 - x = 3$

$2x = 4$

$x = 2$

Substitute $x = 2$ into (3):

$y = 3(2) - 1$

$= 5$

$\therefore x = 2, y = 5$

57. (a) Let the speed of the faster ship and slower ship be x km/h and y km/h respectively.

$x = y + 8$ —(1)

$60x + 60y = 4320$ —(2)

From (2),

$x + y = 72$ —(3)

Substitute (1) into (3):

$y + 8 + y = 72$

$2y = 64$

$y = 32$

Substitute $y = 32$ into (1):

$x = 32 + 8$

$= 40$

$\therefore$ The speeds of the faster ship and slower ship are 40 km/h and 32 km/h respectively.

(b) $\frac{1780}{32} - \frac{1780}{40}$

$= 55.625 - 44.5$

$= 11.125$ h

$= 11$ h 8 min (nearest min)

58. $4x + 4(6) = 40$

$4x = 40 - 24$

$x = 16 \div 4$

$= 4$

Since the rectangles are of equal area,

$6z = 39x$

$z = 39(4) \div 6$

$= 26$

$y = 39 - z$

$= 39 - 26$

$= 13$

$\therefore x = 4$ cm, $y = 13$ cm and $z = 26$ cm

59. At x-axis, $y = 0$

$3x = 30$

$x = 10$

At y-axis, $x = 0$

$-5y = 30$

$y = -6$

$\therefore$ The coordinates of P are (10, 0) and of Q are (0, –6).

60. (i) $4x - 6 = 5y - 7$ (isos. trapezium)

$4x - 5y = -1$ —(1)

$(4x - 6) + (5x + 6y + 33) = 180$ (int. $\angle$s)

$9x + 6y = 153$

$3x + 2y = 51$ —(2)

(ii) (1) × 3: $12x - 15y = -3$ —(3)

(2) × 4: $12x + 8y = 204$ —(4)

(4) – (3): $23y = 207$

$y = 9$

$\hat{B} = \hat{C}$

$= [5(9) - 7]°$

$= 38°$

$\hat{A} = 180° - \hat{B}$

$= 180° - 38°$

$= 142°$

$\therefore \hat{A} = 142°$ and $\hat{B} = 38°$

61. **(a)** $4x - 2y - 5 = 0$

$$2y = 4x - 5$$
$$y = 2x - 2\frac{1}{2}$$

(i) Gradient of line $l = 2$

(ii) y-intercept of line $l = -2\frac{1}{2}$

(b) $2x + 3y = -5$ —(1)

$4x - 2y = 5$ —(2)

(1) × 2: $4x + 6y = -10$ —(3)

(3) – (2): $8y = -15$

$$y = -1\frac{7}{8}$$

Substitute $y = -1\frac{7}{8}$ into (1):

$$2x + 3\left(-1\frac{7}{8}\right) = -5$$
$$2x - 5\frac{5}{8} = -5$$
$$2x = \frac{5}{8}$$
$$x = \frac{5}{16}$$

$\therefore$ The coordinates of C are $\left(\frac{5}{16}, -1\frac{7}{8}\right)$.

62. **(i)** $y = 7 - 2x$ —(1)

$y = x + 10$ —(2)

Substitute $x = -9$ into (1):

$y = 7 - 2(-9)$
$= 7 + 18$
$= 25$

Substitute $x = -9$ into (2):

$y = -9 + 10$
$= 1$

$\therefore$ The coordinates of A are $(-9, 25)$ and of B are $(-9, 1)$.

(ii) $y = 7 - 2x$

From the equation, gradient of the line $= -2$.

(iii) $(0, k)$ lies on the perpendicular bisector of AB.

$\therefore k = \frac{1+25}{2}$
$= 13$

Chapter 3 Expansion and Factorisation of Quadratic Expressions

Basic

1. **(a)** $5a^2 + 2a - 3a^2 - a$
$= 2a^2 + a$

(b) $b^2 - 3b + 4 - 2b^2 + 3b - 7$
$= -b^2 - 3$

(c) $c^2 + 4c + 3 + (-2c^2) + (-c) - 2$
$= c^2 + 4c + 3 - 2c^2 - c - 2$
$= -c^2 + 3c + 1$

(d) $4d^2 - d - 5 - (-2d^2) - (-d) + 6$
$= 4d^2 - d - 5 + 2d^2 + d + 6$
$= 6d^2 + 1$

(e) $8e^2 + 8e + 9 - (5e^2 + 2e - 3)$
$= 8e^2 + 8e + 9 - 5e^2 - 2e + 3$
$= 3e^2 + 6e + 12$

(f) $6f^2 - 4f - 1 - (2f^2 - 7f)$
$= 6f^2 - 4f - 1 - 2f^2 + 7f$
$= 4f^2 + 3f - 1$

(g) $-(2 + g - g^2) + (6g - g^2)$
$= -2 - g + g^2 + 6g - g^2$
$= 5g - 2$

(h) $-(1 + 5h - 3h^2) - (2h^2 + 4h - 7)$
$= -1 - 5h + 3h^2 - 2h^2 - 4h + 7$
$= h^2 - 9h + 6$

2. **(a)** $8 \times 2h$
$= 16h$

(b) $3h \times 4h$
$= 12h^2$

(c) $(-5h) \times 6h$
$= -30h^2$

(d) $(-10h) \times (-7h)$
$= 70h^2$

3. **(a)** $5(2a + 3)$
$= 10a + 15$

(b) $-4(5b + 1)$
$= -20b - 4$

(c) $8(c^2 + 2c - 3)$
$= 8c^2 + 16c - 24$

(d) $-2(4 - 6d^2)$
$= -8 + 12d^2$
$= 12d^2 - 8$

(e) $3e(8e + 7)$
$= 24e^2 + 21e$

(f) $-f(9 - f)$
$= -9f + f^2$
$= f^2 - 9f$

(g) $-6g(5g - 1)$
$= -30g^2 + 6g$
$= 6g - 30g^2$

(h) $-2h(-3h - 4)$
$= 6h^2 + 8h$

4. **(a)** $3(a + 2) + 4(2a + 3)$
$= 3a + 6 + 8a + 12$
$= 11a + 18$

(b) $11(5b - 7) + 9(2 - 3b)$
$= 55b - 77 + 18 - 27b$
$= 28b - 59$

(c) $8(5c - 4) + 3(2 - 4c)$
$= 40c - 32 + 6 - 12c$
$= 28c - 26$

(d) $2d(3d + 4) + d(5d - 2)$
$= 6d^2 + 8d + 5d^2 - 2d$
$= 11d^2 + 6d$

(e) $e(6e - 1) + 2e(e - 2)$
$= 6e^2 - e + 2e^2 - 4e$
$= 8e^2 - 5e$

(f) $4f(1 - 2f) + f(3 - f)$
$= 4f - 8f^2 + 3f - f^2$
$= 7f - 9f^2$

5. **(a)** $(x + 5)(x + 7)$
$= x^2 + 7x + 5x + 35$
$= x^2 + 12x + 35$

(b) $(2x + 1)(x + 3)$
$= 2x^2 + 6x + x + 3$
$= 2x^2 + 7x + 3$

(c) $(x + 6)(3x + 4)$
$= 3x^2 + 4x + 18x + 24$
$= 3x^2 + 22x + 24$

(d) $(4x + 3)(5x + 6)$
$= 20x^2 + 24x + 15x + 18$
$= 20x^2 + 39x + 18$

6. **(a)** $a^2 + 20a + 75$

×	a	15
a	a^2	$15a$
5	$5a$	75

$\therefore a^2 + 20a + 75 = (a + 15)(a + 5)$

(b) $b^2 + 19b + 18$

×	b	18
b	b^2	$18b$
1	b	18

$\therefore b^2 + 19b + 18 = (b + 18)(b + 1)$

(c) $c^2 - 11c + 28$

×	c	-7
c	c^2	$-7c$
-4	$-4c$	28

$\therefore c^2 - 11c + 28 = (c - 7)(c - 4)$

(d) $d^2 - 21d + 68$

×	d	-17
d	d^2	$-17d$
-4	$-4d$	68

$\therefore d^2 - 21d + 68 = (d - 17)(d - 4)$

(e) $e^2 + 4e - 77$

×	e	11
e	e^2	$11e$
-7	$-7e$	-77

$\therefore e^2 + 4e - 77 = (e + 11)(e - 7)$

(f) $f^2 + 3f - 154$

×	f	14
f	f^2	$14f$
-11	$-11f$	-154

$\therefore f^2 + 3f - 154 = (f + 14)(f - 11)$

(g) $g^2 - 2g - 35$

×	g	-7
g	g^2	$-7g$
5	$5g$	-35

$\therefore g^2 - 2g - 35 = (g - 7)(g + 5)$

(h) $h^2 - 10h - 171$

×	h	-19
h	h^2	$-19h$
9	$9h$	-171

$\therefore h^2 - 10h - 171 = (h - 19)(h + 9)$

7. (a) $6a^2 + 31a + 5$

×	$6a$	1
a	$6a^2$	a
5	$30a$	5

$\therefore 6a^2 + 31a + 5 = (6a + 1)(a + 5)$

(b) $8b^2 + 30b + 27$

×	$4b$	9
$2b$	$8b^2$	$18b$
3	$12b$	27

$\therefore 8b^2 + 30b + 27 = (4b + 9)(2b + 3)$

(c) $4c^2 - 25c + 6$

×	$4c$	-1
c	$4c^2$	$-c$
-6	$-24c$	6

$\therefore 4c^2 - 25c + 6 = (4c - 1)(c - 6)$

(d) $9d^2 - 36d + 32$

×	$3d$	-8
$3d$	$9d^2$	$-24d$
-4	$-12d$	32

$\therefore 9d^2 - 36d + 32 = (3d - 8)(3d - 4)$

(e) $15e^2 + 2e - 1$

×	$5e$	-1
$3e$	$15e^2$	$-3e$
1	$5e$	-1

$\therefore 15e^2 + 2e - 1 = (5e - 1)(3e + 1)$

(f) $2g^2 - 5g - 3$

×	$2g$	1
g	$2g^2$	g
-3	$-6g$	-3

$\therefore 2g^2 - 5g - 3 = (2g + 1)(g - 3)$

(g) $12h^2 - 31h - 15$

×	$12h$	5
h	$12h^2$	$5h$
-3	$-36h$	-15

$\therefore 12h^2 - 31h - 15 = (12h + 5)(h - 3)$

Intermediate

8. **(a)** $6(a + 3) - 5(a - 4)$
$= 6a + 18 - 5a + 20$
$= a + 38$

(b) $13(5b + 7) - 6(3b - 5)$
$= 65b + 91 - 18b + 30$
$= 47b + 121$

(c) $9(3c - 2) - 5(2 + c)$
$= 27c - 18 - 10 - 5c$
$= 22c - 28$

(d) $8(5 - 4d) - 7(7 - 5d)$
$= 40 - 32d - 49 + 35d$
$= 3d - 9$

(e) $7(12 - 5e) - 3(9 - 7e)$
$= 84 - 35e - 27 + 21e$
$= 57 - 14e$

(f) $5f(f + 3) - 4f(5 - f)$
$= 5f^2 + 15f - 20f + 4f^2$
$= 9f^2 - 5f$

(g) $-2g(4 - g) - 3g(2g + 1)$
$= -8g + 2g^2 - 6g^2 - 3g$
$= -4g^2 - 11g$

(h) $-5h(3h + 7) - 4h(-h - 2)$
$= -15h^2 - 35h + 4h^2 + 8h$
$= -11h^2 - 27h$

9. **(a)** $(y + 7)(y - 11)$
$= y^2 - 11y + 7y - 77$
$= y^2 - 4y - 77$

(b) $(y - 6)(y + 8)$
$= y^2 + 8y - 6y - 48$
$= y^2 + 2y - 48$

(c) $(y - 9)(y - 4)$
$= y^2 - 4y - 9y + 36$
$= y^2 - 13y + 36$

(d) $(2y + 3)(4y - 5)$
$= 8y^2 - 10y + 12y - 15$
$= 8y^2 + 2y - 15$

(e) $(5y - 9)(6y - 1)$
$= 30y^2 - 5y - 54y + 9$
$= 30y^2 - 59y + 9$

(f) $(4y - 1)(3 - 4y)$
$= 12y - 16y^2 - 3 + 4y$
$= -16y^2 + 16y - 3$

(g) $(7 - 2y)(4 + y)$
$= 28 + 7y - 8y - 2y^2$
$= 28 - y - 2y^2$

(h) $(7 - 3y)(8 - 5y)$
$= 56 - 35y - 24y + 15y^2$
$= 56 - 59y + 15y^2$

10. **(a)** $4 + (a + 2)(a + 5)$
$= 4 + a^2 + 5a + 2a + 10$
$= a^2 + 7a + 14$

(b) $6b + (3b + 1)(b - 2)$
$= 6b + 3b^2 - 6b + b - 2$
$= 3b^2 + b - 2$

(c) $(7c + 2)(3c - 8) + 9c(c + 1)$
$= 21c^2 - 56c + 6c - 16 + 9c^2 + 9c$
$= 30c^2 - 41c - 16$

(d) $(4d - 5)(8d - 7) + (2d + 3)(d - 3)$
$= 32d^2 - 28d - 40d + 35 + 2d^2 - 6d + 3d - 9$
$= 34d^2 - 71d + 26$

11. **(a)** $-x^2 - 4x + 21$

×	$-x$	3
x	$-x^2$	$3x$
7	$-7x$	21

$\therefore -x^2 - 4x + 21 = (-x + 3)(x + 7)$

(b) $-6x^2 + 2x + 20 = -2(3x^2 - x - 10)$

×	$3x$	5
x	$3x^2$	$5x$
-2	$-6x$	-10

$\therefore -6x^2 + 2x + 20 = -2(3x + 5)(x - 2)$

(c) $12hx^2 - 25hx + 12h = h(12x^2 - 25x + 12)$

×	$4x$	-3
$3x$	$12x^2$	$-9x$
-4	$-16x$	12

$\therefore 12hx^2 - 25hx + 12h = h(4x - 3)(3x - 4)$

12. $3x^2 + 26x + 51$

×	$3x$	17
x	$3x^2$	$17x$
3	$9x$	51

$\therefore 3x^2 + 26x + 51 = (3x + 17)(x + 3)$
$32\ 651 = 3(100)^2 + 26(100) + 51$
Let $x = 100$.
$32\ 651 = 317 \times 103$
$\therefore$ The factors are 317 and 103.

13. $4x^2 + 13x + 3$

×	$4x$	1
x	$4x^2$	x
3	$12x$	3

$\therefore 4x^2 + 13x + 3 = (4x + 1)(x + 3)$

$41\ 303 = 4(100)^2 + 13(100) + 3$

Let $x = 100$.

$41\ 303 = 401 \times 103$

$\therefore$ The prime factors are 401 and 103.

Advanced

14. **(a)** $9a^2 - (4a - 1)(a + 2)$

$= 9a^2 - (4a^2 + 8a - a - 2)$

$= 9a^2 - (4a^2 + 7a - 2)$

$= 9a^2 - 4a^2 - 7a + 2$

$= 5a^2 - 7a + 2$

(b) $3b(2 - b) - (1 + b)(1 - b)$

$= 6b - 3b^2 - (1 - b + b - b^2)$

$= 6b - 3b^2 - (1 - b^2)$

$= 6b - 3b^2 - 1 + b^2$

$= -2b^2 + 6b - 1$

(c) $(5c + 6)(6c - 5) - (3 - 2c)(1 - 15c)$

$= 30c^2 - 25c + 36c - 30 - (3 - 45c - 2c + 30c^2)$

$= 30c^2 + 11c - 30 - (3 - 47c + 30c^2)$

$= 30c^2 + 11c - 30 - 3 + 47c - 30c^2$

$= 58c - 33$

(d) $(2d - 8)\left(\frac{1}{2}d - 4\right) - (3d - 6)\left(\frac{1}{3}d + 1\right)$

$= d^2 - 8d - 4d + 32 - (d^2 + 3d - 2d - 6)$

$= d^2 - 12d + 32 - (d^2 + d - 6)$

$= d^2 - 12d + 32 - d^2 - d + 6$

$= 38 - 13d$

15. **(i)** $3x^2 + 48x + 189 = 3(x^2 + 16x + 63)$

$\times$	x	7
x	x^2	$7x$
9	$9x$	63

$\therefore 3x^2 + 48x + 189 = 3(x + 7)(x + 9)$

(ii) $969 = 3(10)^2 + 48(10) + 189$

Let $x = 10$.

$969 = 3 \times 17 \times 19$

$\therefore$ Sum $= 3 + 17 + 19$

$= 39$

16. $2x^2 - 2.9x - 3.6 = 0.1(20x^2 - 29x - 36)$

$\times$	$5x$	4
$4x$	$20x^2$	$16x$
-9	$-45x$	-36

$\therefore 2x^2 - 2.9x - 3.6 = 0.1(5x + 4)(4x - 9)$

i.e. $p = 5, q = 4, r = 4, s = -9$

$\therefore p + q + r + s = 5 + 4 + 4 - 9$

$= 4$

New Trend

17. **(a)** $16a^2 - 9b^2$

$\times$	$4a$	$3b$
$14a$	$16a^2$	$12ab$
$-3b$	$-12ab$	$-9b^2$

$\therefore 16a^2 - 9b^2 = (4a + 3b)(4a - 3b)$

(b) $3f^2 + 11f - 20$

$\times$	$3f$	-4
f	$3f^2$	$-4f$
5	$15f$	-20

$\therefore 3f^2 + 11f - 20 = (3f - 4)(f + 5)$

(c) $9x^2 - 15x - 6 = 3(3x^2 - 5x - 2)$

$\times$	$3x$	1
x	$3x^2$	x
-2	$-6x$	-2

$\therefore 9x^2 - 15x - 6 = 3(3x + 1)(x - 2)$

18. **(i)** Since x is a positive integer, $2x$ is a positive even number and $(2x - 1)$ is one less than an even number.

$\therefore 2x - 1$ is not divisible by 2.

Hence, $2x - 1$ is an odd number.

(ii) $(2x - 1) + 2 = 2x + 1$

(iii) $(2x - 1)^2 = 4x^2 - 4x + 1$

$(2x + 1)^2 = 4x^2 + 4x + 1$

(iv) $(4x^2 + 4x + 1) - (4x^2 - 4x + 1)$

$= 4x + 4x$

$= 8x$

Since $8x$ has a factor of 8, it is always divisible by 8.

Chapter 4 Further Expansion and Factorisation of Algebraic Expressions

Basic

1. **(a)** $7a \times 3b$
$= 21ab$

(b) $5c \times (-4d)$
$= -20cd$

(c) $(-10e) \times (-2f)$
$= 20ef$

(d) $\frac{1}{6}g \times 24h$
$= 4gh$

2. **(a)** $5a(2a + 3b)$
$= 10a^2 + 15ab$

(b) $8c(5c - 2d)$
$= 40c^2 - 16cd$

(c) $9e(-4e + 7f)$
$= -36e^2 + 63ef$

(d) $4h(-2g - 3h)$
$= -8gh - 12h^2$

(e) $-6j(k - 4j)$
$= -6jk + 24j^2$

(f) $-4m(2n + 5m)$
$= -8mn - 20m^2$

(g) $-7p(-3p + 4q)$
$= 21p^2 - 28pq$

(h) $-3r(-2r - s)$
$= 6r^2 + 3rs$

(i) $2u(5u + v - w)$
$= 10u^2 + 2uv - 2uw$

(j) $-6x(3x - 2y + z)$
$= -18x^2 + 12xy - 6xz$

3. **(a)** $4a(3a - b) + 2a(a - 5b)$
$= 12a^2 - 4ab + 2a^2 - 10ab$
$= 14a^2 - 14ab$

(b) $2c(4d - 3c) + 5c(5c - 2d)$
$= 8cd - 6c^2 + 25c^2 - 10cd$
$= 19c^2 - 2cd$

(c) $3f(2e - 7f) + 2e(6f - 5e)$
$= 6ef - 21f^2 + 12ef - 10e^2$
$= -10e^2 + 18ef - 21f^2$

(d) $5h(-2h - 3g) + 2h(-h + 3g)$
$= -10h^2 - 15gh - 2h^2 + 6gh$
$= -12h^2 - 9gh$

4. **(a)** $(x + y)(x + 4y)$
$= x^2 + 4xy + xy + 4y^2$
$= x^2 + 5xy + 4y^2$

(b) $(2x + y)(3x + y)$
$= 6x^2 + 2xy + 3xy + y^2$
$= 6x^2 + 5xy + y^2$

(c) $(x^2 + 1)(x + 1)$
$= x^3 + x^2 + x + 1$

(d) $(4x^2 + 3)(2x + 3)$
$= 8x^3 + 12x^2 + 6x + 9$

5. **(a)** $(a + 5)^2$
$= a^2 + 10a + 25$

(b) $(2b + 3)^2$
$= 4b^2 + 12b + 9$

(c) $(c + 6d)^2$
$= c^2 + 12cd + 36d^2$

(d) $(7e + 4f)^2$
$= 49e^2 + 56ef + 16f^2$

6. **(a)** $(a - 8)^2$
$= a^2 - 16a + 64$

(b) $(4b - 1)^2$
$= 16b^2 - 8b + 1$

(c) $(c - 3d)^2$
$= c^2 - 6cd + 9d^2$

(d) $(9e - 2f)^2$
$= 81e^2 - 36ef + 4f^2$

7. **(a)** $(a + 6)(a - 6)$
$= a^2 - 36$

(b) $(4b + 3)(4b - 3)$
$= 16b^2 - 9$

(c) $(9 + 4c)(9 - 4c)$
$= 81 - 16c^2$

(d) $(5d + e)(5d - e)$
$= 25d^2 - e^2$

8. **(a)** 904^2
$= (900 + 4)^2$
$= 900^2 + 2(900)(4) + 4^2$
$= 810\ 000 + 7200 + 16$
$= 817\ 216$

(b) 791^2
$= (800 - 9)^2$
$= 800^2 - 2(800)(9) + 9^2$
$= 640\ 000 - 14\ 400 + 81$
$= 625\ 681$

(c) 603×597
$= (600 + 3)(600 - 3)$
$= 600^2 - 3^2$
$= 360\ 000 - 9$
$= 359\ 991$

(d) 99×101
$= (100 - 1)(100 + 1)$
$= 100^2 - 1^2$
$= 10\,000 - 1$
$= 9999$

9. $(a + b)^2 = a^2 + 2ab + b^2$
$73 = a^2 + b^2 + 2(65)$
$= a^2 + b^2 + 130$
$a^2 + b^2 = 73 - 130$
$= -57$

10. (a) $a^2 + 12a + 36$
$= (a + 6)^2$
(b) $9b^2 + 12b + 4$
$= (3b + 2)^2$
(c) $4c^2 + 4cd + d^2$
$= (2c + d)^2$
(d) $16e^2 + 40ef + 25f^2$
$= (4e + 5f)^2$

11. (a) $a^2 - 18a + 81$
$= (a - 9)^2$
(b) $25b^2 - 20b + 4$
$= (5b - 2)^2$
(c) $9c^2 - 6cd + d^2$
$= (3c - d)^2$
(d) $49e^2 - 28ef + 4f^2$
$= (7e - 2f)^2$

12. (a) $a^2 - 196$
$= a^2 - 14^2$
$= (a + 14)(a - 14)$
(b) $4b^2 - 81$
$= (2b)^2 - 9^2$
$= (2b + 9)(2b - 9)$
(c) $289 - 36c^2$
$= 17^2 - (6c)^2$
$= (17 + 6c)(17 - 6c)$
(d) $9d^2 - e^2$
$= (3d)^2 - e^2$
$= (3d + e)(3d - e)$

13. (a) $abc - a^2bc^3$
$= abc(1 - ac^2)$
(b) $2a^2b^3c - 8ab^2c^3$
$= 2ab^2c(ab - 4c^2)$
(c) $6k^2 + 8k^3 - 10k^5$
$= 2k^2(3 + 4k - 5k^3)$
(d) $m^2n - mn^2 + m^2n^2$
$= mn(m - n + mn)$
(e) $p^2q - 2pq^2 + 4p^2q^2$
$= pq(p - 2q + 4pq)$
(f) $2s - 4s^2 + 8st^2$
$= 2s(1 - 2s + 4t^2)$
(g) $12x^3 - 9x^2y^2 + 6xy^3$
$= 3x(4x^2 - 3xy^2 + 2y^3)$
(h) $5y^2z - 3y^3z^2 + 6y^2z^2$
$= y^2z(5 - 3yz + 6z)$

14. (a) $4a(x + y) + 7(x + y)$
$= (4a + 7)(x + y)$
(b) $5b(6x + y) - c(y + 6x)$
$= (5b - c)(6x + y)$
(c) $8d(x - 3y) - e(3y - x)$
$= 8d(x - 3y) + e(x - 3y)$
$= (8d + e)(x - 3y)$
(d) $(x + 5)(x - 1) + a(x + 5)$
$= (x - 1 + a)(x + 5)$

Intermediate

15. (a) $\frac{1}{2}a \times \frac{2}{3}b$
$= \frac{1}{3}ab$
(b) $\frac{2}{5}c \times \left(-\frac{3}{8}d\right)$
$= -\frac{3}{20}cd$
(c) $\left(-\frac{1}{4}e\right) \times \frac{12}{13}f$
$= -\frac{3}{13}ef$
(d) $\left(-\frac{6}{7}g\right) \times \left(-\frac{7}{12}h\right)$
$= \frac{1}{2}gh$
(e) $0.2p \times 12q$
$= 2.4pq$
(f) $3r \times 0.9s$
$= 2.7rs$
(g) $4w^2x \times 5wx^3$
$= 20w^3x^4$
(h) $(-8xy^2z) \times (-2xz^3)$
$= 16x^2y^2z^4$

16. $\frac{4}{5}a^2bc^3 \times \frac{15}{16}ab^2$
$= \frac{3}{4}a^3b^3c^3$

17. (a) $5ab(a - 4b)$
$= 5a^2b - 20ab^2$
(b) $-3c(2c^2d + d^2)$
$= -6c^3d - 3cd^2$
(c) $-8ef(6f - e^2)$
$= -48ef^2 + 8e^3f$
(d) $-10h^2(-7g^2h - 9h^3)$
$= 70g^2h^3 + 90h^5$

18. **(a)** $4a(3b + 5c) - 3b(8c - 9a)$
$= 12ab + 20ac - 24bc + 27ab$
$= 39ab + 20ac - 24bc$

(b) $5d(2d + 5e) - 3e(2e - 7d)$
$= 10d^2 + 25de - 6e^2 + 21de$
$= 10d^2 + 46de - 6e^2$

(c) $7f(2f + 3g) - 3f(-4g + 3f)$
$= 14f^2 + 21fg + 12fg - 9f^2$
$= 5f^2 + 33fg$

(d) $4h(-3h + k) - 2h(-5k + h)$
$= -12h^2 + 4hk + 10hk - 2h^2$
$= -14h^2 + 14hk$

19. **(a)** $(a + 6b)(a - 2b)$
$= a^2 - 2ab + 6ab - 12b^2$
$= a^2 + 4ab - 12b^2$

(b) $(4c + 5d)(5c + 7d)$
$= 20c^2 + 28cd + 25cd + 35d^2$
$= 20c^2 + 53cd + 35d^2$

(c) $(4e - 3f)(2e + 7f)$
$= 8e^2 + 28ef - 6ef - 21f^2$
$= 8e^2 + 22ef - 21f^2$

(d) $(2g - 3h)(g - 2h)$
$= 2g^2 - 4gh - 3gh + 6h^2$
$= 2g^2 - 7gh + 6h^2$

(e) $(m^2 - 4)(2m + 3)$
$= 2m^3 + 3m^2 - 8m - 12$

(f) $(2n - 4)(n^2 + 3)$
$= 2n^3 + 6n - 4n^2 - 12$
$= 2n^3 - 4n^2 + 6n - 12$

(g) $(2p - 3q)(2p - 5r)$
$= 4p^2 - 10pr - 6pq + 15qr$

(h) $(xy - 5)(xy + 8)$
$= x^2y^2 + 8xy - 5xy - 40$
$= x^2y^2 + 3xy - 40$

20. **(a)** $(a + 1)(a - 3) + (2a - 3)(5 - 7a)$
$= a^2 - 3a + a - 3 + 10a - 14a^2 - 15 + 21a$
$= -13a^2 + 29a - 18$

(b) $(7b + 1)(b - 5) - 3(4 - 2b - b^2)$
$= 7b^2 - 35b + b - 5 - 12 + 6b + 3b^2$
$= 10b^2 - 28b - 17$

(c) $(3c - 8)(c + 1) - (2c - 1)(5 - c)$
$= 3c^2 + 3c - 8c - 8 - (10c - 2c^2 - 5 + c)$
$= 3c^2 - 5c - 8 - (-2c^2 + 11c - 5)$
$= 3c^2 - 5c - 8 + 2c^2 - 11c + 5$
$= 5c^2 - 16c - 3$

(d) $(d + 3e)(d - 3e) - 2(d + 2e)(d - e)$
$= d^2 - 9e^2 - 2(d^2 - de + 2de - 2e^2)$
$= d^2 - 9e^2 - 2(d^2 + de - 2e^2)$
$= d^2 - 9e^2 - 2d^2 - 2de + 4e^2$
$= -d^2 - 2de - 5e^2$

21. **(a)** $(a + 3)(a^2 + 3a + 9)$
$= a^3 + 3a^2 + 9a + 3a^2 + 9a + 27$
$= a^3 + 6a^2 + 18a + 27$

(b) $(b + c)(b^2 + bc + c^2)$
$= b^3 + b^2c + bc^2 + b^2c + bc^2 + c^3$
$= b^3 + 2b^2c + 2bc^2 + c^3$

(c) $(5 + 2d)(2 + 3d + d^2)$
$= 10 + 15d + 5d^2 + 4d + 6d^2 + 2d^3$
$= 2d^3 + 11d^2 + 19d + 10$

(d) $(2e + f)(3e - 4f + g)$
$= 6e^2 - 8ef + 2eg + 3ef - 4f^2 + fg$
$= 6e^2 - 5ef - 4f^2 + 2eg + fg$

22. **(a)** $a^2 + 7ab + 6b^2$

×	a	b
a	a^2	ab
$6b$	$6ab$	$6b^2$

$\therefore a^2 + 7ab + 6b^2 = (a + b)(a + 6b)$

(b) $c^2 + 11cd - 12d^2$

×	c	$12d$
c	c^2	$12cd$
$-d$	$-cd$	$-12d^2$

$\therefore c^2 + 11cd - 12d^2 = (c + 12d)(c - d)$

(c) $2d^2 - de - 15e^2$

×	$2d$	$5e$
d	$2d^2$	$5de$
$-3e$	$-6de$	$-15e^2$

$\therefore 2d^2 - de - 15e^2 = (2d + 5e)(d - 3e)$

(d) $6f^2 - 29fg + 28g^2$

×	$3f$	$-4g$
$2f$	$6f^2$	$-8fg$
$-7g$	$-21fg$	$28g^2$

$\therefore 6f^2 - 29fg + 28g^2 = (3f - 4g)(2f - 7g)$

(e) $2m^2 + 2mn - 12n^2 = 2(m^2 + mn - 6n^2)$

×	m	$-2n$
m	m^2	$-2mn$
$3n$	$3mn$	$-6n^2$

$\therefore 2m^2 + 2mn - 12n^2 = 2(m - 2n)(m + 3n)$

(f) $px^2 - 11pxy + 24py^2 = p(x^2 - 11xy + 24y^2)$

×	x	$-3y$
x	x^2	$-3xy$
$-8y$	$-8xy$	$24y^2$

$\therefore px^2 - 11pxy + 24py^2 = p(x - 3y)(x - 8y)$

23. $12x^2 + xy - 20y^2$

×	$4x$	$-5y$
$3x$	$12x^2$	$-15xy$
$4y$	$16xy$	$-20y^2$

$12x^2 + xy - 20y^2 = (4x - 5y)(3x + 4y)$

$\therefore$ Breadth of rectangle $= \dfrac{(4x - 5y)(3x + 4y)}{4x - 5y}$

$= (3x + 4y)$ cm

24. (a) $\left(a + \dfrac{b}{3}\right)^2$

$= a^2 + \dfrac{2ab}{3} + \dfrac{b^2}{9}$

(b) $(0.5c + d)^2$

$= 0.25c^2 + cd + d^2$

(c) $(ef + 2)^2$

$= e^2f^2 + 4ef + 4$

(d) $\left(g + \dfrac{2}{g}\right)^2$

$= g^2 + 4 + \dfrac{4}{g^2}$

(e) $(h^2 + 3)^2$

$= h^4 + 6h^2 + 9$

(f) $(k^3 + 4)^2$

$= k^6 + 8k^3 + 16$

(g) $\left(\dfrac{2}{p} + \dfrac{3}{q}\right)^2$

$= \dfrac{4}{p^2} + \dfrac{12}{pq} + \dfrac{9}{q^2}$

(h) $\left(\dfrac{x}{y} + 3y\right)^2$

$= \dfrac{x^2}{y^2} + 6x + 9y^2$

25. (a) $\left(3a - \dfrac{1}{4}b\right)^2$

$= 9a^2 - \dfrac{3}{2}ab + \dfrac{1}{16}b^2$

(b) $(10c - 0.1d)^2$

$= 100c^2 - 2cd + 0.01d^2$

(c) $(2ef - 1)^2$

$= 4e^2f^2 - 4ef + 1$

(d) $\left(2h - \dfrac{1}{h}\right)^2$

$= 4h^2 - 4 + \dfrac{1}{h^2}$

(e) $(p^4 - 2)^2$

$= p^8 - 4p^4 + 4$

(f) $\left(\dfrac{x}{y} - \dfrac{y}{x}\right)^2$

$= \dfrac{x^2}{y^2} - 2 + \dfrac{y^2}{x^2}$

26. (a) $\left(\dfrac{1}{2}a + b\right)\left(\dfrac{1}{2}a - b\right)$

$= \dfrac{1}{4}a^2 - b^2$

(b) $(0.2c + d)(d - 0.2c)$

$= (d + 0.2c)(d - 0.2c)$

$= d^2 - 0.04c^2$

(c) $(3ef + 4)(3ef - 4)$

$= 9e^2f^2 - 16$

(d) $\left(\dfrac{g}{2} - \dfrac{h}{4}\right)\left(\dfrac{h}{4} + \dfrac{g}{2}\right)$

$= \left(\dfrac{g}{2} + \dfrac{h}{4}\right)\left(\dfrac{g}{2} - \dfrac{h}{4}\right)$

$= \dfrac{g^2}{4} - \dfrac{h^2}{16}$

27. $x^2 - y^2 = 6$

$(x + y)(x - y) = 6$

$2(x + y) = 6$

$x + y = 3$

$\therefore (x + y)^2 = 9$

28. (i) $(x + y)^2 = x^2 + 2xy + y^2$

$= 43 + 24$

$= 67$

(ii) $(2x - 2y)^2 = 4x^2 - 8xy + 4y^2$

$= 4(43) - 2(48)$

$= 76$

29. (i) $x^2 - 4y^2 = (x + 2y)(x - 2y)$

$= (-2)(18)$

$= -36$

(ii) $x + 2y = -2$ —(1)

$x - 2y = 18$ —(2)

(1) + (2): $2x = 16$

$x = 8$

(1) – (2): $4y = -20$

$y = -5$

$\therefore x^2 + 4y^2 = 8^2 + 4(-5)^2$

$= 164$

30. **(i)** $a^2 - b^2 = (a + b)(a - b)$

(ii) $2030^2 - 2029^2 + 2028^2 - 2027^2$

$= (2030 + 2029)(2030 - 2029) + (2028 + 2027)(2028 - 2027)$

$= 2030 + 2029 + 2028 + 2027$

$= 8114$

31. **(a)** $4a^2 + 32a + 64$

$= 4(a^2 + 8a + 16)$

$= 4(a + 4)^2$

(b) $\frac{1}{4}b^2 + 4bc + 16c^2$

$= \left(\frac{1}{2}b + 4c\right)^2$

(c) $\frac{1}{9}d^2 + \frac{4}{15}de + \frac{4}{25}e^2$

$= \left(\frac{1}{3}d + \frac{2}{5}e\right)^2$

(d) $f^4 + 8f^2 + 16$

$= (f^2 + 4)^2$

32. **(a)** $3a^2 - 36a + 108$

$= 3(a^2 - 12a + 36)$

$= 3(a - 6)^2$

(b) $64b^2 - 4bc + \frac{1}{16}c^2$

$= \left(8b - \frac{1}{4}c\right)^2$

(c) $e^2f^2 - 10ef + 25$

$= (ef - 5)^2$

(d) $\frac{1}{4}g^2 - \frac{1}{4}gh + \frac{1}{16}h^2$

$= \left(\frac{1}{2}g - \frac{1}{4}h\right)^2$

33. **(a)** $\frac{1}{4}a^2 - b^2$

$= \left(\frac{1}{2}a + b\right)\left(\frac{1}{2}a - b\right)$

(b) $4c^3 - 49c$

$= c(4c^2 - 49)$

$= c(2c + 7)(2c - 7)$

(c) $81ef^2 - 4eg^2$

$= e(81f^2 - 4g^2)$

$= e(9f + 2g)(9f - 2g)$

(d) $18h^3 - 8hk^2$

$= 2h(9h^2 - 4k^2)$

$= 2h(3h + 2k)(3h - 2k)$

(e) $81m^5n^3 - 121m^3n^5$

$= m^3n^3(81m^2 - 121n^2)$

$= m^3n^3(9m + 11n)(9m - 11n)$

(f) $p^4 - 81q^4$

$= (p^2 + 9q^2)(p^2 - 9q^2)$

$= (p^2 + 9q^2)(p + 3q)(p - 3q)$

(g) $(t^2 - 1)^2 - 9$

$= (t^2 - 1 + 3)(t^2 - 1 - 3)$

$= (t^2 + 2)(t^2 - 4)$

$= (t^2 + 2)(t + 2)(t - 2)$

(h) $9 - (a - b)^2$

$= (3 + a - b)(3 - a + b)$

(i) $(d + 2c)^2 - c^2$

$= (d + 2c + c)(d + 2c - c)$

$= (d + 3c)(d + c)$

(j) $(e - 3)^2 - 16f^2$

$= (e - 3 + 4f)(e - 3 - 4f)$

(k) $(3g - h)^2 - g^2$

$= (3g - h + g)(3g - h - g)$

$= (4g - h)(2g - h)$

(l) $4j^2 - (k - 2)^2$

$= (2j + k - 2)(2j - k + 2)$

(m) $9m^2 - (3m - 2n)^2$

$= (3m + 3n - 2n)(3m - 3m + 2n)$

$= (6m - 2n)(2n)$

$= 4n(3m - n)$

(n) $9p^2 - 4(p - 2q)^2$

$= (3p)^2 - (2p - 4q)^2$

$= (3p + 2p - 4q)(3p - 2p + 4q)$

$= (5p - 4q)(p + 4q)$

(o) $(3x - 2y)^2 - (2x - 3y)^2$

$= (3x - 2y + 2x - 3y)(3x - 2y - 2x + 3y)$

$= (5x - 5y)(x + y)$

$= 5(x + y)(x - y)$

34. **(a)** $41^2 + 738 + 81$

$= 41^2 + 2(41)(9) + 9^2$

$= (41 + 9)^2$

$= 50^2$

$= 2500$

(b) $65^2 + 650 + 25$

$= 65^2 + 2(65)(5) + 5^2$

$= (65 + 5)^2$

$= 70^2$

$= 4900$

(c) $92^2 - 368 + 4$

$= 92^2 - 2(92)(2) + 2^2$

$= (92 - 2)^2$

$= 90^2$

$= 8100$

(d) $201^2 - 402 + 1$

$= 201^2 - 2(201)(1) + 1^2$

$= (201 - 1)^2$

$= 200^2$

$= 40\ 000$

(e) $201^2 - 99^2$
$= (201 + 99)(201 - 99)$
$= (300)(102)$
$= 30\ 600$

(f) $1.013^2 - 0.013^2$
$= (1.013 + 0.013)(1.013 - 0.013)$
$= 1.026$

35. (a) $(2a + b)(x + y) + (a + b)(x + y)$
$= (2a + b + a + b)(x + y)$
$= (3a + 2b)(x + y)$

(b) $(4c + 3d)^2 + (4c + 3d)(c + d)$
$= (4c + 3d)(4c + 3d + c + d)$
$= (4c + 3d)(5c + 4d)$

(c) $2p(5r - 7s) + 3q(7s - 5r)$
$= 2p(5r - 7s) - 3q(5r - 7s)$
$= (2p - 3q)(5r - 7s)$

(d) $9w(y - x) - 8z(x - y)$
$= 9w(y - x) + 8z(y - x)$
$= (9w + 8z)(y - x)$

36. (a) $p^2 + pq + 3qr + 3pr$
$= p(p + q) + 3r(q + p)$
$= (p + 3r)(p + q)$

(b) $3xy + 6y - 5x - 10$
$= 3y(x + 2) - 5(x + 2)$
$= (3y - 5)(x + 2)$

(c) $x^2z - 4y - x^2y + 4z$
$= x^2z - x^2y + 4z - 4y$
$= x^2(z - y) + 4(z - y)$
$= (x^2 + 4)(z - y)$

(d) $x^3 + xy - 3x^2y - 3y^2$
$= x(x^2 + y) - 3y(x^2 + y)$
$= (x^2 + y)(x - 3y)$

(e) $x - 4x^2 - 4 + x^3$
$= x^3 + x - 4x^2 - 4$
$= x(x^2 + 1) - 4(x^2 + 1)$
$= (x^2 + 1)(x - 4)$

(f) $h^2 - 1 + hk + k$
$= (h + 1)(h - 1) + k(h + 1)$
$= (h - 1 + k)(h + 1)$

(g) $m - n - m^2 + n^2$
$= (m - n) - (m^2 - n^2)$
$= (m - n) - (m + n)(m - n)$
$= (1 - m - n)(m - n)$

(h) $a^2 - 3bc - ab + 3ac$
$= a^2 - ab + 3ac - 3bc$
$= a(a - b) + 3c(a - b)$
$= (a + 3c)(a - b)$

(i) $x^2y - 3y - 6 + 2x^2$
$= y(x^2 - 3) - 2(3 - x^2)$
$= y(x^2 - 3) + 2(x^2 - 3)$
$= (y + 2)(x^2 - 3)$

(j) $a^2x - 12by - 3bx + 4a^2y$
$= a^2x + 4a^2y - 3bx - 12by$
$= a^2(x + 4y) - 3b(x + 4y)$
$= (x + 4y)(a^2 - 3b)$

Advanced

37. (a) $(2h + 3)(h - 7) - (h + 4)(h^2 - 1)$
$= 2h^2 - 14h + 3h - 21 - (h^3 - h + 4h^2 - 4)$
$= 2h^2 - 11h - 21 - h^3 + h - 4h^2 + 4$
$= -h^3 - 2h^2 - 10h - 17$

(b) $(3p^2 + q)(2p - q) - (2p + q)(3p^2 - q)$
$= 6p^3 - 3p^2q + 2pq - q^2 - (6p^3 - 2pq + 3p^2q - q^2)$
$= 6p^3 - 3p^2q + 2pq - q^2 - 6p^3 + 2pq - 3p^2q + q^2$
$= 4pq - 6p^2q$

38. (a) $(2a + 1)(a^2 - 3a - 4)$
$= 2a^3 - 6a^2 - 8a + a^2 - 3a - 4$
$= 2a^3 - 5a^2 - 11a - 4$

(b) $(b + 2)(3b^2 - 5b + 6)$
$= 3b^3 - 5b^2 + 6b + 6b^2 - 10b + 12$
$= 3b^3 + b^2 - 4b + 12$

(c) $(7 - c)(5c^2 - 2c + 1)$
$= 35c^2 - 14c + 7 - 5c^3 + 2c^2 - c$
$= -5c^3 + 37c^2 - 15c + 7$

(d) $(d^2 - 4)(d^2 - 2d + 1)$
$= d^4 - 2d^3 + d^2 - 4d^2 + 8d - 4$
$= d^4 - 2d^3 - 3d^2 + 8d - 4$

(e) $(h - 2k)(2h + 3k - 1)$
$= 2h^2 + 3hk - h - 4hk - 6k^2 + 2k$
$= 2h^2 - hk - 6k^2 - h + 2k$

(f) $(m - n)(m^2 + mn + n^2)$
$= m^3 + m^2n + mn^2 - m^2n - mn^2 - n^3$
$= m^3 - n^3$

(g) $(p + 1)(p^3 - p^2 + p - 1)$
$= p^4 - p^3 + p^2 - p + p^3 - p^2 + p - 1$
$= p^4 - 1$

(h) $(q - 1)(q^3 - 3q^2 + 3q - 1)$
$= q^4 - 3q^3 + 3q^2 - q - q^3 + 3q^2 - 3q + 1$
$= q^4 - 4q^3 + 6q^2 - 4q + 1$

39. (a) $2a^2b^2 + 4ab - 48 = 2(a^2b^2 + 2ab - 24)$

$\times$	ab	6
ab	a^2b^2	$6ab$
-4	$-4ab$	-24

$\therefore\ 2a^2b^2 + 4ab - 48 = 2(ab + 6)(ab - 4)$

(b) $15c^2d^2e - 77cde + 10e = e(15c^2d^2 - 77cd + 10)$

×	$15cd$	-2
cd	$15c^2d^2$	$-2cd$
-5	$-75cd$	10

$\therefore 15c^2d^2e - 77cde + 10e = e(15cd - 2)(cd - 5)$

(c) $12p^2q^2r - 34pqr - 28r = 2r(6p^2q^2 - 17pq - 14)$

×	$3pq$	2
$2pq$	$6p^2q^2$	$4pq$
-7	$-21pq$	-14

$\therefore 12p^2q^2r - 34pqr - 28r = 2r(3pq + 2)(2pq - 7)$

(d) $3x^2 + 7xy + \frac{15}{4}y^2 = \frac{1}{4}(12x^2 + 28xy + 15y^2)$

×	$6x$	$5y$
$2x$	$12x^2$	$10xy$
$3y$	$18xy$	$15y^2$

$\therefore 3x^2 + 7xy + \frac{15}{4}y^2 = \frac{1}{4}(6x + 5y)(2x + 3y)$

40. $(x^2 - y)(x^2 + y)(x^4 + y^2)$
$= (x^4 - y^2)(x^4 + y^2)$
$= x^8 - y^4$

41. (a) $10^2 - 9^2 + 8^2 - 7^2 + 6^2 - 5^2 + 4^2 - 3^2 + 2^2 - 1^2$
$= (10 + 9)(10 - 9) + (8 + 7)(8 - 7) + (6 + 5)(6 - 5)$
$+ (4 + 3)(4 - 3) + (2 + 1)(2 - 1)$
$= 19 + 15 + 11 + 7 + 3$
$= 55$

(b) $2008^2 - 2007^2 + 2006^2 - 2005^2 + 2004^2 - 2003^2$
$= (2008 + 2007)(2008 - 2007)$
$+ (2006 + 2005)(2006 - 2005)$
$+ (2004 + 2003)(2004 - 2003)$
$= 2008 + 2007 + 2006 + 2005 + 2004 + 2003$
$= 12\,033$

42. (a) $a(b - c) + bc - a^2$
$= ab - ac + bc - a^2$
$= ab + bc - a^2 - ac$
$= b(a + c) - a(a + c)$
$= (b - a)(a + c)$

(b) $25x^4 + \frac{9}{4}y^2z^2 - x^2z^2 - \frac{225}{4}x^2y^2$

$= \frac{1}{4}[100x^4 + 9y^2z^2 - 4x^2z^2 - 225x^2y^2]$

$= \frac{1}{4}[100x^4 - 4x^2z^2 + 9y^2z^2 - 225x^2y^2]$

$= \frac{1}{4}[4x^2(25x^2 - z^2) + 9y^2(z^2 - 25x^2)]$

$= \frac{1}{4}[4x^2(25x^2 - z^2) - 9y^2(25x^2 - z^2)]$

$= \frac{1}{4}(4x^2 - 9y^2)(25x^2 - z^2)$

$= \frac{1}{4}(2x + 3y)(2x - 3y)(5x + z)(5x - z)$

43. (i) $\frac{1}{3}xy + \frac{1}{4}x^2y - y^2 - \frac{1}{12}x^3$

$= \frac{1}{12}[4xy + 3x^2y - 12y^2 - x^3]$

$= \frac{1}{12}[4xy - 12y^2 + 3x^2y - x^3]$

$= \frac{1}{12}[4y(x - 3y) + x^2(3y - x)]$

$= \frac{1}{12}[4y(x - 3y) - x^2(x - 3y)]$

$= \frac{1}{12}(4y - x^2)(x - 3y)$

(ii) Let $x = 22$ and $y = 9$:

$\frac{1}{3} \times 22 \times 9 + \frac{1}{4} \times 484 \times 9 - 81 - \frac{1}{12} \times 10\,648$

$= \frac{1}{12}[4(9) - 22^2][22 - 3(9)]$

$= 186\frac{2}{3}$

New Trend

44. (a) $2ax - 4ay + 3bx - 6by$
$= 2a(x - 2y) + 3b(x - 2y)$
$= (2a + 3b)(x - 2y)$

(b) $5ax - 10ay - 3bx + 6by$
$= 5a(x - 2y) - 3b(x - 2y)$
$= (5a - 3b)(x - 2y)$

(c) $8ab - 6bc + 15cd - 20ad$
$= 2b(4a - 3c) + 5d(3c - 4a)$
$= 2b(4a - 3c) - 5d(4a - 3c)$
$= (2b - 5d)(4a - 3c)$

45. (a) $27d^3 - 48d$
$= 3d(9d^2 - 16)$
$= 3d(3d + 4)(3d - 4)$

(b) $3x^2 - 75y^2$
$= 3(x^2 - 25y^2)$
$= 3(x + 5y)(x - 5y)$

Revision Test A1

1. **(a)** $2x - y = 1$ —(1)

$8x - 3y = 9$ —(2)

$(1) \times 3: 6x - 3y = 3$ —(3)

$(2) - (3): 2x = 6$

$x = 3$

Substitute $x = 3$ into (1):

$2(3) - y = 1$

$6 - y = 1$

$y = 5$

$\therefore x = 3, y = 5$

(b) $y = \frac{1}{2}x + 1$ —(1)

$x + y = 4$ —(2)

Substitute (1) into (2):

$x + \frac{1}{2}x + 1 = 4$

$\frac{3}{2}x = 3$

$x = 3 \times \frac{2}{3}$

$= 2$

Substitute $x = 2$ into (1):

$y = \frac{1}{2}(2) + 1$

$= 2$

$\therefore x = 2, y = 2$

2. **(a)** $(2x - 3)^2 - 3x(x + 7)$

$= 4x^2 - 12x + 9 - 3x^2 - 21x$

$= x^2 - 33x + 9$

(b) $3z(z + y - 4) - (y + 3)(z + 1)$

$= 3z^2 + 3yz - 12z - (yz + y + 3z + 3)$

$= 3z^2 + 3yz - 12z - yz - y - 3z - 3$

$= 3z^2 + 2yz - y - 15z - 3$

3. **(a)** $8a^2 - 12a + 12ab - 18b$

$= 4a(2a - 3) + 6b(2a - 3)$

$= (2a - 3)(4a + 6b)$

$= 2(2a - 3)(2a + 3b)$

(b) $2(m - n)^2 - 2m + 2n$

$= 2(m - n)^2 - 2(m - n)$

$= 2(m - n)(m - n - 1)$

(c) $343p^4 - 7q^2$

$= 7(49p^4 - q^2)$

$= 7(7p^2 + q)(7p^2 - q)$

4. **(i)** $18x^2 - 102x + 60 = 6(3x^2 - 17x + 10)$

$\times$	$3x$	-2
x	$3x^2$	$-2x$
-5	$-15x$	10

$\therefore 18x^2 - 102x + 60 = 6(3x - 2)(x - 5)$

(ii) Breadth of rectangle $= \frac{6(3x - 2)(x - 5)}{3x - 2}$

$= (6x - 30)$ cm

5. **(i)** $y = k\sqrt{x}$

When $x = 16, y = 20$,

$20 = k\sqrt{16}$

$= 4k$

$k = \frac{20}{4}$

$= 5$

$\therefore y = 5\sqrt{x}$

(ii) When $x = 25$,

$y = 5\sqrt{25}$

$= 25$

(iii) When $y = 8$,

$8 = 5\sqrt{x}$

$\sqrt{x} = \frac{8}{5}$

$x = \frac{64}{25}$

$= 2\frac{14}{25}$

6. **(a)** $P = \frac{k}{r^2}$

When $r = 0.1, P = 10$,

$10 = \frac{k}{0.1^2}$

$k = 10 \times 0.1^2$

$= 0.1$

$\therefore P = \frac{0.1}{r^2}$

(b) When $r = 0.2$,

$P = \frac{0.1}{0.2^2}$

$= 2.5$

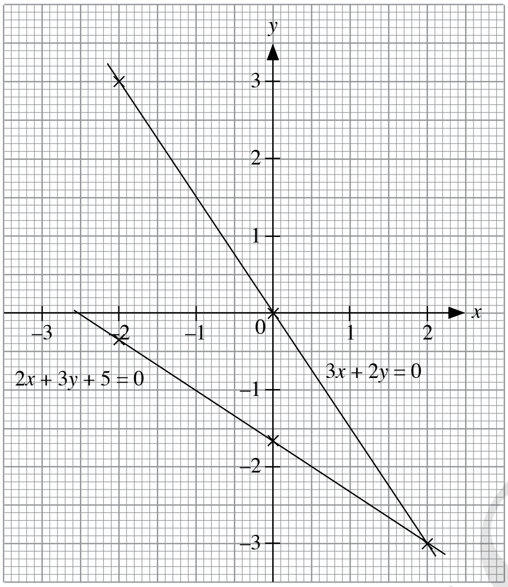
$\therefore$ The pressure exerted is 2.5 pascals.

7. $2x + 3y + 5 = 0$

x	–2	0	2
y	–0.33	–1.67	–3

$3x + 2y = 0$

x	–2	0	2
y	3	0	–3



From the graph,
$x = 2$ and $y = –3$.

8. **(a)** $R = a + bV^2$

When $V = 27, R = 281$,

$281 = a + b(27)^2$

$a + 729b = 281$ —(1)

When $V = 36, R = 344$,

$344 = a + b(36)^2$

$a + 1296b = 344$ —(2)

(b) (2) – (1): $567b = 63$

$b = \frac{63}{567}$

$= \frac{1}{9}$

Substitute $b = \frac{1}{9}$ into (1):

$a + 729\left(\frac{1}{9}\right) = 281$

$a + 81 = 281$

$a = 200$

$\therefore a = 200, b = \frac{1}{9}$

(c) **(i)** $R = 200 + \frac{1}{9}V^2$

When $V = 63$,

$R = 200 + \frac{1}{9}(63)^2$

$= 641$

$\therefore$ The resistance is 641 newtons.

(ii) When $R = 425$,

$425 = 200 + \frac{1}{9}V^2$

$\frac{1}{9}V^2 = 225$

$V^2 = 2025$

$V = \sqrt{2025}$

$= 45$

$\therefore$ The speed is 45 km/h.

Revision Test A2

1. **(a)** $7(a + 5) - 3(2 - 2a)$
$= 7a + 35 - 6 + 6a$
$= 13a + 29$

(b) $-4b(2b + 1) - 3b(5 - 3b)$
$= -8b^2 - 4b - 15b + 9b^2$
$= b^2 - 19b$

(c) $(x + 3)(x^2 + x + 2)$
$= x^3 + x^2 + 2x + 3x^2 + 3x + 6$
$= x^3 + 4x^2 + 5x + 6$

(d) $(3y + 2z)(3y - 2z) - (y - z)^2$
$= 9y^2 - 4z^2 - (y^2 - 2yz + z^2)$
$= 9y^2 - 4z^2 - y^2 + 2yz - z^2$
$= 8y^2 + 2yz - 5z^2$

2. **(a)** $2a^4 - 32b^2c^2$
$= 2(a^4 - 16b^2c^2)$
$= 2(a^2 + 4bc)(a^2 - 4bc)$

(b) $64m^2n^2 - 16mn + 1$
$= (8mn - 1)^2$

(c) $p^2 - 4q^2 + 3(p - 2q)$
$= (p + 2q)(p - 2q) + 3(p - 2q)$
$= (p + 2q + 3)(p - 2q)$

3. $2x^2 + 25x + 63$

×	$2x$	7
x	$2x^2$	$14x$
9	$18x$	63

$\therefore 2x^2 + 25x + 63 = (2x + 7)(x + 9)$
$22\,563 = 2(100)^2 + 25(100) + 63$
Let $x = 100$:
$22\,563 = 207 \times 109$
$\therefore$ The factors are 207 and 109.

4. $\frac{x}{3} = \frac{2y + 1}{5} + 2$ —(1)
$\frac{x + y}{x - y} = 2\frac{3}{4}$ —(2)
From (1),
$5x = 6y + 3 + 30$
$5x - 6y = 33$ —(3)
From (2),
$\frac{x + y}{x - y} = \frac{11}{4}$
$4x + 4y = 11x - 11y$
$15y = 7x$
$y = \frac{7}{15}x$ —(4)

Substitute (4) into (3):
$5x - 6\left(\frac{7}{15}x\right) = 33$
$5x - \frac{14}{5}x = 33$
$\frac{11}{5}x = 33$
$x = \frac{5}{11} \times 33$
$= 15$
Substitute $x = 15$ into (4):
$y = \frac{7}{15}(15)$
$= 7$
$\therefore x = 15, y = 7$

5. **(a)** $y = k(x + 3)^2$
When $x = 0$, $y = 36$,
$36 = 9k$
$k = \frac{36}{9}$
$= 4$
$\therefore y = 4(x + 3)^2$
When $x = 2$,
$y = 4(5)^2$
$= 100$

(b) $H = \frac{k}{(2p - 3)^3}$
When $p = 1$, $H = -5$,
$-5 = \frac{k}{(-1)^3}$
$k = (-5)(-1)$
$= 5$
$\therefore H = \frac{5}{(2p - 3)^3}$

(i) When $p = 2.5$,
$H = \frac{5}{(5 - 3)^3}$
$= \frac{5}{8}$

(ii) When $H = \frac{5}{27}$,
$\frac{5}{27} = \frac{5}{(2p - 3)^3}$
$(2p - 3)^3 = 27$
$2p - 3 = 3$
$2p = 6$
$p = 3$

6. **(a)** Time taken to fry 1 pancake $= \frac{8}{12}$

$= \frac{2}{3}$ minutes

Time taken to fry 50 pancakes $= \frac{2}{3} \times 50$

$= 33\frac{1}{3}$ minutes

(b)

	Number of men		Number of design projects		Number of hours	
	8		12		9	
$\div 8$	↓				↓	$\times 8$
	1		12		72	
$\times 6$	↓				↓	$\div 6$
	6		12		12	
			↓	$\times \frac{8}{3}$	↓	$\times \frac{8}{3}$
	6		32		32	

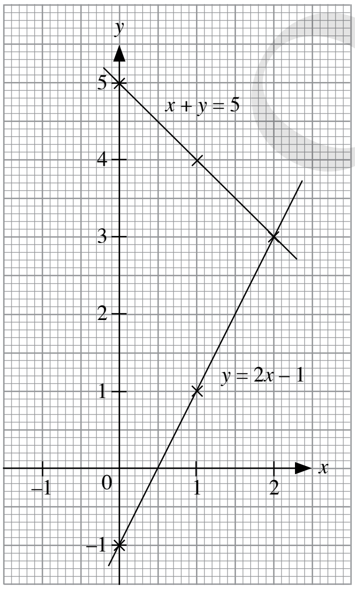
$\therefore$ 6 men take 32 hours to complete 32 design projects.

7. $x + y = 5$

x	0	1	2
y	5	4	3

$y = 2x - 1$

x	0	1	2
y	–1	1	3



From the graph,
$x = 2$ and $y = 3$.

8. $x + y = 14\frac{1}{2}$ —(1)

$40x + 50y = 660$ —(2)

From (1),

$y = 14\frac{1}{2} - x$ —(3)

From (2),

$4x + 5y = 66$ —(4)

Substitute (3) into (4):

$4x + 5\left(14\frac{1}{2} - x\right) = 66$

$4x + 72\frac{1}{2} - 5x = 66$

$x = 6\frac{1}{2}$

Substitute $x = 6\frac{1}{2}$ into (3):

$y = 14\frac{1}{2} - 6\frac{1}{2}$

$= 8$

$\therefore$ Machine A was used for $6\frac{1}{2}$ hours and Machine B was used for 8 hours.

Chapter 5 Quadratic Equations and Graphs

Basic

1. **(a)** $a(a - 6) = 0$

$a = 0$ or $a = 6$

(b) $b(b + 4) = 0$

$b = 0$ or $b = -4$

(c) $3c(c - 5) = 0$

$c = 0$ or $c = 5$

(d) $5d(3d + 2) = 0$

$d = 0$ or $d = -\frac{2}{3}$

(e) $-7e(9e - 4) = 0$

$e = 0$ or $e = \frac{4}{9}$

(f) $-\frac{8}{3}f(7 - 5f) = 0$

$f = 0$ or $f = \frac{7}{5}$

$= 1\frac{2}{5}$

2. **(a)** $(a - 5)(2a - 7) = 0$

$a = 5$ or $a = \frac{7}{2}$

$= 3\frac{1}{2}$

(b) $(7c - 5)(2 - 9c) = 0$

$c = \frac{5}{7}$ or $c = \frac{2}{9}$

(c) $(6 - 5d)(15 + 11d) = 0$

$d = \frac{6}{5}$ or $d = -\frac{15}{11}$

$= 1\frac{1}{5}$ $\quad = -1\frac{4}{11}$

(d) $\frac{1}{2}(e + 1)(2e - 5) = 0$

$e = -1$ or $e = \frac{5}{2}$

$= 2\frac{1}{2}$

(e) $-\frac{3}{4}(5f - 4)(1 + f) = 0$

$f = \frac{4}{5}$ or $f = -1$

3. **(a)** $a^2 + 7a = 0$

$a(a + 7) = 0$

$a = 0$ or $a = -7$

(b) $b^2 - 16b = 0$

$b(b - 16) = 0$

$b = 0$ or $b = 16$

(c) $2c^2 + 5c = 0$

$c(2c + 5) = 0$

$c = 0$ or $c = -\frac{5}{2}$

$= -2\frac{1}{2}$

(d) $3d^2 - 12d = 0$

$3d(d - 4) = 0$

$d = 0$ or $d = 4$

(e) $7e - 8e^2 = 0$

$e(7 - 8e) = 0$

$e = 0$ or $e = \frac{7}{8}$

(f) $-8f - 16f^2 = 0$

$-8f(1 + 2f) = 0$

$f = 0$ or $f = -\frac{1}{2}$

4. **(a)** $a^2 + 10a + 25 = 0$

$(a + 5)^2 = 0$

$a = -5$

(b) $b^2 - 20b + 100 = 0$

$(b - 10)^2 = 0$

$b = 10$

(c) $c^2 - 49 = 0$

$(c + 7)(c - 7) = 0$

$c = -7$ or $c = 7$

(d) $9d^2 + 48d + 64 = 0$

$(3d + 8)^2 = 0$

$d = -\frac{8}{3}$

$= -2\frac{2}{3}$

(e) $36e^2 - 132e + 121 = 0$

$(6e - 11)^2 = 0$

$e = \frac{11}{6}$

$= 1\frac{5}{6}$

(f) $2f^2 - 288 = 0$

$f^2 - 144 = 0$

$(f + 12)(f - 12) = 0$

$f = -12$ or $f = 12$

5. (a) $a^2 + 10a + 24 = 0$
$(a + 4)(a + 6) = 0$
$a = -4$ or $a = -6$

(b) $5b^2 - 17b + 6 = 0$
$(b - 3)(5b - 2) = 0$
$b = 3$ or $b = \frac{2}{5}$

(c) $2c^2 + 7c - 4 = 0$
$(2c - 1)(c + 4) = 0$
$c = \frac{1}{2}$ or $c = -4$

(d) $12d^2 - d - 6 = 0$
$(4d - 3)(3d + 2) = 0$
$d = \frac{3}{4}$ or $d = -\frac{2}{3}$

(e) $3 - 4e - 7e^2 = 0$
$7e^2 + 4e - 3 = 0$
$(e + 1)(7e - 3) = 0$
$e = -1$ or $e = \frac{3}{7}$

(f) $8 - 5f^2 - 18f = 0$
$5f^2 + 18f - 8 = 0$
$(f + 4)(5f - 2) = 0$
$f = -4$ or $f = \frac{2}{5}$

6. Let the number be x.
$x + 2x^2 = 36$
$2x^2 + x - 36 = 0$
$(2x + 9)(x - 4) = 0$
$x = -\frac{9}{2}$ or $x = 4$
$= -4\frac{1}{2}$
(rejected)
$\therefore$ The number is 4.

7. Let the numbers be x and $x + 5$.
$x^2 + (x + 5)^2 = 193$
$x^2 + x^2 + 10x + 25 = 193$
$2x^2 + 10x - 168 = 0$
$x^2 + 5x - 84 = 0$
$(x + 12)(x - 7) = 0$
$x = -12$ or $x = 7$
(rejected) $x + 5 = 12$
$\therefore$ The numbers are 7 and 12.

8. Let the numbers be x and $x + 3$.
$x(x + 3) = 154$
$x^2 + 3x - 154 = 0$
$(x + 14)(x - 11) = 0$
$x = -14$ or $x = 11$
$x + 3 = -11$ $x + 3 = 14$
$\therefore$ The numbers are –14 and –11 or 11 and 14.

9. (i) $(4x + 7)(5x - 4) = 209$
$20x^2 - 16x + 35x - 28 = 209$
$20x^2 + 19x - 237 = 0$
$(20x + 79)(x - 3) = 0$
$x = -\frac{79}{20}$ or $x = 3$
$= -3\frac{19}{20}$
(rejected)
$\therefore x = 3$

(ii) Perimeter of rectangle
$= 2[4(3) + 7 + 5(3) - 4]$
$= 60$ cm

10. $\frac{1}{2}(x + 3 + x + 9)(3x - 4) = 80$
$\frac{1}{2}(2x + 12)(3x - 4) = 80$
$(x + 6)(3x - 4) = 80$
$3x^2 - 4x + 18x - 24 = 80$
$3x^2 + 14x - 104 = 0$
$(x - 4)(3x + 26) = 0$
$x = 4$ or $x = -\frac{26}{3}$
$= -8\frac{2}{3}$
(rejected)
$\therefore x = 4$

11. (a) When $x = 1$, $y = a$,

$a = 1^2 - 5(1) + 5$

$= 1$

When $x = 3$, $y = b$,

$b = 3^2 - 5(3) + 5,$

$= -1$

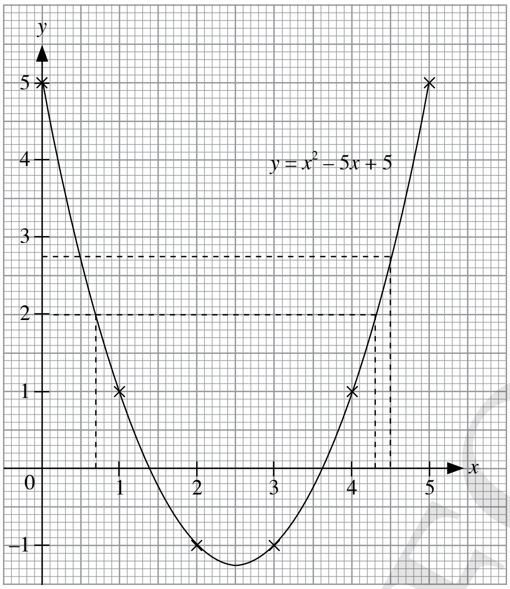
When $x = 4$, $y = c$,

$c = 4^2 - 5(4) + 5$

$= 1$

$\therefore a = 1, b = -1, c = 1$

(b)



(c) (i) When $x = 4.5$, $y = 2.75$

(ii) When $y = 2$, $x = 0.7$ or $x = 4.3$

12. **(a)** When $x = -2$, $y = a$,

$a = (-2 + 3)(-2 - 2)$

$= -4$

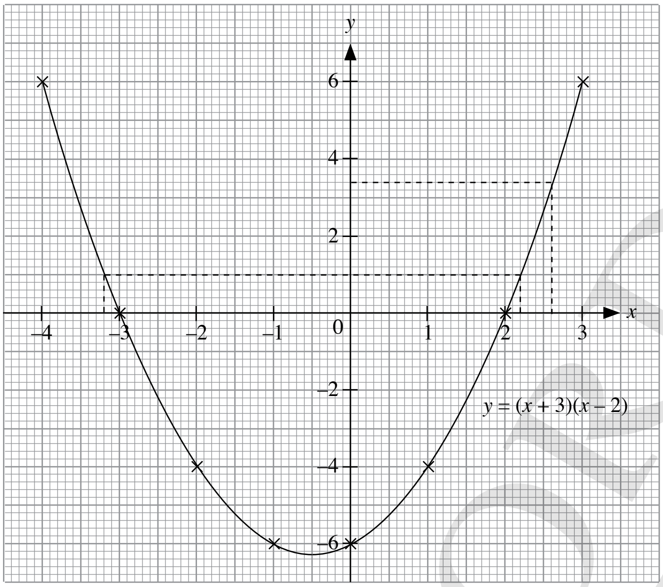
When $x = -1$, $y = b$.

$b = (-1 + 3(-1 - 2)$

$= -6$

$\therefore\ a = -4, b = -6$

(b)



(c) **(i)** When $x = 2.6$, $y = 3.4$

(ii) When $y = 1$, $x = -3.2$ or $x = 2.2$

13. **(a)** When $x = -1$, $y = a$,

$a = (-1)^2 + 1$

$= 2$

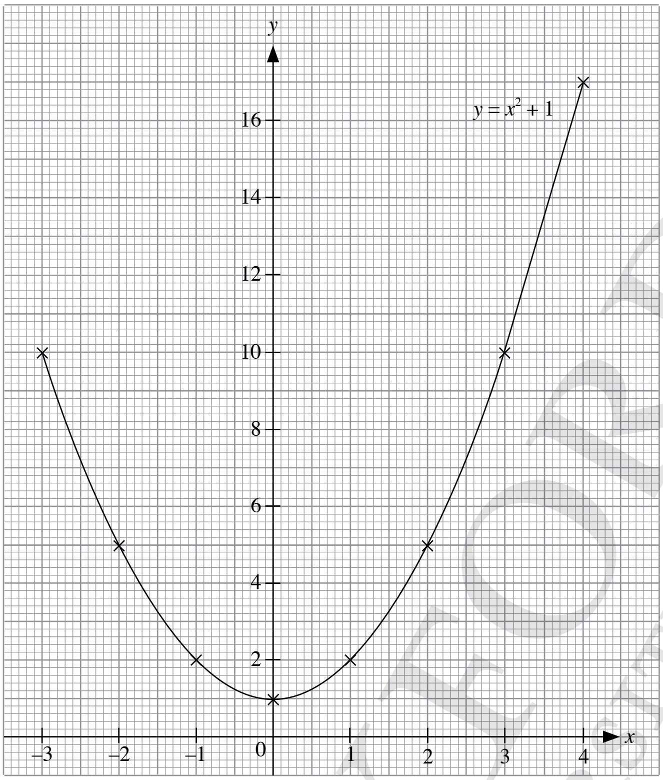
When $x = 3$, $y = b$,

$b = (3)^2 + 1$

$= 10$

$\therefore a = 2, b = 10$

(b)



(c) The minimum point is (0, 1).

(d) The equation of line of symmetry of the graph is $x = 0$.

14. **(a)** When $x = -1$, $y = p$,

$p = (-1)^2 - 4(-1)$

$= 5$

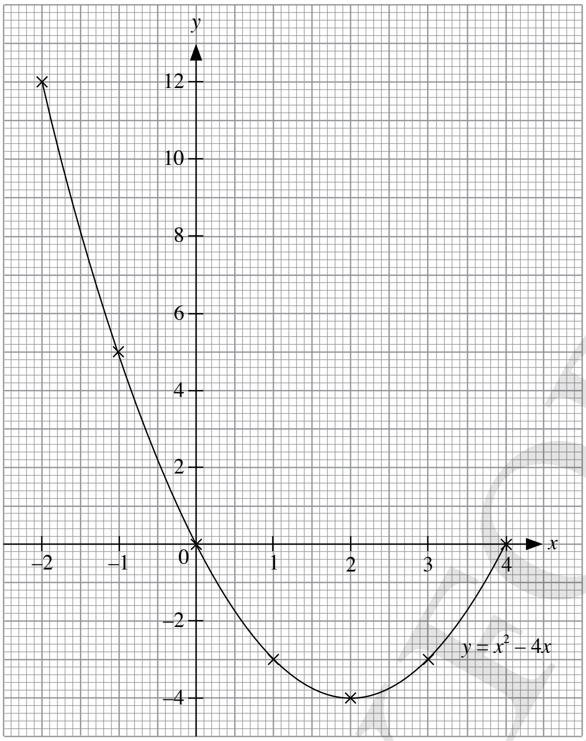
When $x = 3$, $y = q$,

$q = 3^2 - 4(3)$

$= -3$

$\therefore p = 5, q = -3$

(b)



(c) The minimum point is (2, –4).

(d) The equation of line of symmetry of the graph is $x = 2$.

15. **(a)** When $x = 2$, $y = a$,

$a = 6(2) - 2^2$

$= 8$

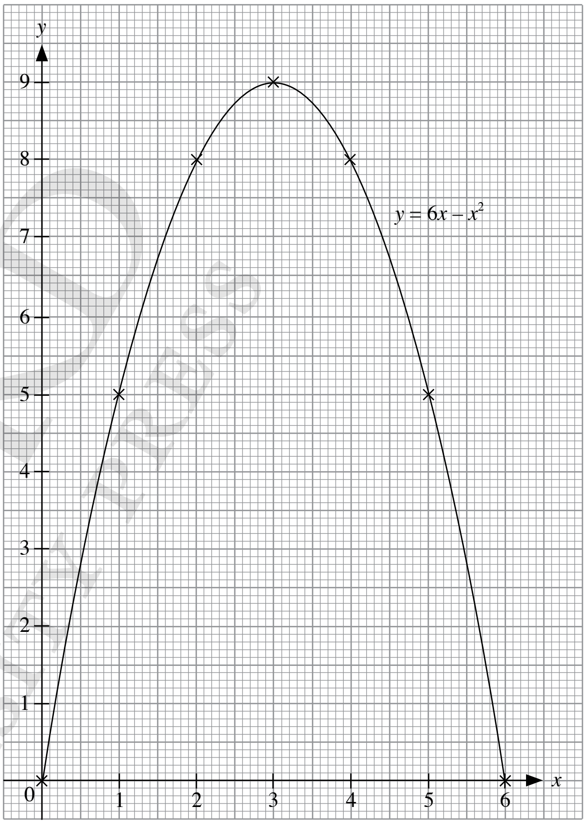
When $x = 5$, $y = b$,

$b = 6(5) - 5^2$

$= 5$

$\therefore a = 8, b = 5$

(b)



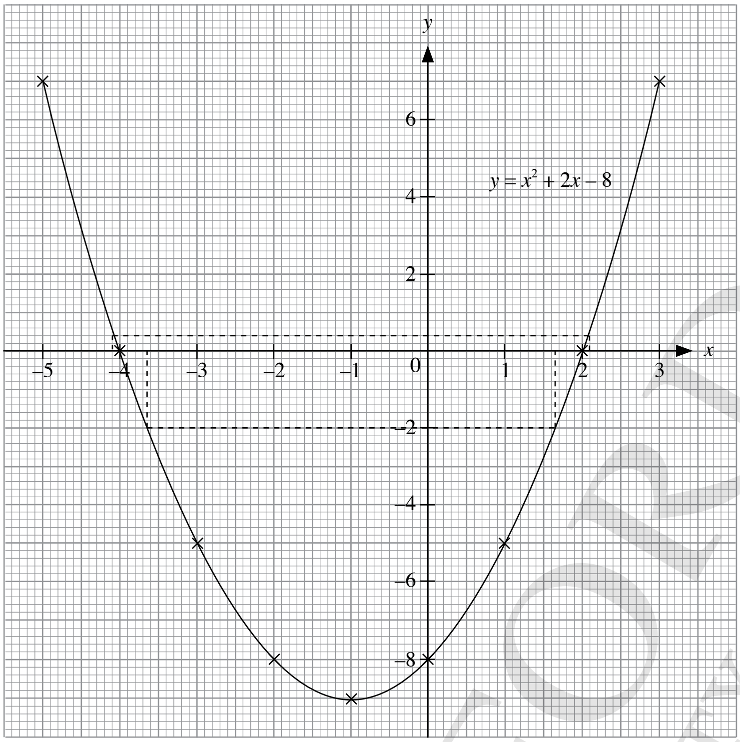
(c) (3, 9), maximum point

(d) The equation of the line of symmetry of the graph is $x = 3$.

16. (a)

x	–5	–4	–3	–2	–1	0	1	2	3
y	7	0	–5	–8	–9	–8	–5	0	7

(b)



(c) **(i)** When $y = 0$, $x = -4$ or $x = 2$

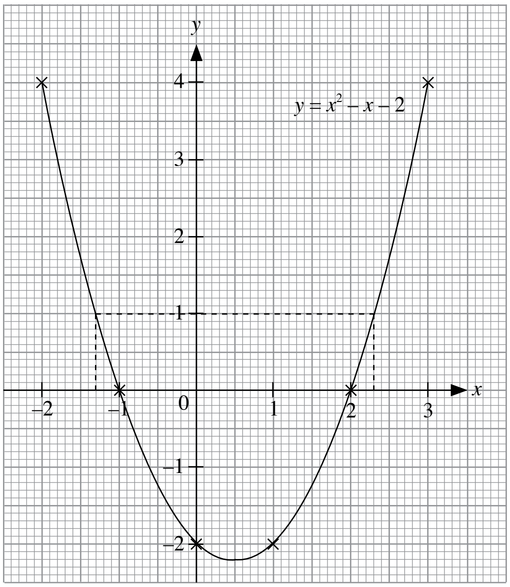
(ii) When $y = -2$, $x = -3.65$ or $x = 1.65$

(iii) When $y = \frac{1}{2}$, $x = -4.1$ or $x = 2.1$

(d) The equation of the line of symmetry of the graph is $x = -1$.

(e) Minimum value of $y = -9$,
Minimum value of y occurs when $x = -1$.

17. (a)



(b) (i) When $y = 1$, $x = -1.3$ or $x = 2.3$.

(ii) Minimum value of y occurs when $x = 0.5$.

18. (i) $x = -2$ or $x = -1$

(ii) $x = \dfrac{-2 + (-1)}{2}$

$= -1.5$

$\therefore$ Equation of line of symmetry is $x = -1.5$

19. (i) G_1: $y = -x^2$

G_2: $y = -x^2 - 2$

(ii) G_3: $y = -x^2 - 4$

Equation of line of symmetry is $x = 0$

Coordinates of maximum point are $(0, -4)$

Intermediate

20. (a)

$$2a^2 = 3a + 14$$
$$2a^2 - 3a - 14 = 0$$
$$(2a - 7)(a + 2) = 0$$
$$a = \frac{7}{2} \quad \text{or} \quad a = -2$$
$$= 3\frac{1}{2}$$

(b)

$$12b^2 - 12 = 7b$$
$$12b^2 - 7b - 12 = 0$$
$$(3b - 4)(4b + 3) = 0$$
$$b = \frac{4}{3} \quad \text{or} \quad b = -\frac{3}{4}$$
$$= 1\frac{1}{3}$$

(c)

$$c^2 + 4 = 8c - 8$$
$$c^2 - 8c + 12 = 0$$
$$(c - 6)(c - 2) = 0$$
$$c = 6 \quad \text{or} \quad c = 2$$

(d)

$$d^2 = \frac{d + 15}{6}$$
$$6d^2 = d + 15$$
$$6d^2 - d - 15 = 0$$
$$(3d - 5)(2d + 3) = 0$$
$$d = \frac{5}{3} \quad \text{or} \quad d = -\frac{3}{2}$$
$$= 1\frac{2}{3} \qquad = -1\frac{1}{2}$$

(e)

$$e(2e + 5) = 3$$
$$2e^2 + 5e - 3 = 0$$
$$(2e - 1)(e + 3) = 0$$
$$e = \frac{1}{2} \quad \text{or} \quad e = -3$$

(f)

$$3f(3f - 1) = 20$$
$$9f^2 - 3f - 20 = 0$$
$$(3f - 5)(3f + 4) = 0$$
$$f = \frac{5}{3} \quad \text{or} \quad f = -\frac{4}{3}$$
$$= 1\frac{2}{3} \qquad = -1\frac{1}{3}$$

(g)

$$9g^2 = 6(g + 20)$$
$$3g^2 = 2(g + 20)$$
$$= 2g + 40$$
$$3g^2 - 2g - 40 = 0$$
$$(g - 4)(3g + 10) = 0$$
$$g = 4 \quad \text{or} \quad g = -\frac{10}{3}$$
$$= -3\frac{1}{3}$$

(h)

$$(6h + 5)(h - 1) = -3$$
$$6h^2 - 6h + 5h - 5 = -3$$
$$6h^2 - h - 2 = 0$$
$$(3h - 2)(2h + 1) = 0$$
$$h = \frac{2}{3} \quad \text{or} \quad h = -\frac{1}{2}$$

21. Let the numbers be $2x$, $2x + 2$ and $2x + 4$.

Sum $= 2x + 2x + 2 + 2x + 4$

$= 6x + 6$

$= 6(x + 1)$, which is divisible by 6

22. Let the numbers be $2x + 1$, $2x + 3$

$2x + 5$ and $2x + 7$.

Sum $= 2x + 1 + 2x + 3 + 2x + 5 + 2x + 7$

$= 8x + 16$

$= 8(x + 2)$, which is divisible by 8

23. Let the integers be x and $x + 2$.

$$x^2 + (x + 2)^2 = 340$$
$$x^2 + x^2 + 4x + 4 = 340$$
$$2x^2 + 4x - 336 = 0$$
$$x^2 + 2x - 168 = 0$$
$$(x - 12)(x + 14) = 0$$
$$x = 12 \quad \text{or} \quad x = -14$$
$$x + 2 = 14 \quad\quad x + 2 = -12$$

$\therefore$ The integers are 12 and 14 or –14 and –12.

24. Let the integers be $x - 1$, x and $x + 1$.

$$(x - 1)^2 + x^2 + (x + 1)^2 = 245$$
$$x^2 - 2x + 1 + x^2 + x^2 + 2x + 1 = 245$$
$$3x^2 = 243$$
$$x^2 = 81$$
$$x = 9$$
$$x + 1 = 10$$

$\therefore$ The largest number is 10.

25. Let the numbers be x and $x + 2$.

$$(x + x + 2)^2 - [x^2 + (x + 2)^2] = 126$$
$$(2x + 2)^2 - (x^2 + x^2 + 4x + 4) = 126$$
$$4x^2 + 8x + 4 - 2x^2 - 4x - 4 = 126$$
$$2x^2 + 4x - 126 = 0$$
$$x^2 + 2x - 63 = 0$$
$$(x + 9)(x - 7) = 0$$
$$x = -9 \quad \text{or} \quad x = 7$$
$$\text{(rejected)} \quad\quad x + 2 = 9$$

$\therefore$ The numbers are 7 and 9.

26. $S = \frac{1}{2}n(n + 1)$

When $S = 325$,

$$\frac{1}{2}n(n + 1) = 325$$
$$n^2 + n = 650$$
$$n^2 + n - 650 = 0$$
$$(n + 26)(n - 25) = 0$$
$$n = -26 \quad \text{or} \quad n = 25$$
$$\text{(rejected)}$$

$\therefore$ 25 integers must be taken.

27. Let Huixian's age be x years.

$$x(x + 5) = 234$$
$$x^2 + 5x - 234 = 0$$
$$(x - 13)(x + 18) = 0$$
$$x = 13 \quad \text{or} \quad x = -18 \text{ (rejected)}$$

$\therefore$ Huixian's current age is 13 years.

28.
$$\frac{8p + 5}{5p} = \frac{3p + 4}{2p}$$
$$16p^2 + 10p = 15p^2 + 20p$$
$$p^2 - 10p = 0$$
$$p(p - 10) = 0$$
$$p = 0 \quad \text{or} \quad p = 10$$

$\therefore p = 10$

29. (i)
$$(x + 2)^2 + (5x - 1)^2 = (5x)^2$$
$$x^2 + 4x + 4 + 25x^2 - 10x + 1 = 25x^2$$
$$x^2 - 6x + 5 = 0 \text{ (shown)}$$

(ii)
$$x^2 - 6x + 5 = 0$$
$$(x - 1)(x - 5) = 0$$
$$x = 1 \quad \text{or} \quad x = 5$$

(iii) Perimeter $= x + 2 + 5x - 1 + 5x$

$= 11x + 1$

Area $= \frac{1}{2}(x + 2)(5x - 1)$

When $x = 1$,

Perimeter = 12 cm

Area = 6 cm^2

When $x = 5$,

Perimeter = 56 cm

Area = 84 cm^2

30. (i)
$$(3x + 1)(2x + 1) = 117$$
$$6x^2 + 3x + 2x + 1 = 117$$
$$6x^2 + 5x - 116 = 0$$
$$(x - 4)(6x + 29) = 0$$
$$x = 4 \quad \text{or} \quad x = -\frac{29}{6}$$
$$= -4\frac{5}{6}$$

$\therefore x = 4$

(ii) Perimeter $= 2(3x + 1 + 2x + 1)$

$= 2(5x + 2)$

When $x = 4$,

Perimeter = 44 cm

31. (i)
$$5x(4x + 2) = (6x + 3)(3x + 1)$$
$$20x^2 + 10x = 18x^2 + 6x + 9x + 3$$
$$2x^2 - 5x - 3 = 0$$
$$(2x + 1)(x - 3) = 0$$
$$x = -\frac{1}{2} \quad \text{or} \quad x = 3$$

$\therefore x = 3$

(ii) Perimeter of $A = 2(5x + 4x + 2)$

$= 2(9x + 2)$

Perimeter of $B = 2(6x + 3 + 3x + 1)$

$= 2(9x + 4)$

$\therefore$ B has a greater perimeter.

32. (i) Let the breadth of the original rectangle be x cm.

$$x(x - 8) - \frac{x}{2}(x - 8 + 6) = 36$$
$$2x(x - 8) - x(x - 2) = 72$$
$$2x^2 - 16x - x^2 + 2x = 72$$
$$x^2 - 14x - 72 = 0$$
$$(x - 18)(x + 4) = 0$$
$$x = 18 \quad \text{or} \quad x = -4$$

$\therefore$ The length of the original rectangle is 18 cm.

(ii) Perimeter of original rectangle
$= 2(18 + 18 - 8)$
$= 56$ cm

33. (i) Let the length of the shorter side be x m.

$$x(x + 7) = 450$$
$$x^2 + 7x - 450 = 0$$
$$(x - 18)(x + 25) = 0$$
$$x = 18 \quad \text{or} \quad x = -25$$

$\therefore$ The length of the shorter side is 18 m.

(ii) Perimeter of field $= 2(18 + 18 + 7)$
$= 86$ m

34. Let the length of the smaller field be $3x$ m.

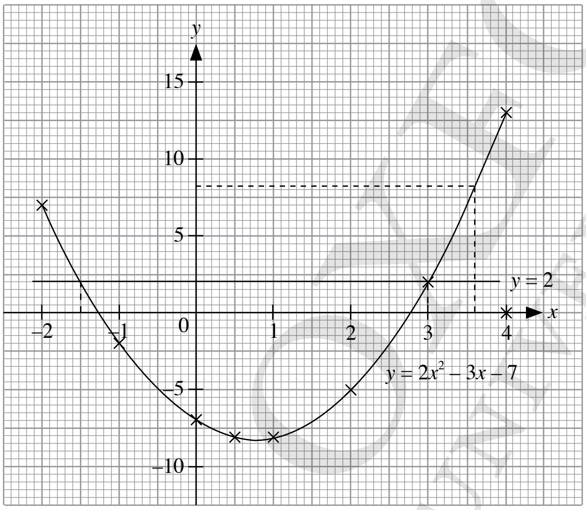
$$(5x)^2 - (3x)^2 = 576$$
$$25x^2 - 9x^2 = 576$$
$$16x^2 = 576$$
$$x^2 = 36$$
$$x = 6$$

Area of smaller field $= [3(6)]^2$
$= 324 \text{ m}^2$

35. (a)

x	–2	–1	0	0.5	1	2	3	4
y	7	–2	–7	–8	–8	–5	2	13

(b)



(c) $y = 8.1$

(d) $2x^2 - 3x - 7 = 2$

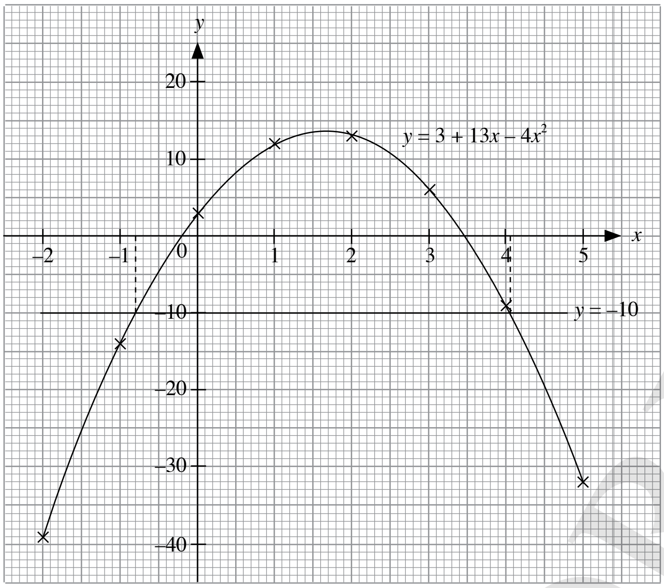
Draw $y = 2$.

$x = -1.5$ or $x = 3$

36. (a)

x	–2	–1	0	1	2	3	4	5
y	–39	–14	3	12	13	6	–9	–32

(b)



(c) $x = -0.2$ or $x = 3.5$

(d) $x = -0.8$ or $x = 4.05$

37. (i) $y = (x + 2)(x - 4)$

When $y = 0$,

$(x + 2)(x - 4) = 0$

$x = -2$ or $x = 4$

$A(-2, 0)$, $C(4, 0)$

When $x = 0$,

$y = (0 + 2)(0 - 4)$

$= -8$

$B(0, -8)$

$\therefore$ $A(-2, 0)$, $B(0, -8)$, $C(4, 0)$

(ii) $x = \dfrac{-2 + 4}{2}$

$= 1$

$\therefore$ Equation of line of symmetry is $x = 1$.

(iii) When $x = 1$,

$y = (1 + 2)(1 - 4)$

$= -9$

$\therefore$ Coordinates of minimum point are $(1, -9)$.

38. (i) $y = 12 + 4x - x^2$

$= (2 + x)(6 - x)$

When $y = 0$,

$(2 + x)(6 - x) = 0$

$x = -2$ or $x = 6$

$A(-2, 0)$, $D(6, 0)$

When $x = 0$,

$y = (2 + 0)(6 - 0)$

$= 12$

$B(0, 12)$

$x = \dfrac{-2 + 6}{2}$

$= 2$

When $x = 2$,

$y = (2 + 2)(6 - 2)$

$= 16$

$C(2, 16)$

$\therefore$ $A(-2, 0)$, $B(0, 12)$, $C(2, 16)$, $D(6, 0)$

(ii) Equation of line of symmetry is $x = 2$

39. **(i)** $y = x(4 - x)$

When $y = 0$,

$x(4 - x) = 0$

$x = 0$ or $x = 4$

$\therefore R(4, 0)$

(ii) Substitute $x = -1$, $y = k$ into $y = x(4 - x)$:

$k = -1[4 - (-1)]$

$= -5$

(iii) $x = \frac{0 + 4}{2}$

$= 2$

$\therefore$ Equation of line of symmetry is $x = 2$.

When $x = 2$,

$y = 2(4 - 2)$

$= 4$

$\therefore M(2, 4)$

(iv) Substitute $x = 3$, $y = p$ into $y = x(4 - x)$:

$p = 3(4 - 3)$

$= 3$

$m = \frac{3 - 0}{3 - 0}$

$= 1$

$\therefore p = 3, m = 1$

Advanced

40. **(i)** Substitute $x = 3$ into $2x^2 + px = 15$:

$2(3)^2 + p(3) = 15$

$18 + 3p = 15$

$3p = -3$

$p = -1$

(ii) $2x^2 - x = 15$

$2x^2 - x - 15 = 0$

$(2x + 5)(x - 3) = 0$

$x = -\frac{5}{2}$ or $x = 3$

$= -2\frac{1}{2}$

$\therefore$ The other solution is $x = -2\frac{1}{2}$.

41. $x^2 = 12(x - 3) + 1$

$= 12x - 36 + 1$

$x^2 - 12x + 35 = 0$

$(x - 5)(x - 7) = 0$

$x = 5$ or $x = 7$

42. Case I is true.

Case II is not true.

Case III is not true.

Case IV is not true.

43. **(d)** is true.

New Trend

44. $(5b + 9)(8 - 3b) = 0$

$b = -\frac{9}{5}$ or $b = \frac{8}{3}$

$= -1\frac{4}{5}$ $\quad = 2\frac{2}{3}$

45. **(i)** $T_{14} = \frac{14(15)}{6}$

$= 35$

(ii) $\frac{n(n+1)}{6} = 57$

$n^2 + n = 342$

$n^2 + n - 342 = 0$

$(n + 19)(n - 18) = 0$

$n = -19$ (rejected) or $n = 18$

$\therefore$ The 18^{th} term has the value 57.

46. **(a)** **(i)** Next line is the 6^{th} line: $6^2 - 6 = 30$.

(ii) 8^{th} line: $8^2 - 8 = 56$

(iii) From the number pattern, we observe that

$1^2 - 1 = 1(1 - 1)$

$2^2 - 2 = 2(2 - 1)$

$3^2 - 3 = 3(3 - 1)$

$4^2 - 4 = 4(4 - 1)$

$5^2 - 5 = 5(5 - 1)$

$\vdots$

n^{th} line: $n^2 - n = n(n - 1)$

(b) **(i)** $139^2 - 139 = 139(139 - 1) = 19\ 182$

(ii) $x^2 - x = 2450$

$x(x - 1) = 2450$

We need to find the product of two numbers where the difference of the two numbers is 1 that gives 2450. By trial and error, $2450 = 50 \times 51$ and $x = 50$.

(c) $P_n = 3n - 8$

(d) $\frac{3n - 8}{n^2 - n} = \frac{1}{3}$

$9n - 24 = n^2 - n$

$n^2 - 10n + 24 = 0$

$(n - 4)(n - 6) = 0$

$\therefore n = 4$ or $n = 6$

Chapter 6 Algebraic Fractions and Formulae

Basic

1. (a) $\dfrac{45a^2b}{3a} = 15ab$

(b) $\dfrac{35c^7d^3}{7cd^4} = \dfrac{5c^6}{d}$

(c) $\dfrac{64ef^3g^4}{24e^3fg^2} = \dfrac{8f^2g^2}{3e^2}$

(d) $\dfrac{8h^3jk^4}{(2hjk)^4} = \dfrac{8h^3jk^4}{16h^4j^4k^4}$

$= \dfrac{1}{2hj^3}$

(e) $\dfrac{8mn^2x^3}{(4mnx)^2} = \dfrac{8mn^2x^3}{16m^2n^2x^2}$

$= \dfrac{x}{2m}$

(f) $\dfrac{9p^3q^4r}{(3pq^2r)^3} = \dfrac{9p^3q^4r}{27p^3q^6r^3}$

$= \dfrac{1}{3q^2r^2}$

2. (a) $\dfrac{(5a^3b^4)^3}{25ab^3} = \dfrac{125a^9b^{12}}{25ab^3}$

$= 5a^8b^9$

(b) $\dfrac{(4c^2)^2d^3e}{8cde^4} = \dfrac{16c^4d^3e}{8cde^4}$

$= \dfrac{2c^3d^2}{e^3}$

(c) $\dfrac{(7f^2g)^2h^4}{21gh} = \dfrac{49f^4g^2h^4}{21gh}$

$= \dfrac{7f^4gh^3}{3}$

(d) $\dfrac{(2jkl^2)^4}{8j^2k^3} = \dfrac{16j^4k^4l^8}{8j^2k^3}$

$= 2j^2kl^8$

3. (a) $\dfrac{4a + 8b}{6a + 12b} = \dfrac{4(a + 2b)}{6(a + 2b)}$

$= \dfrac{2}{3}$

(b) $\dfrac{8c^2 - 16cd}{5c - 10d} = \dfrac{8c(c - 2d)}{5(c - 2d)}$

$= \dfrac{8c}{5}$

(c) $\dfrac{e^2 + ef}{ef + f^2} = \dfrac{e(e + f)}{f(e + f)}$

$= \dfrac{e}{f}$

(d) $\dfrac{gh - h^2}{(g - h)^2} = \dfrac{h(g - h)}{(g - h)^2}$

$= \dfrac{h}{g - h}$

(e) $\dfrac{j^2 - jk}{k^2 - jk} = \dfrac{j(j - k)}{k(k - j)}$

$= \dfrac{j(j - k)}{-k(j - k)}$

$= -\dfrac{j}{k}$

(f) $\dfrac{4mn - 8m^2}{6m^2} = \dfrac{4m(n - 2m)}{6m^2}$

$= \dfrac{2(n - 2m)}{3m}$

4. (a) $\dfrac{a^2 - b^2}{(a - b)^2} = \dfrac{(a + b)(a - b)}{(a - b)^2}$

$= \dfrac{a + b}{a - b}$

(b) $\dfrac{c^2 - 4c}{c^2 - 16} = \dfrac{c(c - 4)}{(c + 4)(c - 4)}$

$= \dfrac{c}{c + 4}$

(c) $\dfrac{d^2 + 4d + 4}{d^2 + 2d} = \dfrac{(d + 2)^2}{d(d + 2)}$

$= \dfrac{d + 2}{d}$

(d) $\dfrac{e - 2}{e^2 - 5e + 6} = \dfrac{e - 2}{(e - 2)(e - 3)}$

$= \dfrac{1}{e - 3}$

(e) $\dfrac{5f - 15}{3f^2 - 13f + 12} = \dfrac{5(f - 3)}{(3f - 4)(f - 3)}$

$= \dfrac{5}{3f - 4}$

(f) $\dfrac{gh + h}{g^2 + 7g + 6} = \dfrac{h(g + 1)}{(g + 1)(g + 6)}$

$= \dfrac{h}{g + 6}$

5. (a) $\dfrac{6ab^2}{7c} \times \dfrac{56a^3}{48bc} = \dfrac{a^4b}{c^2}$

(b) $\dfrac{5a^2b^4}{3bc^4} \times \dfrac{9b^2}{10a^3} = \dfrac{3b^5}{2ac^4}$

(c) $\dfrac{4d^2e}{3ef} \times \dfrac{27e^2f^3}{16d^4} = \dfrac{9e^2f^2}{4d^2}$

(d) $\dfrac{16d^2e^4}{7ef^2} \times \dfrac{21e^4f^3}{24d^3e^3} = \dfrac{2e^4f}{d}$

(e) $\dfrac{9h^3x^2}{4ky^2} \times \dfrac{5k^2y^4}{12h^2y} = \dfrac{15hkx^2y}{16}$

(f) $\dfrac{16xy^3}{15abc^2} \times \dfrac{25a^3bc}{8x^2yz} = \dfrac{10a^2y^2}{3cxz}$

6. (a) $\dfrac{2a^2b}{3c} \div \dfrac{3abc}{8c^3}$

$= \dfrac{2a^2b}{3c} \times \dfrac{8c^3}{3abc}$

$= \dfrac{16ac}{9}$

(b) $\dfrac{18d^4e^3}{14d^2e} \div \dfrac{27de^5}{21ef^2}$

$= \dfrac{18d^4e^3}{14d^2e} \times \dfrac{21ef^2}{27de^5}$

$= \dfrac{df^2}{e^2}$

(c) $\dfrac{14a^3b}{6xy} \div \dfrac{21abc}{12x^2y^3}$

$= \dfrac{14a^3b}{6xy} \times \dfrac{12x^2y^3}{21abc}$

$= \dfrac{4a^2xy^2}{3c}$

(d) $\dfrac{81a^3x^3}{16bxy} \div \dfrac{63ax^2}{24b^2y^3}$

$= \dfrac{81a^3x^3}{16bxy} \times \dfrac{24b^2y^3}{63ax^2}$

$= \dfrac{27a^2by^2}{14}$

7. **(a)** $\dfrac{4(a+3b)}{a-3b} \times \dfrac{3(a-3b)}{25(a+3b)}$

$= \dfrac{12}{25}$

(b) $\dfrac{7c-28d}{e^2} \times \dfrac{e}{2c-8d}$

$= \dfrac{7(c-4d)}{e^2} \times \dfrac{e}{2(c-4d)}$

$= \dfrac{7}{2e}$

(c) $\dfrac{3(g+h)}{10f} \div \dfrac{8g+8h}{5f^3}$

$= \dfrac{3(g+h)}{10f} \times \dfrac{5f^3}{8(g+h)}$

$= \dfrac{3f^2}{16}$

(d) $\dfrac{2(j+k+5)}{9} \div (3j+3k+15)$

$= \dfrac{2(j+k+5)}{9} \times \dfrac{1}{3(j+k+5)}$

$= \dfrac{2}{27}$

8. **(a)** $\dfrac{3}{5a} + \dfrac{1}{4a}$

$= \dfrac{12+5}{20a}$

$= \dfrac{17}{20a}$

(b) $\dfrac{1}{2b} + \dfrac{3}{4b} - \dfrac{1}{6b}$

$= \dfrac{6+9-2}{12b}$

$= \dfrac{13}{12b}$

(c) $\dfrac{2}{7c} - \dfrac{1}{7d}$

$= \dfrac{2d-c}{7cd}$

(d) $\dfrac{4ef}{3g} + \dfrac{ef}{g} - \dfrac{2ef}{5g}$

$= \dfrac{20ef+15ef-6ef}{15g}$

$= \dfrac{29ef}{15g}$

(e) $\dfrac{h+j}{2k} + \dfrac{3h-j}{3k} - \dfrac{j-h}{5k}$

$= \dfrac{15(h+j)+10(3h-j)-6(j-h)}{30k}$

$= \dfrac{15h+15j+30h-10j-6j+6h}{30k}$

$= \dfrac{51h-j}{30k}$

(f) $\dfrac{2(p-q)}{r} + \dfrac{3(p+2q)}{4r} - \dfrac{5(p-4q)}{6r}$

$= \dfrac{24(p-q)+9(p+2q)-10(p-4q)}{12r}$

$= \dfrac{24p-24q+9p+18q-10p+40q}{12r}$

$= \dfrac{23p+34q}{12r}$

(g) $\dfrac{u}{2v+3} + \dfrac{6u}{4v+6}$

$= \dfrac{u}{2v+3} + \dfrac{6u}{2(2v+3)}$

$= \dfrac{u}{2v+3} + \dfrac{3u}{2v+3}$

$= \dfrac{4u}{2v+3}$

(h) $\dfrac{z+1}{x-2y} - \dfrac{2z-3}{2x-4y} + \dfrac{z}{3x-6y}$

$= \dfrac{z+1}{x-2y} - \dfrac{2z-3}{2(x-2y)} + \dfrac{z}{3(x-2y)}$

$= \dfrac{6(z+1)-3(2z-3)+2z}{6(x-2y)}$

$= \dfrac{6z+6-6z+9+2z}{6(x-2y)}$

$= \dfrac{2z+15}{6(x-2y)}$

9. **(a)** $\dfrac{2}{a} + \dfrac{5}{2(a-1)}$

$= \dfrac{4(a-1)+5a}{2a(a-1)}$

$= \dfrac{4a-4+5a}{2a(a-1)}$

$= \dfrac{9a-4}{2a(a-1)}$

(b) $\frac{2}{3d+1} - \frac{1}{5d+3}$

$= \frac{2(5d+3)-(3d+1)}{(3d+1)(5d+3)}$

$= \frac{10d+6-3d-1}{(3d+1)(5d+3)}$

$= \frac{7d+5}{(3d+1)(5d+3)}$

10. (a) $a + x = b$

$a = b - x$

(b) $b - k = h$

$b = h + k$

(c) $c - d = e + f$

$c = d + e + f$

(d) $g - h - j = k^2$

$g = h + j + k^2$

(e) $q - p + r = s^3$

$-p = -q - r + s^3$

$p = q + r - s^3$

(f) $7k + h - q = 2q$

$3q = 7k + h$

$q = \frac{7k+h}{3}$

(g) $5pq - r = p^2 - q$

$r = 5pq + q - p^2$

(h) $3ab + s = a^2b$

$s = a^2b - 3ab$

(i) $wx = 3y$

$w = \frac{3y}{x}$

(j) $2xy = 3ak$

$x = \frac{3ak}{2y}$

(k) $x^2y = 5k - 4$

$y = \frac{5k-4}{x^2}$

(l) $kz = p - q + k$

$z = \frac{p-q+k}{k}$

11. (a) $ah = b - c + k^3$

$a = \frac{b-c+k^3}{h}$

(b) $5by^2 = x^4 - y$

$b = \frac{x^4-y}{5y^2}$

(c) $cx = \frac{A}{B}$

$c = \frac{A}{Bx}$

(d) $3dk = \frac{15x}{7y}$

$d = \frac{5x}{7ky}$

(e) $12mk = \frac{3ek^2}{5x+y}$

$3ek^2 = 12mk(5x+y)$

$e = \frac{12mk(5x+y)}{3k^2}$

$= \frac{4m(5x+y)}{k}$

(f) $\frac{5f}{2y} = \frac{3bk}{4x}$

$f = \frac{3bk}{4x} \times \frac{2y}{5}$

$= \frac{3bky}{10x}$

(g) $\frac{eg}{t} = \frac{t}{5n+4}$

$g = \frac{t^2}{e(5n+4)}$

(h) $\sqrt{v} = m(h+c)$

$h + c = \frac{\sqrt{v}}{m}$

$h = \frac{\sqrt{v}}{m} - c$

12. (a) $\frac{5}{a} = \frac{6}{7}$

$6a = 35$

$a = \frac{35}{6}$

$= 5\frac{5}{6}$

(b) $\frac{7}{2b} = 3$

$6b = 7$

$b = \frac{7}{6}$

$= 1\frac{1}{6}$

(c) $\frac{3}{c-2} = \frac{1}{2}$

$c - 2 = 6$

$c = 8$

(d) $\frac{5}{d-4} - 3 = 0$

$\frac{5}{d-4} = 3$

$3(d-4) = 5$

$d - 4 = \frac{5}{3}$

$d = 4 + \frac{5}{3}$

$= 5\frac{2}{3}$

(e) $\dfrac{9}{5-2e}+7=0$

$$\begin{aligned}\frac{9}{5-2e}&=-7\\ -7(5-2e)&=9\\ -35+14e&=9\\ 14e&=44\\ e&=\frac{44}{14}\\ &=3\frac{1}{7}\end{aligned}$$

(f) $\dfrac{2f}{5f-4}+\dfrac{1}{3}=0$

$$\begin{aligned}\frac{2f}{5f-4}&=-\frac{1}{3}\\ 6f&=-(5f-4)\\ &=-5f+4\\ 11f&=4\\ f&=\frac{4}{11}\end{aligned}$$

13. (a) $\dfrac{2}{5a}=\dfrac{4}{a-1}$

$$\begin{aligned}2(a-1)&=4(5a)\\ 2a-2&=20a\\ 18a&=-2\\ a&=-\frac{2}{18}\\ &=-\frac{1}{9}\end{aligned}$$

(b) $\dfrac{7}{2b-1}=\dfrac{3}{b-4}$

$$\begin{aligned}7(b-4)&=3(2b-1)\\ 7b-28&=6b-3\\ b&=25\end{aligned}$$

(c) $\dfrac{3}{c+2}=\dfrac{14}{2c-1}$

$$\begin{aligned}3(2c-1)&=14(c+2)\\ 6c-3&=14c+28\\ 8c&=-31\\ c&=-\frac{31}{8}\\ &=-3\frac{7}{8}\end{aligned}$$

(d) $\dfrac{7}{2d-5}=\dfrac{9}{3d+4}$

$$\begin{aligned}7(3d+4)&=9(2d-5)\\ 21d+28&=18d-45\\ 3d&=-73\\ d&=-\frac{73}{3}\\ &=-24\frac{1}{3}\end{aligned}$$

(e) $\dfrac{3}{e+1}+\dfrac{1}{2e+1}=0$

$$\begin{aligned}\frac{3}{e+1}&=-\frac{1}{2e+1}\\ 3(2e+1)&=-(e+1)\\ 6e+3&=-e-1\\ 7e&=-4\\ e&=-\frac{4}{7}\end{aligned}$$

(f) $\dfrac{5}{2f-5}-\dfrac{4}{7f+1}=0$

$$\begin{aligned}\frac{5}{2f-5}&=\frac{4}{7f+1}\\ 5(7f+1)&=4(2f-5)\\ 35f+5&=8f-20\\ 27f&=-25\\ f&=-\frac{25}{27}\end{aligned}$$

14. (a) $\dfrac{4}{x}+1\dfrac{1}{2}=\dfrac{5}{2x}$

$$\begin{aligned}\frac{4}{x}-\frac{5}{2x}&=-1\frac{1}{2}\\ \frac{8-5}{2x}&=-\frac{3}{2}\\ \frac{3}{2x}&=-\frac{3}{2}\\ 6&=-6x\\ x&=-1\end{aligned}$$

(b) $\dfrac{1}{3}\left(\dfrac{1}{5y}-3\right)=\dfrac{1}{2}\left(2-\dfrac{1}{y}\right)$

$$\begin{aligned}\frac{1}{15y}-1&=1-\frac{1}{2y}\\ \frac{1}{15y}+\frac{1}{2y}&=2\\ \frac{2+15}{30y}&=2\\ \frac{17}{30y}&=2\\ 17&=2(30y)\\ 60y&=17\\ y&=\frac{17}{60}\end{aligned}$$

15. $\dfrac{5}{2x-7}-\dfrac{6}{x-7}=0$

$$\begin{aligned}\frac{5}{2x-7}&=\frac{6}{x-7}\\ 5(x-7)&=6(2x-7)\\ 5x-35&=12x-42\\ 7x&=7\\ x&=1\end{aligned}$$

Intermediate

16. (a) $\dfrac{(-3a)^2 b^3 c}{27abc^4}$

$= \dfrac{9a^2 b^3 c}{27abc^4}$

$= \dfrac{ab^2}{3c^3}$

(b) $\dfrac{(-3d^2 e^4)^3}{9d^2 e^5}$

$= \dfrac{-27d^6 e^{12}}{9d^2 e^5}$

$= -3d^4 e^7$

(c) $\dfrac{(-4fg^3 h)^3}{-16f^4 gh^5}$

$= \dfrac{-64f^3 g^9 h^3}{-16f^4 gh^5}$

$= \dfrac{4g^8}{fh^2}$

(d) $\dfrac{(-9j^4 kl)^3}{(27jkl)^2}$

$= \dfrac{-729j^{12}k^3 l^3}{729j^2 k^2 l^2}$

$= -j^{10}kl$

17. (a) $\dfrac{(2a-3b)^2}{6a^2-9ab}$

$= \dfrac{(2a-3b)^2}{3a(2a-3b)}$

$= \dfrac{2a-3b}{3a}$

(b) $\dfrac{5c^3 d(x+y)}{10c(x+y)^2}$

$= \dfrac{c^2 d}{2(x+y)}$

(c) $\dfrac{15x^3(e-f)^2}{35xy(e-f)^2}$

$= \dfrac{3x^2}{7y}$

(d) $\dfrac{g^2+g-6}{g^2-9g+14}$

$= \dfrac{(g+3)(g-2)}{(g-7)(g-2)}$

$= \dfrac{g+3}{g-7}$

(e) $\dfrac{6h^2-13h-5}{6h^2+17h+5}$

$= \dfrac{(3h+1)(2h-5)}{(3h+1)(2h+5)}$

$= \dfrac{2h-5}{2h+5}$

(f) $\dfrac{6-11k+4k^2}{3k^2+k-14}$

$= \dfrac{(4k-3)(k-2)}{(3k+7)(k-2)}$

$= \dfrac{4k-3}{3k+7}$

(g) $\dfrac{(p+q)^2-r^2}{(q+r)^2-p^2}$

$= \dfrac{(p+q+r)(p+q-r)}{(q+r+p)(q+r-p)}$

$= \dfrac{p+q-r}{q+r-p}$

(h) $\dfrac{(3x+y)^2-4z^2}{15x^2+5xy+10xz}$

$= \dfrac{(3x+y)^2-(2z)^2}{15x^2+5xy+10xz}$

$= \dfrac{(3x+y+2z)(3x+y-2z)}{5x(3x+y+2z)}$

$= \dfrac{3x+y-2z}{5x}$

(i) $\dfrac{6xz+3yz}{6x^2-2xz+3xy-yz}$

$= \dfrac{3z(2x+y)}{2x(3x-z)+y(3x-z)}$

$= \dfrac{3z(2x+y)}{(2x+y)(3x-z)}$

$= \dfrac{3z}{3x-z}$

(j) $\dfrac{x^2-2xz+xy-2yz}{x^2+xy-xz-yz}$

$= \dfrac{x(x-2z)+y(x-2z)}{x(x+y)-z(x+y)}$

$= \dfrac{(x+y)(x-2z)}{(x-z)(x+y)}$

$= \dfrac{x-2z}{x-z}$

(k) $\dfrac{2ac+bc-2ad-bd}{cx-3cy-dx+3dy}$

$= \dfrac{c(2a+b)-d(2a+b)}{c(x-3y)-d(x-3y)}$

$= \dfrac{(c-d)(2a+b)}{(c-d)(x-3y)}$

$= \dfrac{2a+b}{x-3y}$

18. (a) $\dfrac{2a}{b} \times \dfrac{3c}{4a} \times \dfrac{8a}{9c}$

$= \dfrac{4a}{3b}$

(b) $\dfrac{3d^2}{ef} \times \dfrac{6e^2}{21ef} \times \dfrac{28f^2}{3de}$

$= \dfrac{8d}{e}$

(c) $\frac{2}{h^2} \times \frac{1}{k^3} \div \frac{2h}{3k}$

$= \frac{2}{h^2} \times \frac{1}{k^3} \times \frac{3k}{2h}$

$= \frac{3}{h^3k^2}$

(d) $\frac{4m^2n^4}{36m} \times \frac{24m}{8m^2n^3} \div \frac{16m}{6mn^2}$

$= \frac{4m^2n^4}{36m} \times \frac{24m}{8m^2n^3} \times \frac{6mn^2}{16m}$

$= \frac{n^3}{8}$

(e) $\frac{3p^3q^3}{8r^4} \times \frac{6q^2r^3}{5p^5} \div \frac{9q^2}{10pr}$

$= \frac{3p^3q^3}{8r^4} \times \frac{6q^2r^3}{5p^5} \times \frac{10pr}{9q^2}$

$= \frac{q^3}{2p}$

(f) $\frac{2x^2y^3}{7az^3} \div \frac{4x^2z}{21a^2z} \times \frac{3a}{8xy}$

$= \frac{2x^2y^3}{7az^3} \times \frac{21a^2z}{4x^2z} \times \frac{3a}{8xy}$

$= \frac{9a^2y^2}{16xz^3}$

(g) $\frac{9b}{21c} \div \left(\frac{3d}{4e} \times \frac{16be}{9a}\right)$

$= \frac{3b}{7c} \div \frac{4bd}{3a}$

$= \frac{3b}{7c} \times \frac{3a}{4bd}$

$= \frac{9a}{28cd}$

(h) $\frac{3x}{4y} \div \left(\frac{7x^2}{15z} \div \frac{3y^2}{10z^2}\right)$

$= \frac{3x}{4y} \div \left(\frac{7x^2}{15z} \times \frac{10z^2}{3y^2}\right)$

$= \frac{3x}{4y} \div \frac{14x^2z}{9y^2}$

$= \frac{3x}{4y} \times \frac{9y^2}{14x^2z}$

$= \frac{27y}{56xz}$

19. (a) $\frac{x^5 - x^4}{ax - a} \div \frac{ax^2}{ax - x}$

$= \frac{x^4(x-1)}{a(x-1)} \div \frac{ax^2}{x(a-1)}$

$= \frac{x^4}{a} \times \frac{a-1}{ax}$

$= \frac{x^3(a-1)}{a^2}$

(b) $\frac{x}{x+1} \div \frac{x^2 - 2x}{x^2 - 2x - 3}$

$= \frac{x}{x+1} \div \frac{x(x-2)}{(x-3)(x+1)}$

$= \frac{x}{x+1} \times \frac{(x-3)(x+1)}{x(x-2)}$

$= \frac{x-3}{x-2}$

20. (a) $\frac{3}{a+1} + \frac{a+4}{(a+1)(a+2)}$

$= \frac{3(a+2) + a + 4}{(a+1)(a+2)}$

$= \frac{3a + 6 + a + 4}{(a+1)(a+2)}$

$= \frac{4a + 10}{(a+1)(a+2)}$

(b) $\frac{2}{b-1} - \frac{1}{b-2} + \frac{3(b+2)}{(b-1)(b-2)}$

$= \frac{2(b-2) - (b-1) + 3(b+2)}{(b-1)(b-2)}$

$= \frac{2b - 4 - b + 1 + 3b + 6}{(b-1)(b-2)}$

$= \frac{4b + 3}{(b-1)(b-2)}$

(c) $\frac{4}{(c-2)(c-4)} - \frac{2}{(c-2)(c-3)}$

$= \frac{4(c-3) - 2(c-4)}{(c-2)(c-3)(c-4)}$

$= \frac{4c - 12 - 2c + 8}{(c-2)(c-3)(c-4)}$

$= \frac{2c - 4}{(c-2)(c-3)(c-4)}$

$= \frac{2(c-2)}{(c-2)(c-3)(c-4)}$

$= \frac{2}{(c-3)(c-4)}$

(d) $\frac{d-4}{(d+1)(d-5)} - \frac{d+5}{(d+1)(d+3)}$

$= \frac{(d-4)(d+3) - (d+5)(d-5)}{(d+1)(d+3)(d-5)}$

$= \frac{(d^2 + 3d - 4d - 12) - (d^2 - 25)}{(d+1)(d+3)(d-5)}$

$= \frac{d^2 - d - 12 - d^2 + 25}{(d+1)(d+3)(d-5)}$

$= \frac{13 - d}{(d+1)(d+3)(d-5)}$

(e) $\dfrac{e^2}{(e+f)(e-3f)} - \dfrac{e-f}{e-3f}$

$= \dfrac{e^2-(e+f)(e-f)}{(e+f)(e-3f)}$

$= \dfrac{e^2-(e^2-f^2)}{(e+f)(e-3f)}$

$= \dfrac{e^2-e^2+f^2}{(e+f)(e-3f)}$

$= \dfrac{f^2}{(e+f)(e-3f)}$

(f) $\dfrac{3g}{g-3} - \dfrac{g}{g^2-9}$

$= \dfrac{3g}{g-3} - \dfrac{g}{(g+3)(g-3)}$

$= \dfrac{3g(g+3)-g}{(g+3)(g-3)}$

$= \dfrac{3g^2+9g-g}{(g+3)(g-3)}$

$= \dfrac{3g^2+8g}{(g+3)(g-3)}$

(g) $\dfrac{h}{h^2-4} - \dfrac{1}{h+2}$

$= \dfrac{h}{(h+2)(h-2)} - \dfrac{1}{h+2}$

$= \dfrac{h-(h-2)}{(h+2)(h-2)}$

$= \dfrac{h-h+2}{(h+2)(h-2)}$

$= \dfrac{2}{(h+2)(h-2)}$

(h) $\dfrac{j+2}{j^2-1} - \dfrac{3}{2(j-1)}$

$= \dfrac{j+2}{(j+1)(j-1)} - \dfrac{3}{2(j-1)}$

$= \dfrac{2(j+2)-3(j+1)}{2(j+1)(j-1)}$

$= \dfrac{2j+4-3j-3}{2(j+1)(j-1)}$

$= \dfrac{-j+1}{2(j+1)(j-1)}$

$= \dfrac{-(j-1)}{2(j+1)(j-1)}$

$= -\dfrac{1}{2(j+1)}$

21. (a) $\dfrac{1}{a-1} + \dfrac{2a}{1-a^2}$

$= \dfrac{1}{a-1} + \dfrac{2a}{(1+a)(1-a)}$

$= \dfrac{1}{a-1} - \dfrac{2a}{(1+a)(a-1)}$

$= \dfrac{1+a-2a}{(a-1)(1+a)}$

$= \dfrac{1-a}{(a-1)(1+a)}$

$= -\dfrac{1}{1+a}$

(b) $\dfrac{3}{b+2} - \dfrac{b}{4-b^2}$

$= \dfrac{3}{b+2} - \dfrac{b}{(2+b)(2-b)}$

$= \dfrac{3}{b+2} + \dfrac{b}{(b+2)(b-2)}$

$= \dfrac{3(b-2)+b}{(b+2)(b-2)}$

$= \dfrac{3b-6+b}{(b+2)(b-2)}$

$= \dfrac{4b-6}{(b+2)(b-2)}$

(c) $\dfrac{2}{c+4} + \dfrac{3}{4-c} - \dfrac{c}{c^2-16}$

$= \dfrac{2}{c+4} - \dfrac{3}{c-4} - \dfrac{c}{(c+4)(c-4)}$

$= \dfrac{2(c-4)-3(c+4)-c}{(c+4)(c-4)}$

$= \dfrac{2c-8-3c-12-c}{(c+4)(c-4)}$

$= \dfrac{-2c-20}{(c+4)(c-4)}$

$= -\dfrac{2c+20}{(c+4)(c-4)}$

(d) $\dfrac{1}{2d+3e} + \dfrac{4d}{9e^2-4d^2} - \dfrac{2}{3e-2d}$

$= \dfrac{1}{2d+3e} - \dfrac{4d}{4d^2-9e^2} + \dfrac{2}{2d-3e}$

$= \dfrac{1}{2d+3e} - \dfrac{4d}{(2d+3e)(2d-3e)} + \dfrac{2}{2d-3e}$

$= \dfrac{2d-3e-4d+2(2d+3e)}{(2d+3e)(2d-3e)}$

$= \dfrac{2d+3e}{(2d+3e)(2d-3e)}$

$= \dfrac{1}{2d-3e}$

22. (a) $\dfrac{2a}{a^2+a-6}+\dfrac{1}{a-2}$

$=\dfrac{2a}{(a+3)(a-2)}+\dfrac{1}{a-2}$

$=\dfrac{2a+a+3}{(a+3)(a-2)}$

$=\dfrac{3a+3}{(a+3)(a-2)}$

(b) $\dfrac{1}{2(b-1)}+\dfrac{b+1}{b^2+b-2}$

$=\dfrac{1}{2(b-1)}+\dfrac{b+1}{(b-1)(b+2)}$

$=\dfrac{b+2+2(b+1)}{2(b-1)(b+2)}$

$=\dfrac{b+2+2b+2}{2(b-1)(b+2)}$

$=\dfrac{3b+4}{2(b-1)(b+2)}$

(c) $\dfrac{4}{c^2+2c-3}-\dfrac{1}{c^2-5c+4}$

$=\dfrac{4}{(c-1)(c+3)}-\dfrac{1}{(c-1)(c-4)}$

$=\dfrac{4(c-4)-(c+3)}{(c-1)(c-4)(c+3)}$

$=\dfrac{4c-16-c-3}{(c-1)(c-4)(c+3)}$

$=\dfrac{3c-19}{(c-1)(c-4)(c+3)}$

(d) $\dfrac{3d-2}{d^2-3d+2}-\dfrac{3d-1}{d^2-2d}$

$=\dfrac{3d-2}{(d-1)(d-2)}-\dfrac{3d-1}{d(d-2)}$

$=\dfrac{d(3d-2)-(3d-1)(d-1)}{d(d-1)(d-2)}$

$=\dfrac{3d^2-2d-(3d^2-3d-d+1)}{d(d-1)(d-2)}$

$=\dfrac{3d^2-2d-3d^2+3d+d-1}{d(d-1)(d-2)}$

$=\dfrac{2d-1}{d(d-1)(d-2)}$

(e) $\dfrac{4e}{e-f}+\dfrac{2e}{e+2f}+\dfrac{1}{e^2+ef-2f^2}$

$=\dfrac{4e}{e-f}+\dfrac{2e}{e+2f}+\dfrac{1}{(e-f)(e+2f)}$

$=\dfrac{4e(e+2f)+2e(e-f)+1}{(e-f)(e+2f)}$

$=\dfrac{4e^2+8ef+2e^2-2ef+1}{(e-f)(e+2f)}$

$=\dfrac{6e^2+6ef+1}{(e-f)(e+2f)}$

(f) $\dfrac{1}{3g+2h}+\dfrac{1}{2g-3h}-\dfrac{1}{6g^2-6h^2-5gh}$

$=\dfrac{1}{3g+2h}+\dfrac{1}{2g-3h}-\dfrac{1}{(3g+2h)(2g-3h)}$

$=\dfrac{2g-3h+3g+2h-1}{(3g+2h)(2g-3h)}$

$=\dfrac{5g-h-1}{(3g+2h)(2g-3h)}$

23. (a) $\dfrac{2}{a+\frac{1}{2}}$

$=\dfrac{4}{2a+1}$

(b) $\dfrac{\frac{1}{4}b}{2+\frac{1}{2}c}$

$=\dfrac{b}{8+2c}$

24. (a) $\left(2x-\dfrac{8}{x}\right)\div\left(1-\dfrac{2}{x}\right)$

$=\dfrac{2x^2-8}{x}\div\dfrac{x-2}{x}$

$=\dfrac{2(x^2-4)}{x}\div\dfrac{x-2}{x}$

$=\dfrac{2(x+2)(x-2)}{x}\times\dfrac{x}{x-2}$

$=2(x+2)$

(b) $\left(\dfrac{1}{x}-\dfrac{1}{y}\right)\div\left(\dfrac{1}{x^2}-\dfrac{1}{y^2}\right)$

$=\dfrac{y-x}{xy}\div\dfrac{y^2-x^2}{x^2y^2}$

$=\dfrac{y-x}{xy}\times\dfrac{x^2y^2}{y^2-x^2}$

$=\dfrac{y-x}{xy}\times\dfrac{x^2y^2}{(y+x)(y-x)}$

$=\dfrac{xy}{x+y}$

25. $\left(\dfrac{1}{x^3}-\dfrac{1}{x}\right)\div\left(\dfrac{1}{x^2}-\dfrac{1}{x}\right)$

$=\dfrac{1-x^2}{x^3}\div\dfrac{1-x}{x^2}$

$=\dfrac{(1+x)(1-x)}{x^3}\times\dfrac{x^2}{1-x}$

$=\dfrac{1+x}{x}$

$\therefore k=1$

26. (a) $(a+p)y = q(2a-q)$
$$ay + py = 2aq - q^2$$
$$2aq - ay = py + q^2$$
$$a(2q - y) = py + q^2$$
$$a = \frac{py + q^2}{2q - y}$$

(c) $\frac{k(m+c)}{m} = \frac{4}{x}$
$$k(m+c) = \frac{4m}{x}$$
$$m + c = \frac{4m}{kx}$$
$$c = \frac{4m}{kx} - m$$

(c) $\frac{1}{d} + c = \frac{b}{d}$
$$\frac{b}{d} - \frac{1}{d} = c$$
$$\frac{b-1}{d} = c$$
$$b - 1 = cd$$
$$d = \frac{b-1}{c}$$

(d) $y = \frac{7bj + k}{7 - 4j}$
$$y(7 - 4j) = 7bj + k$$
$$7y - 4jy = 7bj + k$$
$$7bj + 4jy = 7y - k$$
$$j(7b + 4y) = 7y - k$$
$$j = \frac{7y - k}{7b + 4y}$$

(e) $\frac{x}{k+y} = \frac{y^2}{k}$
$$kx = y^2(k+y)$$
$$= ky^2 + y^3$$
$$kx - ky^2 = y^3$$
$$k(x - y^2) = y^3$$
$$k = \frac{y^3}{x - y^2}$$

(f) $\frac{3}{5} = \frac{n - 4a}{n + 7b}$
$$3(n + 7b) = 5(n - 4a)$$
$$3n + 21b = 5n - 20a$$
$$2n = 20a + 21b$$
$$n = \frac{20a + 21b}{2}$$

(g) $kx + 4 = \frac{2x - 3r}{2r - 5}$
$$(kx + 4)(2r - 5) = 2x - 3r$$
$$2krx - 5kx + 8r - 20 = 2x - 3r$$
$$2krx + 11r = 5kx + 2x + 20$$
$$r(2kx + 11) = 5kx + 2x + 20$$
$$r = \frac{5kx + 2x + 20}{2kx + 11}$$

(h) $k - 3ux = \frac{3uy}{4}$
$$4k - 12ux = 3uy$$
$$12ux + 3uy = 4k$$
$$u(12x + 3y) = 4k$$
$$u = \frac{4k}{12x + 3y}$$

(i) $x = \frac{3kw + 4hx + 4}{5bw - 4xy + 2}$
$$x(5bw - 4xy + 2) = 3kw + 4hx + 4$$
$$5bwx - 4x^2y + 2x = 3kw + 4hx + 4$$
$$5bwx - 3kw = 4x^2y + 4hx - 2x + 4$$
$$w(5bx - 3k) = 4x^2y + 4hx - 2x + 4$$
$$w = \frac{4x^2y + 4hx - 2x + 4}{5bx - 3k}$$

(j) $\frac{1}{x} + \frac{3}{2y} = \frac{4}{5z}$
$$\frac{3}{2y} = \frac{4}{5z} - \frac{1}{x}$$
$$= \frac{4x - 5z}{5xz}$$
$$15xz = 2y(4x - 5z)$$
$$y = \frac{15xz}{2(4x - 5z)}$$

27. (a) $ax^2 + bd^2 + c = 0$
$$bd^2 = -ax^2 - c$$
$$d^2 = \frac{-ax^2 - c}{b}$$
$$= -\frac{ax^2 + c}{b}$$
$$d = \pm\sqrt{-\frac{ax^2 + c}{b}}$$

(b) $k = \frac{2hae^2}{b - e^2}$
$$k(b - e^2) = 2hae^2$$
$$bk - e^2k = 2hae^2$$
$$e^2k + 2hae^2 = bk$$
$$e^2(k + 2ha) = bk$$
$$e^2 = \frac{bk}{k + 2ha}$$
$$e = \pm\sqrt{\frac{bk}{k + 2ha}}$$

(c) $y = \frac{2 - f^2}{2f^2 + 3}$

$y(2f^2 + 3) = 2 - f^2$

$2f^2y + 3y = 2 - f^2$

$2f^2y + f^2 = 2 - 3y$

$f^2(2y + 1) = 2 - 3y$

$f^2 = \frac{2 - 3y}{2y + 1}$

$f = \pm\sqrt{\frac{2 - 3y}{2y + 1}}$

(d) $\frac{1}{a} + \frac{1}{\sqrt{n}} = y$

$\frac{1}{\sqrt{n}} = y - \frac{1}{a}$

$= \frac{ay - 1}{a}$

$a = \sqrt{n}\,(ay - 1)$

$\sqrt{n} = \frac{a}{ay - 1}$

$n = \frac{a^2}{(ay - 1)^2}$

(e) $x = \sqrt{\frac{k^2 - t^2}{2k^2 + 3t^2}}$

$x^2 = \frac{k^2 - t^2}{2k^2 + 3t^2}$

$x^2(2k^2 + 3t^2) = k^2 - t^2$

$2k^2x^2 + 3t^2x^2 = k^2 - t^2$

$3t^2x^2 + t^2 = k^2 - 2k^2x^2$

$t^2(3x^2 + 1) = k^2 - 2k^2x^2$

$t^2 = \frac{k^2 - 2k^2x^2}{3x^2 + 1}$

$t = \pm\sqrt{\frac{k^2 - 2k^2x^2}{3x^2 + 1}}$

(f) $k = \sqrt[3]{\frac{b(x - b)}{h}}$

$k^3 = \frac{b(x - b)}{h}$

$hk^3 = b(x - b)$

$x - b = \frac{hk^3}{b}$

$x = b + \frac{hk^3}{b}$

28. $u + v = m$ —(1)

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ —(2)

From (1),

$v = m - u$ —(3)

Substitute (3) into (2):

$\frac{1}{u} + \frac{1}{m - u} = \frac{1}{f}$

$\frac{m - u + u}{u(m - u)} = \frac{1}{f}$

$\frac{m}{u(m - u)} = \frac{1}{f}$

$fm = u(m - u)$

$= mu - u^2$

$u^2 = mu - fm$

$= m(u - f)$

$m = \frac{u^2}{u - f}$

29. $y = p + \frac{q}{x}$ —(1)

$z = p + \frac{q}{y}$ —(2)

Substitute (1) into (2):

$z = p + \frac{q}{p + \frac{q}{x}}$

$= p + \frac{qx}{px + q}$

$z - p = \frac{qx}{px + q}$

$(z - p)(px + q) = qx$

$pxz + qz - p^2x - pq = qx$

$p^2x + qx - pxz = qz - pq$

$x(p^2 + q - pz) = qz - pq$

$x = \frac{qz - pq}{p^2 + q - pz}$

30. (i) $\frac{1}{a} + \frac{2}{b} + \frac{3}{c} = \frac{4}{d}$

$\frac{2}{b} = \frac{4}{d} - \frac{3}{c} - \frac{1}{a}$

$= \frac{4ac - 3ad - cd}{acd}$

$2acd = b(4ac - 3ad - cd)$

$b = \frac{2acd}{4ac - 3ad - cd}$

(ii) When $a = 6$, $c = 4$, $d = \frac{1}{2}$,

$$b = \frac{2(6)(4)\left(\frac{1}{2}\right)}{4(6)(4) - 3(6)\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)}$$

$= \frac{24}{85}$

31. (i) $x = y + \frac{k^2 y}{gm}$

$gmx = gmy + k^2y$

$= y(gm + k^2)$

$y = \frac{gmx}{gm + k^2}$

(ii) When $x = 5$, $k = 9$, $g = 3$ and $m = 4$,

$y = \frac{(3)(4)(5)}{3(4) + 9^2}$

$= \frac{20}{31}$

32. (i) $\sqrt{x^2 - a^2} = x + a$

$x^2 - a^2 = (x + a)^2$

$= x^2 + 2ax + a^2$

$2ax + 2a^2 = 0$

$2a(x + a) = 0$

$a = 0$ or $x = -a$

$\therefore x = -a$

(ii) When $\sqrt{a} = \frac{3}{4}$,

$a = \frac{9}{16}$

$x = -\frac{9}{16}$

33. (i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{a^2} = 1 - \frac{y^2}{b^2}$

$= \frac{b^2 - y^2}{b^2}$

$x^2 = \frac{a^2}{b^2}(b^2 - y^2)$

$x = \pm\frac{a}{b}\sqrt{b^2 - y^2}$

(ii) When $y = 4$, $a = 2$, $b = 5$,

$x = \pm\frac{2}{5}\sqrt{5^2 - 4^2}$

$= \pm 1\frac{1}{5}$

34. (i) $Q = mc\theta$

$m = \frac{Q}{c\theta}$

(ii) When $c = 4186$, $Q = 12\ 560$, $\theta = 3$,

$m = \frac{12\ 560}{4186(3)}$

$= 1.00$ (to 3 s.f.)

$\therefore$ The mass of the water is 1.00 kg.

35. (i) $P = 5000n^2 - 8000$

$5000n^2 = P + 8000$

$n^2 = \frac{P + 8000}{5000}$

$n = \sqrt{\frac{P + 8000}{5000}}$

(ii) Given that $P > 1\ 000\ 000$,

$5000n^2 - 8000 > 1\ 000\ 000$

$5000n^2 > 1\ 008\ 000$

$n^2 > 201.6$

When $n = 14$, $n^2 = 196 < 201.6$.

When $n = 15$, $n^2 = 225 > 201.6$.

$\therefore$ The minimum number of employees is 15.

36. (i) $T = \frac{l_T - l_0}{l_{100} - l_0} \times 100$

$\frac{T}{100} = \frac{l_T - l_0}{l_{100} - l_0}$

$\frac{T}{100}(l_{100} - l_0) = l_T - l_0$

$l_T = \frac{T}{100}(l_{100} - l_0) + l_0$

(ii) When $T = 80$, $l_0 = 1.5$, $l_{100} = 13.5$,

$l_T = \frac{80}{100}(13.5 - 1.5) + 1.5$

$= 11.1$

$\therefore$ The length of mercury thread is 11.1 cm.

37. (a) $a = \frac{3}{a + 2}$

$a(a + 2) = 3$

$a^2 + 2a = 3$

$a^2 + 2a - 3 = 0$

$(a - 1)(a + 3) = 0$

$a = 1$ or $a = -3$

(b) $b - 2 = \frac{9}{b - 2}$

$(b - 2)^2 = 9$

$b - 2 = 3$ or $b - 2 = -3$

$b = 5$ $\quad b = -1$

(c) $c = 8 - \frac{7}{c}$

$c^2 = 8c - 7$

$c^2 - 8c + 7 = 0$

$(c - 1)(c - 7) = 0$

$c = 1$ or $c = 7$

(d) $\dfrac{6d}{2d-1} = 2d$

$6d = 2d(2d-1)$

$= 4d^2 - 2d$

$4d^2 - 8d = 0$

$4d(d-2) = 0$

$d = 0$ or $d = 2$

(e) $\dfrac{84}{f-4} = 1 + \dfrac{75}{f}$

$84f = f(f-4) + 75(f-4)$

$= f^2 - 4f + 75f - 300$

$f^2 - 13f - 300 = 0$

$(f-25)(f+12) = 0$

$f = 25$ or $f = -12$

(f) $\dfrac{1}{h+3} + \dfrac{4}{5} = \dfrac{h}{4-h}$

$5(4-h) + 4(h+3)(4-h) = 5h(h+3)$

$20 - 5h + 4(4h - h^2 + 12 - 3h) = 5h^2 + 15h$

$20 - 5h + 16h - 4h^2 + 48 - 12h = 5h^2 + 15h$

$9h^2 + 16h - 68 = 0$

$(h-2)(9h+34) = 0$

$h = 2$ or $h = -\dfrac{34}{9}$

$= -3\dfrac{7}{9}$

(g) $\dfrac{1}{j+2} + \dfrac{3}{j+4} = \dfrac{4}{j+3}$

$(j+4)(j+3) + 3(j+2)(j+3) = 4(j+2)(j+4)$

$j^2 + 3j + 4j + 12 + 3(j^2 + 3j + 2j + 6)$

$= 4(j^2 + 4j + 2j + 8)$

$j^2 + 7j + 12 + 3j^2 + 15j + 18 = 4j^2 + 24j + 32$

$2j = -2$

$j = -1$

(h) $\dfrac{3}{k+1} = \dfrac{8}{k+2} - \dfrac{5}{k+3}$

$3(k+2)(k+3) = 8(k+1)(k+3) - 5(k+1)(k+2)$

$3(k^2 + 3k + 2k + 6)$

$= 8(k^2 + 3k + k + 3) - 5(k^2 + 2k + k + 2)$

$3k^2 + 15k + 18 = 8k^2 + 32k + 24 - 5k^2 - 15k - 10$

$2k = 4$

$k = 2$

38. $\dfrac{x}{x-1} + \dfrac{x}{x+1} = 3 + \dfrac{1}{1-x^2}$

$\dfrac{x}{x-1} + \dfrac{x}{x+1} - \dfrac{1}{1-x^2} = 3$

$\dfrac{x}{x-1} + \dfrac{x}{x+1} + \dfrac{1}{x^2-1} = 3$

$\dfrac{x}{x-1} + \dfrac{x}{x+1} + \dfrac{1}{(x+1)(x-1)} = 3$

$\dfrac{x(x+1) + x(x-1) + 1}{(x+1)(x-1)} = 3$

$x^2 + x + x^2 - x + 1 = 3(x+1)(x-1)$

$2x^2 + 1 = 3(x^2 - 1)$

$= 3x^2 - 3$

$x^2 = 4$

$x = 2$ or $x = -2$

39. $y = \dfrac{3}{1+2x}$ —(1)

$y = \dfrac{5}{3+4x}$ —(2)

Substitute (1) into (2):

$\dfrac{3}{1+2x} = \dfrac{5}{3+4x}$

$3(3+4x) = 5(1+2x)$

$9 + 12x = 5 + 10x$

$2x = -4$

$x = -2$

Advanced

40. (a) $\dfrac{15a^n}{25a^{n+3}}$

$= \dfrac{3}{5a^3}$

(b) $\dfrac{49a^{n-1}b^n}{7a^2b^3}$

$= 7a^{n-3}b^{n-3}$

41. $\dfrac{6a^{n+5}b^{n-2}}{16a^4b^{n-4}}$

$= \dfrac{3}{8}a^{n+1}b^2$

$\therefore h = \dfrac{3}{8}, k = 2, n = 8$

42. $\dfrac{16}{a^{n-1}} \times b^{n+3} \div \dfrac{48b^n}{a^n}$

$= \dfrac{16}{a^{n-1}} \times b^{n+3} \times \dfrac{a^n}{48b^n}$

$= \dfrac{ab^3}{3}$

43. $\dfrac{16a^3b^4}{7xy^4} \div \dfrac{4ab^2}{21xy^3} \times \dfrac{27a^{n+1}}{9a^{n-2}}$

$= \dfrac{16a^3b^4}{7xy^4} \times \dfrac{21xy^3}{4ab^2} \times \dfrac{27a^{n+1}}{9a^{n-2}}$

$= \dfrac{36a^5b^2}{y}$

44. $\dfrac{1}{\dfrac{2}{a} + \dfrac{3}{b}}$

$= \dfrac{1}{\dfrac{2b + 3a}{ab}}$

$= \dfrac{ab}{3a + 2b}$

45. $\dfrac{\dfrac{2}{x} + \dfrac{5}{y}}{\dfrac{3}{x}}$

$= \dfrac{\dfrac{2y + 5x}{xy}}{\dfrac{3}{x}}$

$= \dfrac{5x + 2y}{3y}$

46. (i) $\dfrac{x - 3}{5x - 4} - \dfrac{12 - 9x}{4 - 5x}$

$= \dfrac{x - 3}{5x - 4} + \dfrac{12 - 9x}{5x - 4}$

$= \dfrac{x - 3 + 12 - 9x}{5x - 4}$

$= \dfrac{9 - 8x}{5x - 4}$

(ii) $\dfrac{x^2 - 3}{5x^2 - 4} - \dfrac{12 - 9x^2}{4 - 5x^2} = 1$

From **(i)**,

$\dfrac{9 - 8x^2}{5x^2 - 4} = 1$

$9 - 8x^2 = 5x^2 - 4$

$13x^2 = 13$

$x^2 = 1$

$x = \pm 1$

47. (i) $F = \dfrac{mv^2}{r}$

$Fr = mv^2$

$v^2 = \dfrac{Fr}{m}$

$v = \pm\sqrt{\dfrac{Fr}{m}}$

(ii) When $F = 30, m = 2, r = 3$,

$v = \pm\sqrt{\dfrac{30(3)}{2}}$

$= \pm\sqrt{45}$

$= \pm 6.71$ (to 3 s.f.)

48. $y = \dfrac{1}{x - 1} - \dfrac{1}{2x + 1} - \dfrac{1}{2x - 3}$

When $y = 0$,

$\dfrac{1}{x - 1} - \dfrac{1}{2x + 1} - \dfrac{1}{2x - 3} = 0$

$(2x + 1)(2x - 3) - (x - 1)(2x - 3) - (x - 1)(2x + 1) = 0$

$4x^2 - 6x + 2x - 3 - (2x^2 - 3x - 2x + 3) - (2x^2 + x - 2x - 1) = 0$

$4x^2 - 4x - 3 - 2x^2 + 5x - 3 - 2x^2 + x + 1 = 0$

$2x = 5$

$x = \dfrac{5}{2}$

$= 2\dfrac{1}{2}$

$\therefore$ The coordinates are $\left(2\dfrac{1}{2}, 0\right)$.

New Trend

49. $t = 2\pi\sqrt{\dfrac{d}{g}}$

$\dfrac{t}{2\pi} = \sqrt{\dfrac{d}{g}}$

$\dfrac{t^2}{4\pi^2} = \dfrac{d}{g}$

$gt^2 = 4\pi^2 d$

$g = \dfrac{4\pi^2 d}{t^2}$

50. (a) (i) $\dfrac{25d^3e}{49df} \div \dfrac{15de^2}{21d^3f^2}$

$= \dfrac{25d^3e}{49df} \times \dfrac{21d^3f^2}{15de^2}$

$= \dfrac{5d^4f}{7e}$

(ii) $\dfrac{2}{c - 1} - \dfrac{3}{c - 2}$

$= \dfrac{2(c - 2) - 3(c - 1)}{(c - 1)(c - 2)}$

$= \dfrac{2c - 4 - 3c + 3}{(c - 1)(c - 2)}$

$= \dfrac{-c - 1}{(c - 1)(c - 2)}$

$= -\dfrac{c + 1}{(c - 1)(c - 2)}$

(b) $3 - e = \dfrac{8}{e+3}$

$(3+e)(3-e) = 8$

$9 - e^2 = 8$

$e^2 = 1$

$e = 1$ or $e = -1$

51. $\dfrac{2}{x-6} + \dfrac{5x}{(x-6)^2}$

$= \dfrac{2(x-6)+5x}{(x-6)^2}$

$= \dfrac{2x-12+5x}{(x-6)^2}$

$= \dfrac{7x-12}{(x-6)^2}$

52. (a) (i) When $r = 3$ and $h = 5$,

$A = \pi(3)\sqrt{5^2 - 3^2}$

$= 37.7$ (to 3 s.f.)

(ii) $A = \pi r\sqrt{h^2 - r^2}$

$\dfrac{A}{\pi r} = \sqrt{h^2 - r^2}$

$\dfrac{A^2}{\pi^2 r^2} = h^2 - r^2$

$h^2 = \dfrac{A^2}{\pi^2 r^2} + r^2$

$= \dfrac{A^2 + \pi^2 r^4}{\pi^2 r^2}$

$h = \pm\sqrt{\dfrac{A^2 + \pi^2 r^4}{\pi^2 r^2}}$

$= \pm\dfrac{\sqrt{A^2 + \pi^2 r^4}}{\pi r}$

(b) $\dfrac{(u+3v)^2 - 4v^2}{u^2 - 25v^2} = \dfrac{(u+3v+2v)(u+3v-2v)}{(u+5v)(u-5v)}$

$= \dfrac{(u+5v)(u+v)}{(u+5v)(u-5v)}$

$= \dfrac{u+v}{u-5v}$

(c) $\dfrac{2g}{2g-3} + 1 = \dfrac{1}{2-3g}$

$\dfrac{4g-3}{2g-3} = \dfrac{1}{2-3g}$

$(4g-3)(2-3g) = 2g-3$

$8g - 12g^2 - 6 + 9g = 2g - 3$

$12g^2 - 15g + 3 = 0$

$(12g-3)(g-1) = 0$

$g = \dfrac{1}{4}$ or $g = 1$

53. $\dfrac{5x}{(3x-2)^2} - \dfrac{3}{3x-2}$

$= \dfrac{5x - 3(3x-2)}{(3x-2)^2}$

$= \dfrac{5x-9x+6}{(3x-2)^2}$

$= \dfrac{6-4x}{(3x-2)^2}$

54. (i) When $l = 0$, $k = 0.4$, $b = 12$ and $m = 2$

$a = 0.4[0(2) + 12(2)]$

$= 9.6$

(ii) $k(lm + bm) = a$

$lm + bm = \dfrac{a}{k}$

$bm = \dfrac{a}{k} - lm$

$b = \dfrac{a}{km} - l$

55. (a) $\dfrac{32x^2 - 50}{8x^2 + 14x - 30} = \dfrac{2(16x^2-25)}{2(4x^2+7x-15)}$

$= \dfrac{(4x+5)(4x-5)}{(4x-5)(x+3)}$

$= \dfrac{4x+5}{x+3}$

(b) $\dfrac{7}{5-2x} - \dfrac{3}{4-x} = \dfrac{7(4-x)-3(5-2x)}{(5-2x)(4-x)}$

$= \dfrac{28-7x-15+6x}{(5-2x)(4-x)}$

$= \dfrac{13-x}{(5-2x)(4-x)}$

56. (a) $\dfrac{3a^2b}{8ab^3} \div \dfrac{21ac^4}{49abc^2}$

$= \dfrac{3a^2b}{8ab^3} \times \dfrac{49abc^2}{21ac^4}$

$= \dfrac{7a}{8bc^2}$

(b) (i) $V = \dfrac{4\pi}{3}(r^3 + s^3)$

When $r = 2.1$ and $s = 0.9$,

$V = \dfrac{4\pi}{3}(2.1^3 + 0.9^3)$

$= 41.8$ (to 3 s.f.)

(ii) $V = \dfrac{4\pi}{3}(r^3 + s^3)$

$r^3 + s^3 = \dfrac{3V}{4\pi}$

$r^3 = \dfrac{3V}{4\pi} - s^3$

$r = \sqrt[3]{\dfrac{3V}{4\pi} - s^3}$

(c) $\frac{3x-7}{4x+5} = 6$

$3x - 7 = 24x + 30$

$-37 = 21x$

$x = -\frac{37}{21}$

$= -1\frac{6}{21}$

57. (a) $\frac{5m}{3} \div \frac{60m^2}{n}$

$= \frac{5m}{3} \times \frac{n}{60m^2}$

$= \frac{n}{36m}$

(b) $\frac{6x^2-20x-16}{6x^2-5x-6}$

$= \frac{2(3x^2-10x-8)}{(3x+2)(2x-3)}$

$= \frac{2(3x+2)(x-4)}{(3x+2)(2x-3)}$

$= \frac{2(x-4)}{2x-3}$

58. (a) $k = \frac{2m-1}{m+4}$

$k(m+4) = 2m - 1$

$km + 4k = 2m - 1$

$2m - km = 4k + 1$

$m(2 - k) = 4k + 1$

$m = \frac{4k+1}{2-k}$

(b) $\frac{3}{2b+1} + \frac{6}{2b-1}$

$= \frac{3(2b-1)+6(2b+1)}{(2b+1)(2b-1)}$

$= \frac{6b-3+12b+6}{(2b+1)(2b-1)}$

$= \frac{18b+3}{(2b+1)(2b-1)}$

(c) $\frac{9x^2-y^2}{3x^2+xy}$

$= \frac{(3x+y)(3x-y)}{x(3x+y)}$

$= \frac{3x-y}{x}$

Chapter 7 Relations and Functions

Basic

1. **(a)** No, the element a has two images.

(b) Yes.

(c) No, the element c has no image.

(d) Yes.

2. $f(x) = \frac{3}{7}x + 4$

$f(-14) = \frac{3}{7}(-14) + 4 = -2$

$f(28) = \frac{3}{7}(28) + 4 = 16$

$f\left(\frac{7}{8}\right) = \frac{3}{7}\left(\frac{7}{8}\right) + 4 = 4\frac{3}{8}$

$f\left(-\frac{2}{9}\right) = \frac{3}{7}\left(-\frac{2}{9}\right) + 4 = 3\frac{19}{21}$

3. $f(x) = 9 - 4x$

(i) $f(4) = 9 - 4(4)$
$= -7$

(ii) $f(-2) = 9 - 4(-2)$
$= 17$

(iii) $f(0) = 9 - 4(0)$
$= 9$

(iv) $f(1) + f(-5) = [9 - 4(1)] + [9 - 4(-5)]$
$= 34$

4. $g(x) = 3x - 13$

(i) $g(8) = 3(8) - 13$
$= 11$

(ii) $g(-6) = 3(-6) - 13$
$= -31$

(iii) $g\left(4\frac{1}{3}\right) = 3\left(4\frac{1}{3}\right) - 13$
$= 0$

(iv) $g(7) + g(-3) = [3(7) - 13] + [3(-3) - 13]$
$= 8 + (-22)$
$= -14$

(v) $g\left(1\frac{1}{3}\right) - g\left(-\frac{2}{3}\right) = \left[3\left(1\frac{1}{3}\right) - 13\right] - \left[3\left(-\frac{2}{3}\right) - 13\right]$
$= -9 - (-15)$
$= 6$

Intermediate

5. $h(x) = -3x + 12$

(i) $h(3a) - h(2a) = [-3(3a) + 12] - [-3(2a) + 12]$
$= -9a + 12 + 6a - 12$
$= -3a$

(ii) $h(a) = 0$
$\therefore -3a + 12 = 0$
$3a = 12$
$a = 4$

(iii) $h\left(\frac{2}{3}a\right) + h(a) = \left[-3\left(\frac{2}{3}a\right) + 12\right] + [-3a + 12]$
$= -2a + 12 - 3a + 12$
$= -5a + 24$

6. $f(x) = \frac{2}{7}x + 3,\ g(x) = \frac{1}{5}x - 4$

(a) **(i)** $f(7) + g(35) = \left[\frac{2}{7}(7) + 3\right] + \left[\frac{1}{5}(35) - 4\right]$
$= 2 + 3 + 7 - 4$
$= 8$

(ii) $f(-2) - g(-15) = \left[\frac{2}{7}(-2) + 3\right] - \left[\frac{1}{5}(-15) - 4\right]$
$= -\frac{4}{7} + 3 + 3 + 4$
$= 9\frac{3}{7}$

(iii) $3f(2) - 2g(10) = 3\left[\frac{2}{7}(2) + 3\right] - 2\left[\frac{1}{5}(10) - 4\right]$
$= \frac{12}{7} + 9 - 4 + 8$
$= 14\frac{5}{7}$

(iv) $\frac{1}{3}f(-7) - 9g(0) = \frac{1}{3}\left[\frac{2}{7}(-7) + 3\right] - 9\left[\frac{1}{5}(0) - 4\right]$
$= -\frac{2}{3} + 1 - 0 + 36$
$= 36\frac{1}{3}$

(b) **(i)** $f(x) = g(x)$
$\frac{2}{7}x + 3 = \frac{1}{5}x - 4$
$\frac{2}{7}x - \frac{1}{5}x = -4 - 3$
$\frac{3}{35}x = -7$
$x = -81\frac{2}{3}$

(ii) $f(x) = 8$
$\frac{2}{7}x + 3 = 8$
$\frac{2}{7}x = 5$
$x = 17\frac{1}{2}$

7. $f(x) = 12x - 1, g(x) = 9 - 5x$

(i) $f(x) = 23$

$12x - 1 = 23$

$12x = 24$

$x = 2$

(ii) $g(x) = 24$

$9 - 5x = 24$

$5x = -15$

$x = -3$

(iii) $g(x) = 2x$

$9 - 5x = 2x$

$7x = 9$

$x = 1\frac{2}{7}$

(iv) $f(x) = -5x$

$12x - 1 = -5x$

$17x = 1$

$x = \frac{1}{17}$

(v) $f(x) = g(x)$

$12x - 1 = 9 - 5x$

$17x = 10$

$x = \frac{10}{17}$

8. $f(x) = 11x - 7, F(x) = \frac{3}{4}x + 3$

(i) $f(p) = 11p - 7$

(ii) $F\left(8p + \frac{1}{2}\right) = \frac{3}{4}\left(8p + \frac{1}{2}\right) + 3$

$= 6p + \frac{3}{8} + 3$

$= 6p + 3\frac{3}{8}$

(iii) $f\left(\frac{3}{11}p\right) + F(4p - 12)$

$= 11\left(\frac{3}{11}p\right) - 7 + \left[\frac{3}{4}(4p - 12) + 3\right]$

$= 3p - 7 + 3p - 9 + 3$

$= 6p - 13$

Advanced

9. $f(x) = \frac{3x - 7}{8}$

(i) $f(-2) = \frac{3(-2) - 7}{8}$

$= -1\frac{5}{8}$

$f\left(2\frac{5}{6}\right) = \frac{3\left(2\frac{5}{6}\right) - 7}{8}$

$= \frac{3}{16}$

(ii) $f(x) = 5$

$\frac{3x - 7}{8} = 5$

$3x - 7 = 40$

$3x = 47$

$x = 15\frac{2}{3}$

Chapter 8 Congruence and Similarity

Basic

1. A and D, B and H, C and F, E and I

2. **(i)** QP
(ii) PC
(iii) CA
(iv) $\angle QPC$
(v) $\angle CAB$
(vi) $\angle BCA$

3. **(i)** $AB = CD$, $BD = DB$, $AD = CB$
(ii) $\angle ABD = \angle CDB$, $\angle ADB = \angle CBD$, $\angle BAD = \angle DCB$

4. **(i)** $PQ = QP$, $QS = PR$, $PS = QR$
(ii) $\angle PQS = \angle QPR$, $\angle PSQ = \angle QRP$, $\angle QPS = \angle PQR$

5. **(i)** $AB = AC$, $BQ = CP$, $AQ = AP$
(ii) $\angle ABQ = \angle ACP$, $\angle AQB = \angle APC$, $\angle BAQ = \angle CAP$

6. **(a)** $x = 40$, $y = 50$, $z = 50$
(b) $x = 44$, $y = 54$, $z = 82$
(c) $x = 6.75$, $y = 88$
(d) $x = 6.3$

7. **(a)** $\dfrac{2}{900\ 000} = \dfrac{1}{450\ 000}$
(b) $\dfrac{3}{450\ 000} = \dfrac{1}{150\ 000}$
(c) $\dfrac{0.5}{40\ 000} = \dfrac{1}{80\ 000}$
(d) $\dfrac{7.5}{10\ 500\ 000} = \dfrac{1}{1\ 400\ 000}$

8. 4 cm represents 30 m
1 cm represents 7.5 m
$\therefore$ Scale is 1 cm to 7.5 m

9. **(i)** Actual perimeter $= 2(5 + 4)(15)$
$= 270$ m
(ii) Actual area $= (5 \times 15) \times (4 \times 15)$
$= 4500\ m^2$

10. 2 cm represent 3 km
1 cm represents 1.5 km
(a) 24 cm represent 36 km
(b) 10.5 cm represent 15.75 km
(c) 14.2 cm represent 21.3 km
(d) 2.6 cm represent 3.9 km

11. 1 cm represents 0.4 km
0.4 km is represented by 1 cm
(a) 800 m is represented by 2 cm
(b) 0.2 km is represented by 0.5 cm
(c) 3.6 km is represented by 9 cm
(d) 2 km 400 m is represented by 6 cm

12. **(a)** 2 cm represents 3 km
16 cm represents 24 km
(b) 1.2 cm represents 3 km
16 cm represents 40 km
(c) 2.4 cm represents 9 km
16 cm represents 60 km
(d) 0.5 cm represents 0.25 km
16 cm represents 8 km

13. **(a)** 25 km is represented by 2 cm
480 km is represented by 38.4 cm
(b) 75 km is represented by 5 cm
480 km is represented by 32 cm
(c) 25 km is represented by 9 cm
480 km is represented by 172.8 cm
(d) 120 km is represented by 0.5 cm
480 km is represented by 2 cm

14. 1 cm represent 5 km
17.6 cm represents 88 km

15. 1 cm represents 0.25 km
(a) 18 cm represent 4.5 km
(b) 16.5 cm represent 4.125 km
(c) 65 cm represent 16.25 km
(d) 7.4 cm represent 1.85 km

16. 0.5 km is represented by 1 cm
0.25 km^2 is represented by 1 cm^2
20 km^2 is represented by 80 cm^2

17. 1 cm represents 0.2 km
1 cm^2 represents 0.04 km^2
(a) 5 cm^2 represent 0.2 km^2
(b) 18 cm^2 represent 0.72 km^2
(c) 75 cm^2 represent 3 km^2
(d) 124 cm^2 represent 4.96 km^2

18. 1 cm represents 75 km
(a) 12 cm represent 900 km
(b) 8 cm represent 600 km
(c) 20.5 cm represent 1537.5 km
(d) 22 cm represent 1650 km

19. **(i)** 1 cm represents 1.2 km
5.4 cm represent 6.48 km
(ii) 10 km 80 m is represented by 8.4 cm
(iii) 1 cm^2 represents 1.44 km^2
3.6 cm^2 represent 5.184 km^2

20. **(i)** 4 cm represent 3 km
1 cm represents 0.75 km
10.5 cm represent 7.875 km
(ii) $\dfrac{1}{75\ 000}$
(iii) 0.5625 km^2 is represented by 1 cm^2
32.4 km^2 is represented by 57.6 cm^2

21. **(i)** Actual length = 2.6×1.6

$= 4.16$ m

$\therefore$ Actual dimensions are 4.16 m by 4.16 m

(ii) Actual area = $(2.6 \times 1.6) \times (1.8 \times 1.6)$

$= 12.0 \text{ m}^2$ (to 3 s.f.)

(iii) Actual total area = $(6.0 \times 1.6) \times (6.0 \times 1.6)$

$= 92.16$

$= 92 \text{ m}^2$ (to the nearest m^2)

Intermediate

22. **(a)** $c = 48, q = 92, z = 7$

$a = p = 180 - 92 - 48$

$= 40$

$\therefore a = 40, c = 48, p = 40, q = 92, z = 7$

(b) $a = 58, b = 10, q = 8.5, y = 32$

(c) $a = 39, p = 6, q = 66$

$r = 180 - 66 - 39$

$= 75$

$\therefore a = 39, p = 6, q = 66, r = 75$

(d) $b = 6.5, p = 6, r = 7$

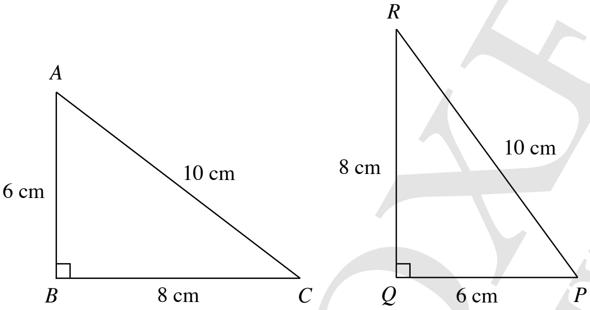
23. **(a)** $b = 102, p = 73, q = 6, s = 7$

(b) $a = 11.5, c = 42, d = 62, x = 41, y = 11$

(c) $a = 7.6, b = 8.0, p = 92, r = 57$

(d) $a = 5.8, b = 7, s = 83, x = 7.2$

24. **(a)**



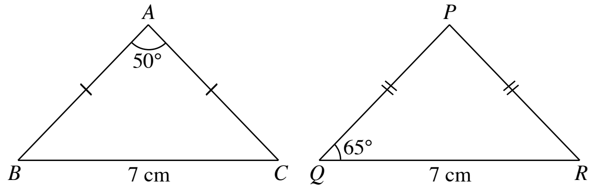
$AB = PQ = 6$ cm

$BC = QR = 8$ cm

$CA = RP = 10$ cm

$\therefore \triangle ABC \equiv \triangle PQR$

(b)



$\angle BAC = \angle QPR = 50°$

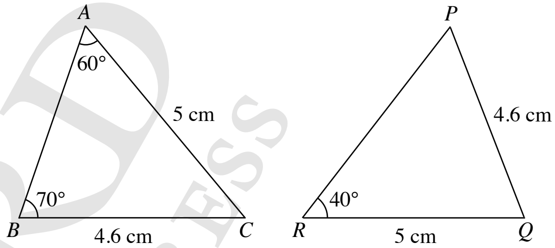
$\angle ABC = \angle PQR = 65°$

$\angle ACB = \angle PRQ = 65°$

$BC = QR = 7$ cm

$\therefore \triangle ABC \equiv \triangle PQR$

(c)



$\angle ACB = 180° - 60° - 70°$

$= 50°$

Since the triangles do not have the same shape and size, they are not congruent.

25. **(i)** $\angle ABP = 180° - 90° - 23°$

$= 67°$

(ii) Using Pythagoras' Theorem,

$AP^2 + 5^2 = 13^2$

$AP^2 = 13^2 - 5^2$

$= 144$

$AP = 12$ cm

Area of $\triangle ABC = \frac{1}{2}(10)(12)$

$= 60 \text{ cm}^2$

26. $\angle BAC = \angle EDF = 60°$

$\angle ABC = \angle DEF = 50°$

$\angle ACB = \angle DFE = 70°$

Since all the corresponding angles are equal, $\triangle ABC$ is similar to $\triangle DEF$.

27. $\frac{PQ}{XY} = \frac{3.9}{2.6} = 1.5$

$\frac{QR}{YZ} = \frac{13.9}{9.1} = 1.53$

$\frac{PR}{XZ} = \frac{12.6}{8.4} = 1.5$

Since not all the ratios of the corresponding sides are equal, $\triangle PQR$ is not similar to $\triangle XYZ$.

28. $\frac{l}{8} = \frac{3.2}{4}$

$l = \frac{3.2}{4} \times 8$

$= 6.4$

$\therefore$ Actual length is 6.4 m.

29. (i) $\frac{PS}{AD} = \frac{PQ}{AB}$

$\frac{PS}{18} = \frac{36}{24}$

$PS = \frac{36}{24} \times 18$

$= 27$

$\therefore$ Width of *PQRS* is 27 cm.

(ii) $\frac{PQ}{AB} = \frac{QR}{BC}$

$\frac{PQ}{24} = \frac{36}{18}$

$PQ = \frac{36}{18} \times 24$

$= 48$

$\therefore$ Length of *PQRS* is 48 cm.

30. (a) $\frac{h}{8.4} = \frac{1.2}{2}$

$h = \frac{1.2}{2} \times 8.4$

$= 5.04$

$\therefore$ Height of the smaller mould is 5.04 cm.

(b) $\frac{l}{7.6} = \frac{2}{1.2}$

$l = \frac{2}{1.2} \times 7.6$

$= 12.7$ (to 3 s.f.)

$\therefore$ Length of the base of the larger mould is 12.7 cm.

31. (a) $\frac{r}{5.5} = \frac{24}{10}$

$r = \frac{24}{10} \times 5.5$

$= 13.2$

$\therefore$ Radius of the larger cone is 13.2 cm.

(b) $\frac{c}{84} = \frac{10}{24}$

$c = \frac{10}{24} \times 84$

$= 35$

$\therefore$ Circumference of the smaller cone is 35 cm.

32. $\frac{7 + x}{7} = \frac{18}{8}$

$7 + x = 15\frac{3}{4}$

$x = 8\frac{3}{4}$

$\frac{y}{24} = \frac{8}{18}$

$y = \frac{8}{18} \times 24$

$= 10\frac{2}{3}$

$\therefore x = 8\frac{3}{4}, y = 10\frac{2}{3}$

33. $\frac{x}{x + 3} = \frac{3}{4}$

$4x = 3x + 9$

$x = 9$

$\frac{y}{y + 2.8} = \frac{3}{4}$

$4y = 3y + 8.4$

$y = 8.4$

$\therefore x = 9, y = 8.4$

34. (i) 5 cm represents 2 km
7 cm represents 2.8 km

(ii) 4 km is represented by 6 cm
2.8 km is represented by 4.2 cm

35. (i) 16 km^2 is represented by 1 cm^2
64 km^2 is represented by 4 cm^2

(ii) 4 km^2 is represented by 1 cm^2
64 km^2 is represented by 16 cm^2

36. 1 cm represents 15 km
14 cm represents 210 km

(a) 7 km is represented by 3 cm
210 km is represented by 90 cm

(b) 35 km is represented by 4 cm
210 km is represented by 24 cm

(c) 10.5 km is represented by 5 cm
210 km is represented by 100 cm

(d) 6 km is represented by 7 cm
210 km is represented by 245 cm

37. (a) 2 cm represent 15 m
4 cm^2 represent 225 m^2
24 cm^2 represent 1350 m^2

(b) 4 cm represent 25 m
16 cm^2 represent 625 m^2
24 cm^2 represent 937.5 m^2

(c) 4 cm represent 600 m
16 cm^2 represent 360 000 m^2
24 cm^2 represent 540 000 m^2

(d) 1.5 cm represent 120 m

2.25 cm^2 represent 14 400 m^2

24 cm^2 represent 153 600 m^2

38. 1 cm represents 0.5 km

1 cm^2 represents 0.25 km^2

36 cm^2 represent 9 km^2

(a) 0.25 km is represented by 1 cm

0.0625 km^2 is represented by 1 cm^2

9 km^2 is represented by 144 cm^2

(b) 0.125 km is represented by 1 cm

0.015 625 km^2 is represented by 1 cm^2

9 km^2 is represented by 576 cm^2

(c) 0.75 km is represented by 1 cm

0.5625 km^2 is represented by 1 cm^2

9 km^2 is represented by 16 cm^2

(d) 2 km is represented by 1 cm

4 km^2 is represented by 1 cm^2

9 km^2 is represented by 2.25 cm^2

39. 100 m^2 is represented by 1 m^2

10 m is represented by 1 m

40 m is represented by 4 m

40. 5 km is represented by 1 cm

25 km^2 is represented by 1 cm^2

225 km^2 is represented by 9 cm^2

Advanced

41. **(a)** False
(b) False
(c) True
(d) False
(e) False
(f) True
(g) False
(h) True
(i) True
(j) True
(k) False
(l) False

42. **(a)**
$$\frac{x+6}{6} = \frac{9+5}{5}$$
$$x + 6 = \frac{14}{5} \times 6$$
$$x = 10\frac{4}{5}$$
$$\frac{y}{4} = \frac{9+5}{5}$$
$$y = \frac{14}{5} \times 4$$
$$= 11\frac{1}{5}$$
$$\therefore x = 10\frac{4}{5}, y = 11\frac{1}{5}$$

(b)
$$\frac{x+9}{9} = \frac{18}{7}$$
$$x + 9 = \frac{18}{7} \times 9$$
$$x = 14\frac{1}{7}$$
$$\frac{y+8}{8} = \frac{18}{7}$$
$$y + 8 = \frac{18}{7} \times 8$$
$$y = 12\frac{4}{7}$$
$$\therefore x = 14\frac{1}{7}, y = 12\frac{4}{7}$$

(c)
$$\frac{x}{12} = \frac{10}{18}$$
$$x = \frac{10}{18} \times 12$$
$$= 6\frac{2}{3}$$
$$\frac{y + 6\frac{2}{3}}{8} = \frac{18}{10}$$
$$y + 6\frac{2}{3} = \frac{18}{10} \times 8$$
$$y = 7\frac{11}{15}$$
$$\therefore x = 6\frac{2}{3}, y = 7\frac{11}{15}$$

(d)
$$\frac{x}{x+5} = \frac{8}{12}$$
$$12x = 8x + 40$$
$$4x = 40$$
$$x = 10$$
$$\frac{y}{15} = \frac{12}{8}$$
$$y = \frac{12}{8} \times 15$$
$$= 22\frac{1}{2}$$
$$\therefore x = 10, y = 22\frac{1}{2}$$

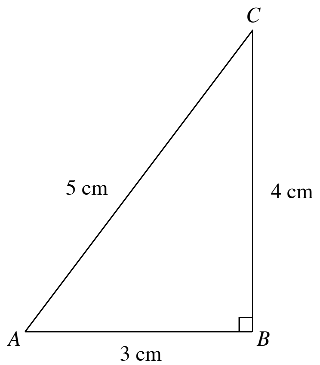
43. **(i)**
$$\frac{CQ+6}{6} = \frac{7}{4}$$
$$CQ + 6 = \frac{7}{4} \times 6$$
$$CQ = 4\frac{1}{2} \text{ cm}$$

(ii) $\dfrac{CR}{12} = \dfrac{4\frac{1}{2}}{4\frac{1}{2} + 6}$

$CR = \dfrac{4\frac{1}{2}}{4\frac{1}{2} + 6} \times 12$

$= 5\frac{1}{7}$ cm

44. Let 1 cm represent 100 m.



5 cm represents 500 m

$\therefore AC = 500$ m

45. (i) 1 cm represents 500 cm

$\therefore$ Scale is 1 : 500

(ii) Perimeter = 2(2.4 + 4)(5)

= 64 m

Area = (2.4 × 5) × (4 × 5)

= 240 m^2

46. (i) $\dfrac{PB}{PQ} = \dfrac{BA}{QR}$

$\dfrac{PB}{18} = \dfrac{16}{24}$

$PB = \dfrac{16}{24} \times 18$

= 12 cm

(ii) $\dfrac{PR}{PA} = \dfrac{QR}{BA}$

$\dfrac{PR}{9} = \dfrac{24}{16}$

$PR = \dfrac{24}{16} \times 9$

= 13.5 cm

$BR = PB + PR$

= 12 + 13.5

= 25.5 cm

47. (a) 1 cm represents 45 000 cm

1 cm represents 0.45 km

$\therefore n = 0.45$

(b) Actual distance = 32.5 × 0.45

= 14.625 km

(c) 1 cm represents 450 m

1 cm^2 represents 202 500 m^2

1 cm^2 represents 202.5 ha

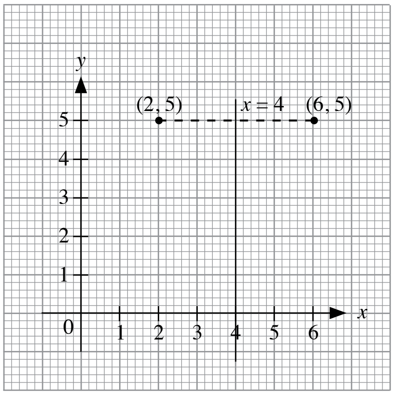
Area on the map = $\dfrac{2227}{202.5}$

= 11.0 cm^2 (to 3 s.f.)

Chapter 9 Geometrical Transformation

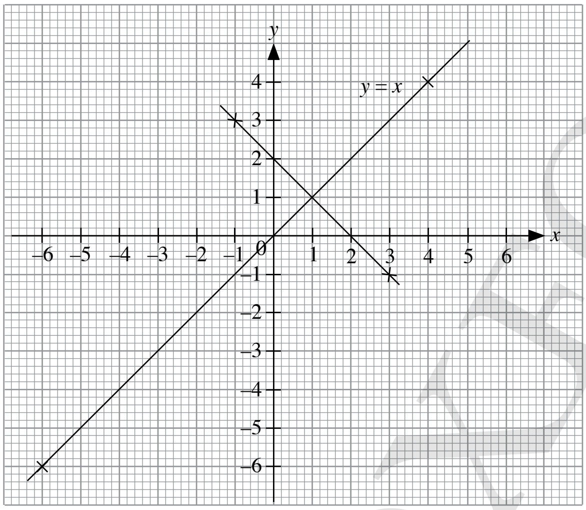
Basic

1.



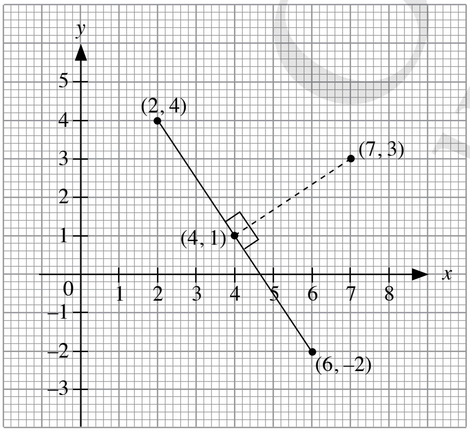
∴ The coordinates of the reflection of the point (2, 5) is (6, 5).

2.



∴ $p = -1, q = 3$

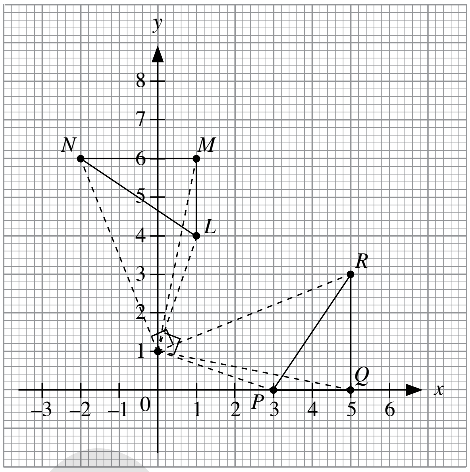
3.



(a) (7, 3)

(b) (6, –2)

4.



5. Let the translation vector T be $\begin{pmatrix} a \\ b \end{pmatrix}$.

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 9 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -7 \\ -4 \end{pmatrix}$$

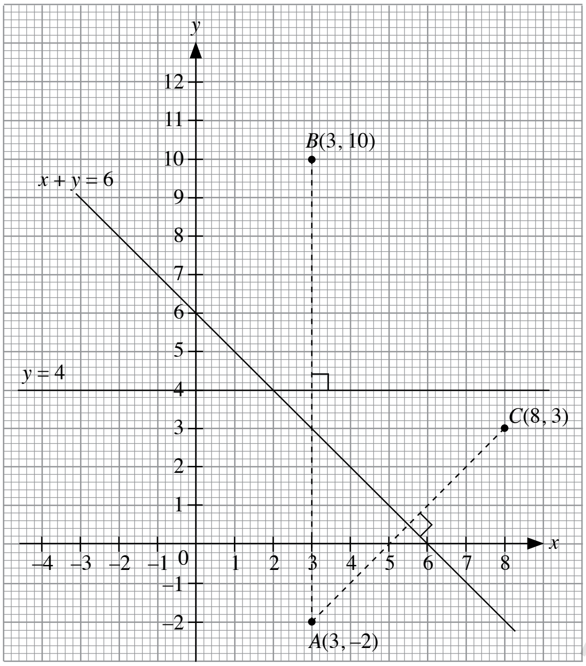
$$\begin{pmatrix} -7 \\ -4 \end{pmatrix} + Q = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$$

$$Q = \begin{pmatrix} -5 \\ 6 \end{pmatrix} - \begin{pmatrix} -7 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 10 \end{pmatrix}$$

∴ The translation vector of T is $\begin{pmatrix} -7 \\ -4 \end{pmatrix}$ and $Q(2, 10)$.

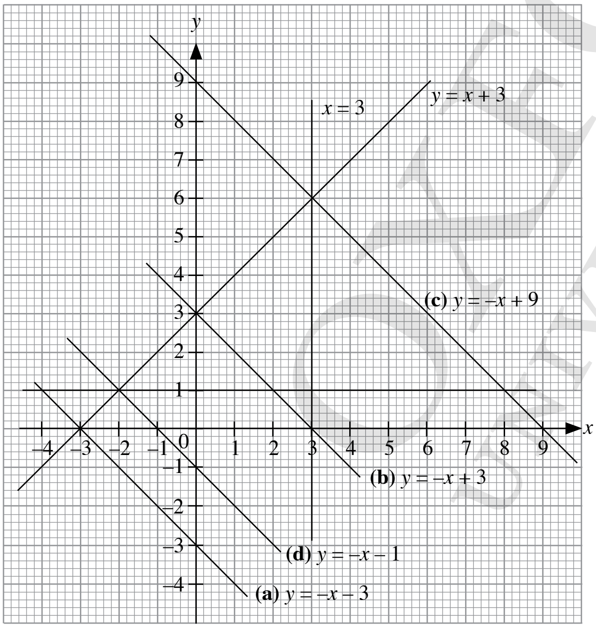
6.



(a) The coordinates of B are (3, 10).

(b) The coordinates of C are (8, 3).

7.



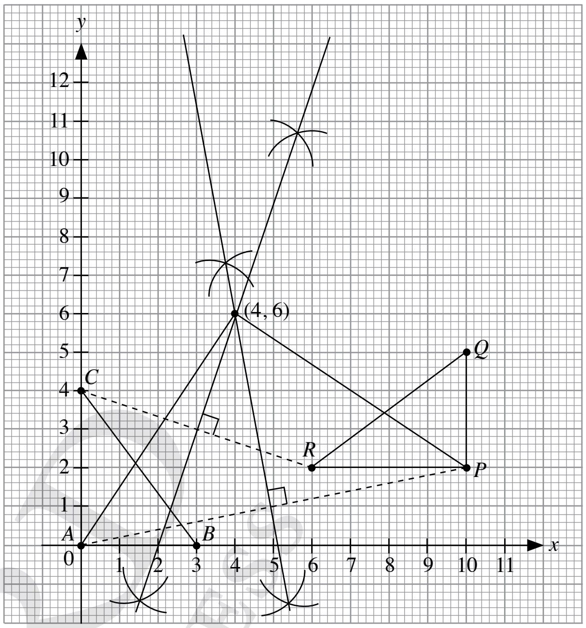
(a) $y = -x - 3$

(b) $y = -x + 3$

(c) $y = -x + 9$

(d) $y = -x - 1$

8.



(ii) The centre of rotation is (4, 6). The angle of rotation is 90° clockwise or 270° anticlockwise.

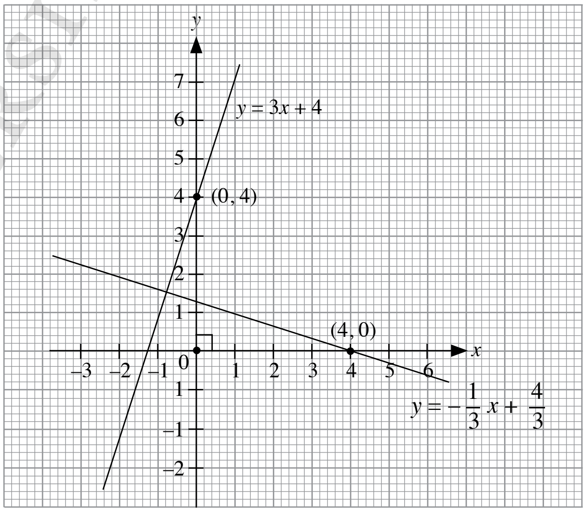
9. R^4 represents (4 × 160°) – 360°

= 280° anticlockwise rotation about the origin.

R^5 represents (5 × 160°) – 720°

= 80° anticlockwise rotation about the origin.

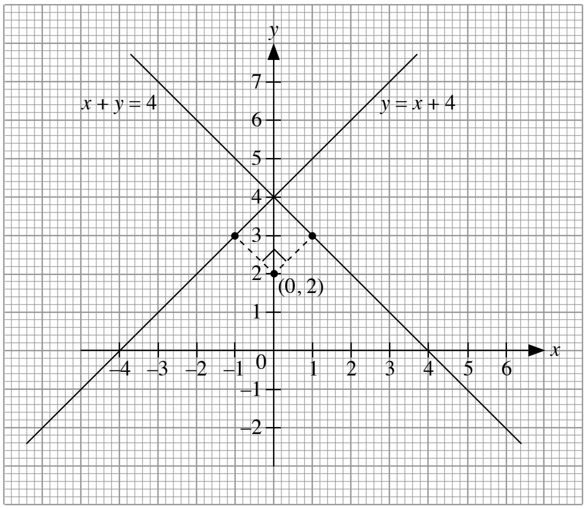
10.



∴ The equation of the image of the line

$y = 3x + 4$ is $y = -\frac{1}{3}x + \frac{4}{3}$.

11.



$\therefore$ The equation of the image of the line $x + y = 4$ is $y = x + 4$.

12. Let the translation vector T be $\begin{pmatrix} p \\ q \end{pmatrix}$.

$$\begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 9 \\ 7 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

(i) $\begin{pmatrix} 8 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ -3 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

$\therefore x = 6, y = 1$

(ii) $\begin{pmatrix} 8 \\ 4 \end{pmatrix} + \begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$$\begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$h = -4, k = 2$

13. (i) $Z\hat{X}X' = 20°$

$XZ = X'Z$

$\therefore Z\hat{X}X' = \dfrac{180° - 20°}{2}$

$= 80°$ (base $\angle$ of isos. $\triangle$)

(ii) $Y\hat{Z}Y' = 20°$

$\tan X'\hat{Z}Y = \dfrac{7}{4}$

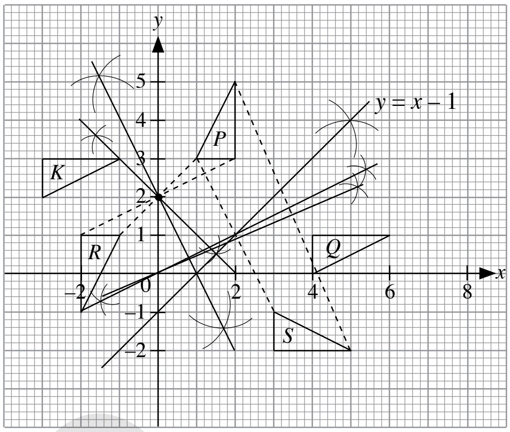
$X'\hat{Z}Y' = \tan^{-1} \dfrac{7}{4}$

$= 60.3°$ (to 1 d.p.)

$\therefore Y\hat{Z}X' = 60.3° - 20°$

$= 40.3°$

14.



(i) The line of reflection is $y = x - 1$.

(ii) The centre of rotation is $(0, 2)$ and the angle of rotation is 180°.

(iii) $\triangle K$ is mapped onto $\triangle Q$ by a translation $\begin{pmatrix} 7 \\ -2 \end{pmatrix}$.

(iv) A 90° anticlockwise rotation about (0, 0).

Revision Test B1

1.
$$x^2 - y^2 = 28$$
$$(x + y)(x - y) = 28$$
$$4(x + y) = 28$$
$$x + y = 7$$
$$(2x + 2y)^2 = 4(x + y)^2$$
$$= 4(7)^2$$
$$= 196$$

2. **(a)** $\dfrac{4b - 1}{a^2 + 3a} \times \dfrac{a + 3}{4b^2 + 11b - 3}$

$= \dfrac{4b - 1}{a(a + 3)} \times \dfrac{a + 3}{(4b - 1)(b + 3)}$

$= \dfrac{1}{a(b + 3)}$

(b)
$$4 - 4x + \frac{3}{x} = 0$$
$$4x - 4x^2 + 3 = 0$$
$$4x^2 - 4x - 3 = 0$$
$$(2x - 3)(2x + 1) = 0$$
$$x = \frac{3}{2} \quad \text{or} \quad x = -\frac{1}{2}$$
$$= 1\frac{1}{2}$$

$\therefore x = 1\frac{1}{2}$ or $x = -\frac{1}{2}$

3. **(i)**
$$(2p - q)(r + 5) = r(p - 1)$$
$$2pr + 10p - qr - 5q = pr - r$$
$$10p - 5q = qr - pr - r$$
$$= r(q - p - 1)$$
$$r = \frac{10p - 5q}{q - p - 1}$$

(ii) When $p = 6$, $q = -3$,

$r = \dfrac{10(6) - 5(-3)}{-3 - 6 - 1}$

$= -7\frac{1}{2}$

4. **(i)** $x = -1$ or $x = 6$

(ii) $x = \dfrac{-1 + 6}{2}$

$= 2\frac{1}{2}$

$\therefore$ Equation of line of symmetry is $x = 2\frac{1}{2}$.

(iii) When $x = 2\frac{1}{2}$,

$y = \left(2\frac{1}{2}\right)^2 - 5\left(2\frac{1}{2}\right) - 6$

$= -12\frac{1}{4}$

$\therefore$ Minimum value of y is $-12\frac{1}{4}$ when $x = 2\frac{1}{2}$.

5. $h(x) = 5x + \dfrac{4}{5}$

$h(-3) = 5(-3) + \dfrac{4}{5}$

$= -14\frac{1}{5}$

$h(4) = 5(4) + \dfrac{4}{5}$

$= 20\frac{4}{5}$

$h\left(\dfrac{7}{25}\right) = 5\left(\dfrac{7}{25}\right) + \dfrac{4}{5}$

$= 2\frac{1}{5}$

$h\left(-\dfrac{9}{10}\right) = 5\left(-\dfrac{9}{10}\right) + \dfrac{4}{5}$

$= -3\frac{7}{10}$

6. **(i)** $\triangle ADO$

(ii) $\triangle COB$

(iii) $\triangle ADC$

7. $\dfrac{x + 8}{8} = \dfrac{21 + 7}{7}$

$x + 8 = \dfrac{28}{7} \times 8$

$= 32$

$x = 24$

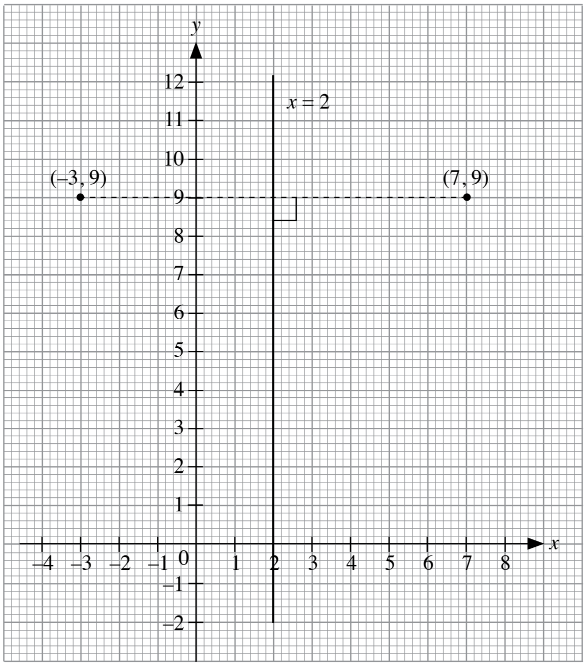
$\dfrac{y}{6} = \dfrac{21 + 7}{7}$

$y = \dfrac{28}{7} \times 6$

$= 24$

$\therefore x = 24, y = 24$

8.



$\therefore$ The coordinates of the reflection of (7, 9) in $x = 2$ is (–3, 9).

9. Let the translation vector T be $\begin{pmatrix} a \\ b \end{pmatrix}$.

$$\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 8 \\ -7 \end{pmatrix} = A + \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 8 \\ -7 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -10 \end{pmatrix}$$

$\therefore$ The coordinates of A are (6, –10).

10. Let h be the height of the larger jar.

$$\frac{h}{12} = \frac{6}{4}$$

$$h = \frac{6}{4} \times 12$$

$$= 18$$

$\therefore$ Height of larger jar is 18 cm.

11. **(i)** 5 km is represented by 4 cm

40 km is represented by 32 cm

(ii) 16 cm^2 represents 25 km^2

12 cm^2 represents 18.75 km^2

18.75 km^2 = 187 500 ha

12. **(i)** $\frac{560}{x}$

(ii) $\frac{560}{x} - \frac{560}{x+1} = 0$

$560(x + 1) - 560x = 10x(x + 1)$

$560x + 560 - 560x = 10x^2 + 10x$

$10x^2 + 10x - 560 = 0$

$x^2 + x - 56 = 0$ (shown)

(iii) $(x + 8)(x - 7) = 0$

$x = -8$ or $x = 7$

(rejected) $\frac{560}{x} = 80$

$\therefore$ Original price of each casing is \$7, number of casings bought is 80.

13. **(a)** When $x = 2$, $y = a$,

$a = 5 + 4(2) - 2^2$

$= 9$

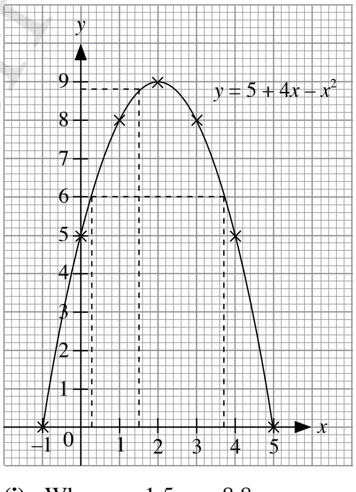
When $x = 3$, $y = b$,

$b = 5 + 4(3) - 3^2$

$= 8$

$\therefore a = 9, b = 8$

(b)



(c) **(i)** When $x = 1.5$, $y = 8.8$

(ii) When $y = 6$, $x = 0.3$ or $x = 3.7$

Revision Test B2

1. **(a)** $20.75^2 - 0.75^2$

$= (20.75 + 0.75)(20.75 - 0.75)$

$= (21.5)(20)$

$= 430$

(b) 1597×1603

$= (1600 - 3)(1600 + 3)$

$= 1600^2 - 3^2$

$= 2\,560\,000 - 9$

$= 2\,559\,991$

2. **(a)** $\dfrac{p^2 - q^2}{5(p^2 + 2pq + q^2)} \div \dfrac{q^2 + p^2 - 2pq}{25(p+q)}$

$= \dfrac{(p+q)(p-q)}{5(p+q)^2} \times \dfrac{25(p+q)}{(p-q)^2}$

$= \dfrac{5}{p-q}$

(b) $\dfrac{w}{2w-5} - \dfrac{1}{10-4w}$

$= \dfrac{w}{2w-5} + \dfrac{1}{2(2w-5)}$

$= \dfrac{2w+1}{2(2w-5)}$

3. **(i)** $\dfrac{5b - ac^2}{3bc^2 - 4a} = \dfrac{2}{3}$

$15b - 3ac^2 = 6bc^2 - 8a$

$3ac^2 + 6bc^2 = 8a + 15b$

$c^2(3a + 6b) = 8a + 15b$

$c^2 = \dfrac{8a + 15b}{3a + 6b}$

$c = \sqrt{\dfrac{8a + 15b}{3a + 6b}}$

(ii) When $a = 2$, $b = 1$,

$c = \sqrt{\dfrac{8(2) + 15(1)}{3(2) + 6(1)}}$

$= 1.61$ (to 3 s.f.)

4. $y = (3 - x)(2x + 3)$

When $y = 0$,

$x = 3$ or $x = -\dfrac{3}{2}$

$= -1\dfrac{1}{2}$

$A\left(-1\dfrac{1}{2}, 0\right)$, $C(3, 0)$

At maximum point,

$x = \dfrac{-1\frac{1}{2} + 3}{2}$

$= \dfrac{3}{4}$

When $x = \dfrac{3}{4}$,

$y = \left(3 - \dfrac{3}{4}\right)\left[2\left(\dfrac{3}{4}\right) + 3\right]$

$= 10\dfrac{1}{8}$

$\therefore$ Coordinates of the maximum point are $\left(\dfrac{3}{4}, 10\dfrac{1}{8}\right)$.

5. Let the lengths of the two squares be x cm and $(72 - x)$ cm.

$\left(\dfrac{x}{4}\right)^2 + \left(\dfrac{72 - x}{4}\right)^2 = 170$

$\dfrac{x^2}{16} + \dfrac{5184 - 144x + x^2}{16} = 170$

$x^2 + 5184 - 144x + x^2 = 2720$

$2x^2 - 144x + 2464 = 0$

$x^2 - 72x + 1232 = 0$

$(x - 28)(x - 44) = 0$

$x = 28$ or $x = 44$

$72 - x = 44$ $\quad 72 - x = 28$

$\therefore$ The length of each part is 28 cm and 44 cm respectively.

6. **(i)** $f(x) = \dfrac{2}{5}x - 4$

$f(a) = \dfrac{2}{5}a - 4$

(ii) $F(x) = 8x + 3$

$F\left(\dfrac{1}{8} - \dfrac{1}{2}a\right) = 8\left(\dfrac{1}{8} - \dfrac{1}{2}a\right) + 3$

$= 1 - 4a + 3$

$= 4 - 4a$

(iii) $f(5a) + F(2a - 3) = \dfrac{2}{5}(5a) - 4 + 8(2a - 3) + 3$

$= 2a - 4 + 16a - 24 + 3$

$= 18a - 25$

7. $\angle BAC = 180° - 90° - 45°$

$= 45°$

Since the triangles do not have the same shape and size, they are not congruent.

8. $\frac{BC}{QR} = \frac{AB}{PQ}$

$\frac{x}{10} = \frac{8}{14}$

$x = \frac{8}{14} \times 10$

$= 5\frac{5}{7}$

$\frac{PR}{AC} = \frac{PQ}{AB}$

$\frac{y}{10} = \frac{14}{8}$

$y = \frac{14}{8} \times 10$

$= 17\frac{1}{2}$

$\therefore x = 5\frac{5}{7}, y = 17\frac{1}{2}$

9. Let h be the height of the smaller rocket.

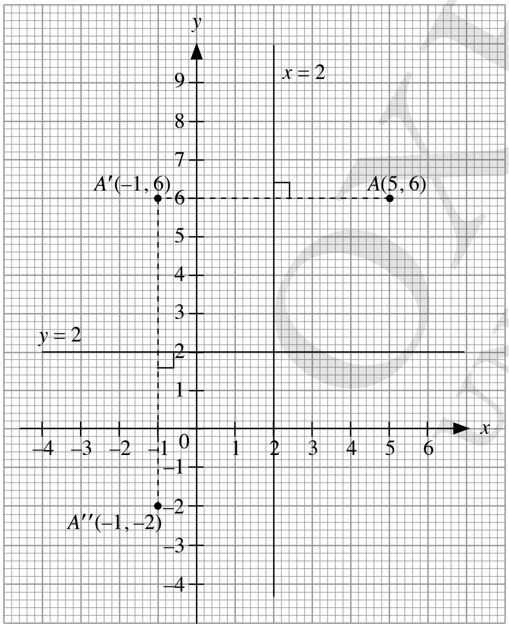
$\frac{h}{24} = \frac{5}{7}$

$h = \frac{5}{7} \times 24$

$= 17\frac{1}{7}$

$\therefore$ Height of smaller rocket is $17\frac{1}{7}$ cm.

10.



(i) The coordinates of A under the two reflections are $(-1, -2)$.

(ii) The point which remains invariant is $(2, 2)$.

11. (i) 1 cm represents 0.75 km

0.75 km is represented by 1 cm

15 km is represented by 20 cm

(ii) 46 cm represent 34.5 km

(iii) 1 cm^2 represents 0.5625 km^2

8 cm^2 represent 4.5 km^2

(iv) 3 cm^2 represent 1.6875 km^2

0.25 km is represented by 1 cm

0.0625 km^2 is represented by 1 cm^2

1.6875 km^2 is represented by 27 cm^2

12. (a) When $x = -2, y = a$,

$a = 3(-2)^2 - 4(-2) - 30$

$= -10$

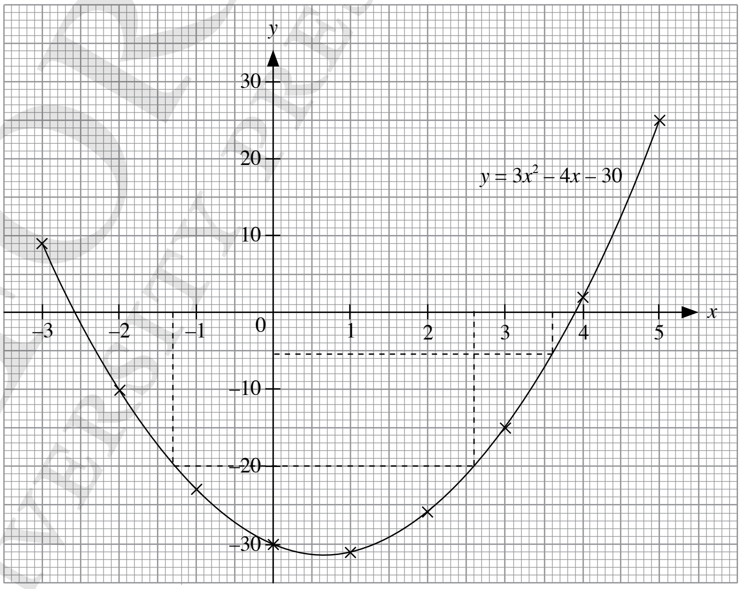
When $x = 1, y = b$,

$b = 3(1)^2 - 4(1) - 30$

$= -31$

$\therefore a = -10, b = -31$

(b)



(c) **(i)** When $x = 3.6, y = -5.5$

(ii) When $y = 0, x = 3.9$ or $x = -2.6$

(iii) When $y = -20, x = 2.6$ or $x = -1.3$

Mid-Year Examination Specimen Paper A

Part I

1. Average speed $= \dfrac{1200\text{ m}}{6 \text{ minutes}}$

$= \dfrac{1200 \div 1000}{6 \div 60}$

$= 12$ km/h

2. **(a)** $5x(x - 3) = 0$

$x = 0 \quad \text{or} \quad x = 3$

(b) $6y^2 + y - 1 = 0$

$(3y - 1)(2y + 1) = 0$

$y = \dfrac{1}{3} \quad \text{or} \quad y = -\dfrac{1}{2}$

3. $3x - y = 13 \quad —(1)$

$\dfrac{x}{3} - \dfrac{y}{4} = 1 \quad —(2)$

$(1) \times 3$: $9x - 3y = 39 \quad —(3)$

$(2) \times 12$: $4x - 3y = 12 \quad —(4)$

$(3) - (4)$: $5x = 27$

$x = 5\dfrac{2}{5}$

Substitute $x = 5\dfrac{2}{5}$ into (1):

$3\left(5\dfrac{2}{5}\right) - y = 13$

$16\dfrac{1}{5} - y = 13$

$y = 16\dfrac{1}{5} - 13$

$y = 3\dfrac{1}{5}$

$\therefore x = 5\dfrac{2}{5}, y = 3\dfrac{1}{5}$

4. **(a)** $40 - 10x^2$

$= 10(4 - x^2)$

$= 10(2 + x)(2 - x)$

(b) $2ac - 2bc - bd + ad$

$= 2c(a - b) + d(a - b)$

$= (a - b)(2c + d)$

5. $3(a^2 + b^2)$

$= 3[(a + b)^2 - 2ab]$

$= 3\left[189 - 2\left(\dfrac{78}{6}\right)\right]$

$= 489$

6. **(i)** $a^2 - b^2 = (a + b)(a - b)$

(ii) $88.74^2 - 11.26^2 = (88.74 + 11.26)(88.74 - 11.26)$

$= (100)(77.48)$

$= 7748$

7. **(i)** $3x - y^2 = ax + b$

$3x - ax = y^2 + b$

$x(3 - a) = y^2 + b$

$x = \dfrac{y^2 + b}{3 - a}$

(ii) When $a = 5, b = 7, y = -1$,

$x = \dfrac{(-1)^2 + 7}{3 - 5}$

$= -4$

8. $(2x - y)(x + 3y) - x(2x - 3y)$

$= 2x^2 + 6xy - xy - 3y^2 - 2x^2 + 3xy$

$= 8xy - 3y^2$

9. **(a)** $\dfrac{a}{3} - \dfrac{a - 2}{6}$

$= \dfrac{2a - (a - 2)}{6}$

$= \dfrac{a + 2}{6}$

(b) $\dfrac{5}{m} - \dfrac{7}{mn}$

$= \dfrac{5n - 7}{mn}$

(c) $\dfrac{2}{3p + 4q} + \dfrac{3p}{16q^2 - 9p^2} - \dfrac{5}{3p - 4q}$

$= \dfrac{2}{3p + 4q} - \dfrac{3p}{(3p + 4q)(3p - 4q)} - \dfrac{5}{3p - 4q}$

$= \dfrac{2(3p - 4q) - 3p - 5(3p + 4q)}{(3p + 4q)(3p - 4q)}$

$= \dfrac{6p - 8q - 3p - 15p - 20q}{(3p + 4q)(3p - 4q)}$

$= \dfrac{-12p - 28q}{(3p + 4q)(3p - 4q)}$

$= \dfrac{12p + 28q}{(4q + 3p)(4q - 3p)}$

10. $\left(1 - \dfrac{25}{4x^2}\right) \div \left(1 - \dfrac{5}{2x}\right)$

$= \dfrac{4x^2 - 25}{4x^2} \div \dfrac{2x - 5}{2x}$

$= \dfrac{(2x + 5)(2x - 5)}{4x^2} \times \dfrac{2x}{2x - 5}$

$= \dfrac{2x + 5}{2x}$

11. $\frac{1}{3-x} + \frac{1}{1-2x} = 6\left(\frac{1}{3-4x}\right)$

$\frac{1-2x+3-x}{(3-x)(1-2x)} = \frac{6}{3-4x}$

$\frac{4-3x}{(3-x)(1-2x)} = \frac{6}{3-4x}$

$(4-3x)(3-4x) = 6(3-x)(1-2x)$

$12 - 16x - 9x + 12x^2 = 6(3 - 6x - x + 2x^2)$

$12 - 25x + 12x^2 = 18 - 42x + 12x^2$

$17x = 6$

$x = \frac{6}{17}$

12. Let the cost of each bracelet and each pair of earrings be \$$x$ and \$$y$ respectively.

$3x + 6y = 1140$ —(1)

$7x + 9y = 1910$ —(2)

$(1) \times \frac{3}{2}: \frac{9}{2}x + 9y = 1710$ —(3)

$(2) - (3): \frac{5}{2}x = 200$

$x = 80$

Substitute $x = 80$ into (1):

$3(80) + 6y = 1140$

$240 + 6y = 1140$

$6y = 900$

$y = 150$

∴ Each bracelet costs \$80 and each pair of earrings costs \$150.

13. $R = \frac{k}{d^2}$

When $d = 2$, $R = 23$,

$23 = \frac{k}{2^2}$

$k = 23 \times 4$

$= 92$

$\therefore R = \frac{92}{d^2}$

When $d = 2.3$,

$R = \frac{92}{2.3^2}$

$= 17.4$ (to 3 s.f.)

∴ The resistance is 17.4 ohms.

14. On map A,

2 cm represent 5 km

4 cm^2 represent 25 km^2

72 cm^2 represent 450 km^2

On map B,

4 km is represented by 3 cm

16 km^2 is represented by 9 cm^2

450 km^2 is represented

by $253\frac{1}{8}$ cm^2

∴ The forest is represented by an area of $253\frac{1}{8}$ cm^2 on map B.

15. **(a)** Yes

(b) No, the relation is not a function since the element 3 in the domain has two images, p and s in the codomain.

16. **(i)** $\triangle ABD$ and $\triangle BCD$

(ii) $\frac{CD}{CB} = \frac{BD}{AB}$

$\frac{CD}{a} = \frac{x}{c}$

$CD = \frac{ax}{c}$

17. **(a)** $\angle ACT = 180° - 56° - 78°$ ($\angle$ sum of $\triangle CAT$)

$= 46°$

$\angle DGO = 180° - 46° - 78°$ ($\angle$ sum of $\triangle OGD$)

$= 56°$

$C \leftrightarrow O$

$A \leftrightarrow G$

$T \leftrightarrow D$

$AT = GD = 9$ cm

$CA = OG = 12.2$ cm

$CT = OD = 10.4$ cm

$\therefore \triangle CAT \equiv \triangle OGD$

(b) $\angle RUN = 180° - 78° - 56°$ ($\angle$ sum of $\triangle RUN$)

$= 46°$

$\angle EPI = 180° - 56° - 46°$ ($\angle$ sum of $\triangle PIE$)

$= 78°$

$R \leftrightarrow P$

$U \leftrightarrow E$

$N \leftrightarrow I$

Since $RU \neq PE$, $UN \neq EI$ and $RN \neq PI$, $\triangle RUN$ is not congruent to $\triangle PIE$.

18. R^3 represents $(3 \times 130°) - 360° = 30°$ anticlockwise about the origin.

R^5 represents $(5 \times 130°) - 360° = 290°$ anticlockwise about the origin.

Part II

Section A

1. $\dfrac{x-1}{2x+3} = y + 4$

$x - 1 = (y + 4)(2x + 3)$

$= 2xy + 3y + 8x + 12$

$2xy + 7x = -3y - 13$

$x(2y + 7) = -(3y + 13)$

$x = -\dfrac{3y+13}{2y+7}$

2. **(a)** $24m^2 - 13m - 2$

$= (8m + 1)(3m - 2)$

(b) $2a^2 - ap - 2ac + pc$

$= a(2a - p) - c(2a - p)$

$= (a - c)(2a - p)$

(c) $64x^2 - 25y^2 - (8x - 5y)$

$= (8x + 5y)(8x - 5y) - (8x - 5y)$

$= (8x - 5y)(8x + 5y - 1)$

3. **(a)** $3x - 4 - 7(3 - 2x) = 0$

$3x - 4 - 21 + 14x = 0$

$17x = 25$

$x = \dfrac{25}{17}$

$= 1\dfrac{8}{17}$

(b) $(8y - 5)^2 = 98 - (y + 9)^2$

$64y^2 - 80y + 25 = 98 - y^2 - 18y - 81$

$65y^2 - 62y + 8 = 0$

$(13y - 2)(5y - 4) = 0$

$y = \dfrac{2}{13}$ or $y = \dfrac{4}{5}$

4. $x - 2y = 3$ —(1)

$6y - 3x = 4$ —(2)

(1) × (–3): $6y - 3x = -9$ —(3)

The two equations represent two parallel lines which do not meet.

5. **(i)** 4 cm represents 5 km

1 cm represents 1.25 km

∴ Scale is 1 : 125 000

(ii) 9.4 cm represents 11.75 km

(iii) 1.5625 km^2 is represented by 1 cm^2

64 km^2 is represented by 40.96 cm^2

6. $V = kr^2$ —(1)

$1.96V = kR^2$ —(2)

(2) ÷ (1): $\dfrac{R^2}{r^2} = 1.96$

$R^2 = 1.96r^2$

$R = 1.4r$

∴ The radius will increase by 40%.

Section B

7. **(a)** $\dfrac{9x-15}{9x^2-25} = \dfrac{3(3x-5)}{(3x+5)(3x-5)}$

$= \dfrac{3}{3x+5}$

(b) $\dfrac{(3y-2)(y-2)-5y}{y-4} = \dfrac{3y^2-6y-2y+4-5y}{y-4}$

$= \dfrac{3y^2-13y+4}{y-4}$

$= \dfrac{(3y-1)(y-4)}{y-4}$

$= 3y - 1$

8. Let the numbers be x and $x + 2$.

$x^2 + (x + 2)^2 = 1460$

$x^2 + x^2 + 4x + 4 = 1460$

$2x^2 + 4x - 1456 = 0$

$x^2 + 2x - 728 = 0$

$(x + 28)(x - 26) = 0$

$x = -28$ or $x = 26$

$x + 2 = 28$

∴ The numbers are 26 and 28.

9. **(a)** $p = a + bq$

When $q = \dfrac{1}{6}$, $p = 6$,

$a + \dfrac{1}{6}b = 6$ —(1)

When $q = \dfrac{1}{3}$, $p = 10$,

$a + \dfrac{1}{3}b = 10$ —(2)

(1) × 6: $6a + b = 36$ —(3)

(2) × 3: $3a + b = 30$ —(4)

(b) (1) – (2): $3a = 6$

$a = 2$

Substitute $a = 2$ into (4):

$3(2) + b = 30$

$6 + b = 30$

$b = 24$

(c) $p = 2 + 24q$

(i) When $q = 2$,

$p = 2 + 24(2)$

$= 50$

(ii) When $p = 0$,

$0 = 2 + 24q$

$24q = -2$

$q = -\dfrac{1}{12}$

10. **(i)** $\dfrac{480}{x}$

(ii) $\dfrac{480}{x-2}$

(iii) $\dfrac{480}{x-2} - \dfrac{480}{x} = 8$

$480x - 480(x-2) = 8x(x-2)$

$480x - 480x + 960 = 8x^2 - 16x$

$8x^2 - 16x - 960 = 0$

$x^2 - 2x - 120 = 0$ (shown)

(iv) $(x-12)(x+10) = 0$

$x = 12$ or $x = -10$

$\dfrac{480}{x} = 40$

∴ Mr Lim's car used 40 l to travel 480 km.

11. **(a)** When $x = -1$, $y = a$,

$a = 5 - (-1) - (1)^2$

$= 5$

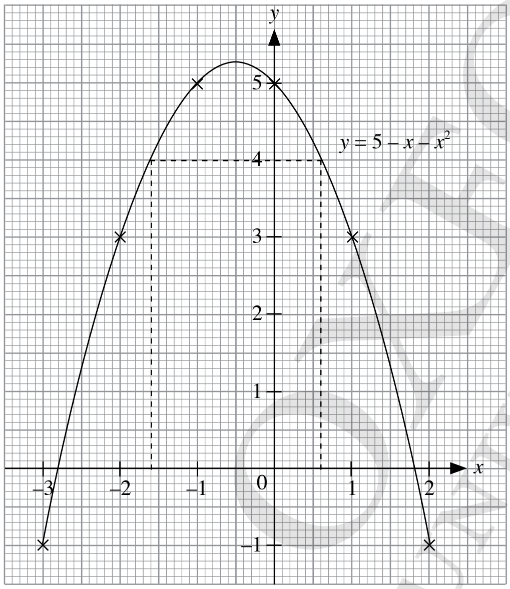
When $x = 2$, $y = b$,

$b = 5 - 2 - 2^2$

$= -1$

$\therefore a = 5, b = -1$

(b)



(c) **(i)** The equation of the line of symmetry of the graph is $x = -0.5$.

(ii) Greatest value of $y = 5.25$.

(iii) When $y = 4$, $x = 0.6$ or $x = -1.6$.

Mid-Year Examination Specimen Paper B

Part I

1. $3x - 4y = 9$ —(1)

$4x + 5y = 43$ —(2)

$(1) \times 4$: $12x - 16y = 36$ —(3)

$(2) \times 3$: $12x + 15y = 129$ —(4)

$(4) - (3)$: $31y = 93$

$y = 3$

Substitute $y = 3$ into (1):

$3x - 4(3) = 9$

$3x - 12 = 9$

$3x = 21$

$x = 7$

$\therefore x = 7, y = 3$

2. **(a)** $(x + 2)(x + 3) - x(x - 3)$

$= x^2 + 3x + 2x + 6 - x^2 + 3x$

$= 8x + 6$

(b) $(2x + y)(x - y) - 2x(x - 2y)$

$= 2x^2 - 2xy + xy - y^2 - 2x^2 + 4xy$

$= 3xy - y^2$

3. **(a)** $12p^3 - 3pq^2$

$= 3p(4p^2 - q^2)$

$= 3p(2p + q)(2p - q)$

(b) $6ax + 3bx - 6ay - 3by$

$= 3(2ax + bx + 2ay - by)$

$= 3[x(2a + b) - y(2a + b)]$

$= 3(2a + b)(x - y)$

4. $6x - \dfrac{6}{x} = 5$

$6x^2 - 6 = 5x$

$6x^2 - 5x - 6 = 0$

$(2x - 3)(3x + 2) = 0$

$x = \dfrac{3}{2}$ or $x = -\dfrac{2}{3}$

$= 1\dfrac{1}{2}$

5. $a^2 - b^2 = 72$

$(a + b)(a - b) = 72$

$6(a - b) = 72$

$a - b = 12$

$b - a = -12$

$3b - 3a = -36$

6. **(a)** $\dfrac{4}{x - 3} - \dfrac{5}{x}$

$= \dfrac{4x - 5(x - 3)}{x(x - 3)}$

$= \dfrac{4x - 5x + 15}{x(x - 3)}$

$= \dfrac{15 - x}{x(x - 3)}$

(b) $\left(\dfrac{x}{y^2 - xy} + \dfrac{y}{x^2 - xy}\right) \div \dfrac{x + y}{xy}$

$= \left(\dfrac{x}{y(y - x)} + \dfrac{y}{x(x - y)}\right) \times \dfrac{xy}{x + y}$

$= \dfrac{y^2 - x^2}{xy(x - y)} \times \dfrac{xy}{x + y}$

$= \dfrac{(x + y)(y - x)}{xy(x - y)} \times \dfrac{xy}{x + y}$

$= -1$

7. $x - \dfrac{2y}{7} = \dfrac{3y}{5a} + 2$

$35ax - 10ay = 21y + 70a$

$10ay + 21y = 35ax - 70a$

$y(10a + 21) = 35ax - 70a$

$y = \dfrac{35ax - 70a}{10a + 21}$

8. **(i)** $y = ax^2 + bx + 5$

When $x = 1$, $y = 0$,

$0 = a(1)^2 + b(1) + 5$

$a + b = -5$ —(1)

When $x = 3$, $y = 2$,

$2 = a(3)^2 + b(3) + 5$

$9a + 3b = -3$

$3a + b = -1$ —(2)

(ii) $(2) - (1)$: $2a = 4$

$a = 2$

Substitute $a = 2$ into (1):

$2 + b = -5$

$b = -7$

$\therefore$ Equation of the curve is $y = 2x^2 - 7x + 5$

9. **(i)** $(2x + 1)(x - 1) = 90$

$2x^2 - 2x + x - 1 = 90$

$2x^2 - x - 91 = 0$ (shown)

(ii) $(x - 7)(2x + 13) = 0$

$x = 7$ or $x = -\dfrac{13}{2}$

$= -6\dfrac{1}{2}$

(iii) Perimeter $= 2[2(7) + 1 + 7 - 1]$

$= 42$ cm

10. 1 cm represents 0.2 km

1 cm^2 represents 0.04 km^2

40 cm^2 represent 1.6 km^2

0.05 km is represented by 1 cm

0.0025 km^2 is represented by 1 cm^2

1.6 km^2 is represented by 640 cm^2

11. (a) $y = kx^3$

When $x = 3$, $y = 108$,

$108 = k(3)^3$

$= 27k$

$k = 4$

$\therefore y = 4x^3$

(b) $y = k(x + 4)$

When $x = 1$, $y = 10$,

$10 = k(1 + 4)$

$= 5k$

$k = 2$

$\therefore y = 2(x + 4)$

When $x = 2$, $y = a$,

$a = 2(2 + 4)$

$= 12$

When $x = b$, $y = 5$,

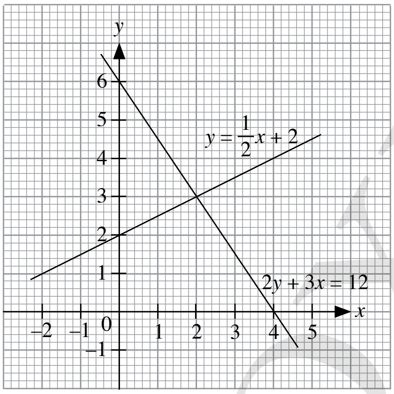
$5 = 2(b + 4)$

$\frac{5}{2} = b + 4$

$b = -1\frac{1}{2}$

$\therefore a = 12, b = -1\frac{1}{2}$

12. (i)



(ii) $x = 2, y = 3$

13. $f(x) = 5x - \frac{2}{3}$

$f(3) = 5(3) - \frac{2}{3}$

$= 14\frac{1}{3}$

$f(-5) = 5(-5) - \frac{2}{3}$

$= -25\frac{2}{3}$

$f\left(\frac{2}{5}\right) = 5\left(\frac{2}{5}\right) - \frac{2}{3}$

$= 1\frac{1}{3}$

$f\left(-\frac{3}{5}\right) = 5\left(-\frac{3}{5}\right) - \frac{2}{3}$

$= -3\frac{2}{3}$

14. $AP = PB = BR = RC = CQ = QA$

$\angle PAQ = \angle APQ = \angle AQP = 60°$

$\angle BPR = \angle PBR = \angle PRB = 60°$

$\angle RQC = \angle QRC = \angle QCR = 60°$

$\angle PRQ = \angle RPQ = \angle PQR = 60°$

Since $\triangle APQ$ and $\triangle RPQ$ are equilateral triangles with sides of equal length, $\triangle APQ$ is congruent to $\triangle RPQ$.

15. $\frac{x + 9}{9} = \frac{5 + 7}{7}$

$x + 9 = \frac{12}{7} \times 9$

$x = 6\frac{3}{7}$

$\frac{y}{6} = \frac{5 + 7}{7}$

$y = \frac{12}{7} \times 6$

$= 10\frac{2}{7}$

$\therefore x = 6\frac{3}{7}, y = 10\frac{2}{7}$

16. (a) Length of $P'Q'$ = Length of PQ

= 4 units

x-coordinate of $Q' = 4 + 4$

$= 8$

The coordinates of Q' are $(8, 2)$.

$\therefore k = 8$

(b) Since $(2, 3.5)$ is 1.5 units away from P, its image will be 1.5 units away from P' i.e. $(5.5, 2)$

(c) Since $(7, 2)$ is 1 unit away from Q', the coordinates of the original point will be 1 unit away from Q i.e. $(2, 5)$.

Part II

Section A

1. (a) (i) $y = k(2x + 1)^2$

When $x = 2$, $y = 75$,

$75 = k[2(2) + 1]^2$

$= 25k$

$k = 3$

$\therefore y = 3(2x + 1)^2$

(ii) When $x = 3$,

$y = 3[2(3) + 1]^2$

$= 147$

(b) (i) $F = \dfrac{k}{\sqrt[3]{R}}$

When $R = 125, F = 4$,

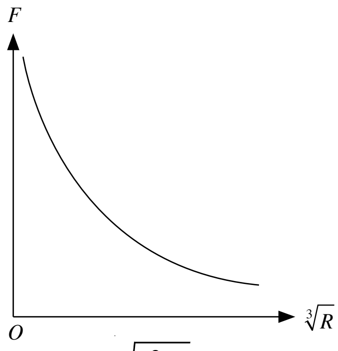
$4 = \dfrac{k}{\sqrt[3]{125}}$

$= \dfrac{k}{5}$

$k = 20$

$\therefore F = \dfrac{20}{\sqrt[3]{R}}$

(ii)



2. (a) (i) $y = \sqrt{\dfrac{2x}{x+3}}$

$y^2 = \dfrac{2x}{x+3}$

$y^2(x + 3) = 2x$

$2x - xy^2 = 3y^2$

$x(2 - y^2) = 3y^2$

$x = \dfrac{3y^2}{2 - y^2}$

(ii) When $y = 1$,

$x = \dfrac{3(1)^2}{2-(1)^2}$

$= 3$

(b) $\dfrac{3}{4x-2y} + \dfrac{5}{y-2x} = \dfrac{3}{2y-4x} + \dfrac{2(5)}{2(y-2x)}$

$= \dfrac{7}{2y-4x}$

3. (i) $A(-1, 0), B(5, 0)$

(ii) $x = \dfrac{-1+5}{2}$

$= 2$

When $x = 2$,

$y = 2(2 + 1)(2 - 5)$

$= -18$

$\therefore$ Coordinates of minimum point are $(2, -18)$.

4. (i) $\dfrac{80}{x}$ h

(ii) $\dfrac{80}{x-3}$ h

(iii) $\dfrac{80}{x-3} - \dfrac{80}{x} = \dfrac{80}{60}$

$\dfrac{1}{x-3} - \dfrac{1}{x} = \dfrac{1}{60}$

$60x - 60(x - 3) = x(x - 3)$

$60x - 60x + 180 = x^2 - 3x$

$x^2 - 3x - 180 = 0$ (shown)

(iv) $(x - 15)(x + 12) = 0$

$x = 15$ or $x = -12$ (rejected)

$\dfrac{80}{x} = 5\dfrac{1}{3}$

$\therefore$ Time taken is 5 h 20 min.

Section B

5. (a) $(a - b)^2 = 87$

$a^2 - 2ab + b^2 = 87$

$a^2 + b^2 - 2(7.5) = 87$

$a^2 + b^2 - 15 = 87$

$a^2 + b^2 = 102$

$3a^2 + 3b^2 = 306$

(b) $xy + 2x - 3y = 6$

$xy + 2x - 3y - 6 = 0$

$x(y + 2) - 3(y + 2) = 0$

$(x - 3)(y + 2) = 0$

$x = 3$ or $y = -2$

6. Let the original fraction be $\dfrac{x}{y}$.

$\dfrac{x-1}{y-1} = \dfrac{1}{6}$ —(1)

$\dfrac{x+3}{y+3} = \dfrac{1}{2}$ —(2)

From (1),

$6x - 6 = y - 1$

$6x - y = 5$ —(3)

From (2),

$2x + 6 = y + 3$

$2x - y = -3$ —(4)

(3) – (4): $4x = 8$

$x = 2$

Substitute $x = 2$ into (4):

$2(2) - y = -3$

$4 - y = -3$

$y = 7$

$\therefore$ The fraction is $\dfrac{2}{7}$.

7. **(i)** 0.25 km is represented by 1 cm

6 km is represented by 24 cm

∴ The length of the line representing the coastline is 24 cm.

(ii) 1 cm^2 represents 0.0625 km^2

60 cm^2 represent 3.75 km^2

∴ The actual area of the marine park is 3.75 km^2.

8. **(i)** $\frac{(x+2)(x-1)}{(x+1)(x-2)} = \frac{10}{7}$

$7(x+2)(x-1) = 10(x+1)(x-2)$

$7(x^2 - x + 2x - 2) = 10(x^2 - 2x + x - 2)$

$7x^2 + 7x - 14 = 10x^2 - 10x - 20$

$3x^2 - 17x - 6 = 0$

$(x-6)(3x+1) = 0$

$x = 6$ or $x = -\frac{1}{3}$ (rejected)

(ii) Perimeter of $A = 2(6 + 2 + 6 - 1)$

$= 26$ cm

Perimeter of $B = 2(6 + 1 + 6 - 2)$

$= 22$ cm

∴ Perimeter of A : Perimeter of B

= 26 : 22

= 13 : 11

9. **(a)** When $x = -1\frac{1}{2}$, $y = a$,

$a = -1\frac{1}{2}\left[3 - 2\left(-1\frac{1}{2}\right)\right]$

$= -9$

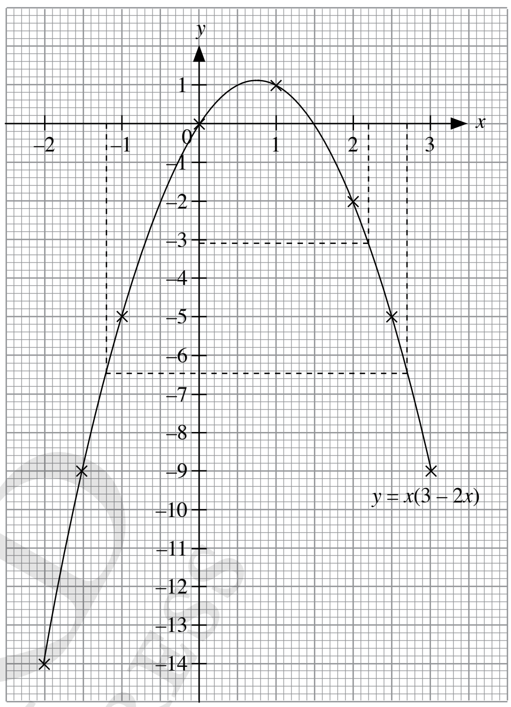
When $x = 2$, $y = b$,

$b = 2[3 - 2(2)]$

$= -2$

∴ $a = -9$, $b = -2$

(b)



(c) **(i)** The equation of the line of symmetry of the graph is $x = 0.75$.

(ii) When $x = 2.2$, $y = -3.1$

(iii) $2x(3 - 3x) = -13$

$x(3 - 2x) = -\frac{13}{2}$

When $y = -\frac{13}{2}$, $x = 2.7$ or $x = -1.2$

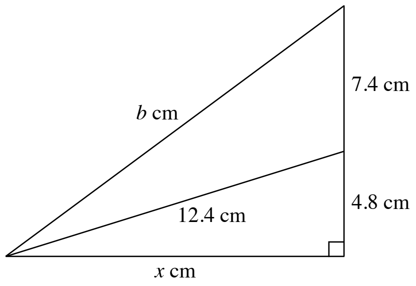
Chapter 10 Pythagoras' Theorem

Basic

1. **(a)** Using Pythagoras' Theorem,

$a^2 = 11.9^2 + 6.8^2$
$= 187.85$
$a = \sqrt{187.85}$
$= 13.7$ (to 3 s.f.)

(b)



Using Pythagoras' Theorem,

$x^2 + 4.8^2 = 12.4^2$
$x^2 = 130.72$
$x = \sqrt{130.72}$

Using Pythagoras' Theorem,

$b^2 = 130.72 + (7.4 + 4.8)^2$
$= 279.56$
$b = \sqrt{279.56}$
$= 16.7$ (to 3 s.f.)

2. **(a)** Using Pythagoras' Theorem,

$(3a)^2 + (2a)^2 = 18.9^2$
$9a^2 + 4a^2 = 357.21$
$13a^2 = 357.21$
$a^2 = 27.47$ (to 4 s.f.)
$a = \sqrt{27.47}$
$= 5.24$ (to 3 s.f.)

(b) Using Pythagoras' Theorem,

$(3b + 4b + 3b)^2 + 16.3^2 = 29.6^2$
$(10b)^2 = 29.6^2 - 16.3^2$
$100b^2 = 610.47$
$b^2 = 6.1047$
$b = \sqrt{6.1047}$
$= 2.47$ (to 3 s.f.)

3. Using Pythagoras' Theorem,

$a^2 = 5^2 + 12^2$
$= 169$
$a = \sqrt{169}$
$= 13$

Using Pythagoras' Theorem,

$b^2 + 12^2 = 21^2$
$b^2 = 21^2 - 12^2$
$= 297$
$b = \sqrt{297}$
$= 17.2$ (to 3 s.f.)

$\therefore a = 13, b = 17.2$

4. Let the length of the square be x cm.

$x^2 = 350$
$x = \sqrt{350}$

Using Pythagoras' Theorem,

Length of diagonal $= \sqrt{350 + 350}$
$= \sqrt{700}$
$= 26.5$ cm (to 3 s.f.)

5. Using Pythagoras' Theorem,

$(x + 1)^2 + (4x)^2 = (4x + 1)^2$
$x^2 + 2x + 1 + 16x^2 = 16x^2 + 8x + 1$
$x^2 - 6x = 0$
$x(x - 6) = 0$
$x = 0$ (rejected) or $x = 6$

6. Let the length of the ladder be x m.

Using Pythagoras' Theorem,

$x^2 = 3.2^2 + 0.8^2$
$= \sqrt{10.88}$
$x = 3.30$ (to 3 s.f.)

$\therefore$ The length of the ladder is 3.30 m.

7. Let the vertical height of the cone be h cm.

Using Pythagoras' Theorem,

$h^2 + 8^2 = 12^2$
$h^2 = 12^2 - 8^2$
$= 80$
$h = \sqrt{80}$
$= 8.94$ (to 3 s.f.)

$\therefore$ The vertical height of the cone is 8.94 cm.

8. Let the length of the diagonal be x m.

Using Pythagoras' Theorem,

$x^2 = 14^2 + 12^2$

$= 340$

$x = \sqrt{340}$

$= 18.4$ (to 3 s.f.)

$\therefore$ The length of the fence is 18.4 m.

9. Let the distance between the tips of the hands be x m.

Using Pythagoras' Theorem,

$x^2 = 3.05^2 + 3.85^2$

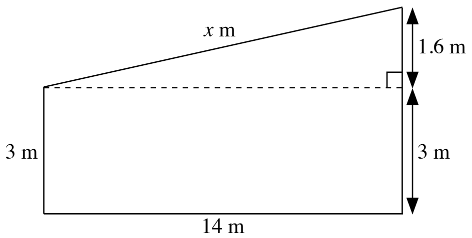
$= 24.125$

$x = \sqrt{24.125}$

$= 4.91$ (to 3 s.f.)

$\therefore$ The distance between the tips of the hands is 4.91 m.

10.



Using Pythagoras' Theorem,

$x^2 = 14^2 + 1.6^2$

$= 198.56$

$x = \sqrt{198.56}$

$= 14.1$ (to 3 s.f.)

$\therefore$ The distance between the top of the two posts is 14.1 m.

11. Using Pythagoras' Theorem,

$\left(\frac{d}{2}\right)^2 + 9^2 = 18^2$

$\left(\frac{d}{2}\right)^2 = 18^2 - 9^2$

$= 243$

$\frac{d}{2} = \sqrt{243}$

$d = 2\sqrt{243}$

$= 31.2$ (to 3 s.f.)

12. **(a)** $AC^2 = 32^2$

$= 1024$

$AB^2 + BC^2 = 24^2 + 28^2$

$= 1360$

Since $AC^2 \neq AB^2 + BC^2$,

$\therefore \triangle ABC$ is not a right-angled triangle.

(b) $DF^2 = 85^2$

$= 7225$

$DE^2 + EF^2 = 13^2 + 84^2$

$= 7225$

Since $DF^2 = DE^2 + EF^2$,

$\therefore \triangle DEF$ is a right-angled triangle with $\angle DEF = 90°$.

(c) $HI^2 = 6.5^2$

$= 42.25^2$

$GH^2 + GI^2 = 3.3^2 + 5.6^2$

$= 42.25$

Since $HI^2 = GH^2 + GI^2$,

$\therefore \triangle GHI$ is a right-angled triangle with $\angle HGI = 90°$.

(d) $KL^2 = \left(2\frac{3}{17}\right)^2$

$= 4\frac{213}{289}$

$JK^2 + JL^2 = \left(\frac{12}{17}\right)^2 + 2^2$

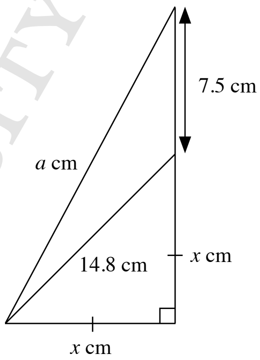
$= 4\frac{144}{289}$

Since $KL^2 \neq JK^2 + JL^2$,

$\therefore \triangle JKL$ is not a right-angled triangle.

Intermediate

13. **(a)**



Using Pythagoras' Theorem,

$x^2 + x^2 = 14.8^2$

$2x^2 = 219.04$

$x^2 = 109.52$

$x = \sqrt{109.52}$

$= 10.47$ (to 4 s.f.)

Using Pythagoras' Theorem,

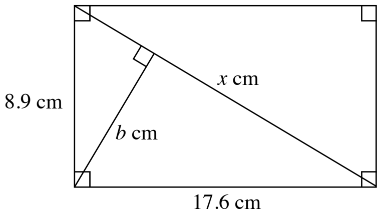
$a^2 = 10.47^2 + (7.5 + 10.47)^2$

$= 432.2$ (to 4 s.f.)

$a = \sqrt{432.2}$

$= 20.8$ (to 3 s.f.)

(b)



Using Pythagoras' Theorem,

$x^2 = 8.9^2 + 17.6^2$

$= 388.97$

$x = \sqrt{388.97}$

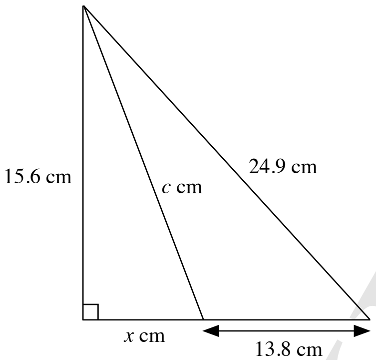
Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$\frac{1}{2} \times \sqrt{388.97} \times b = \frac{1}{2} \times 17.6 \times 8.9$

$b = \frac{17.6 \times 8.9}{\sqrt{388.97}}$

$= 7.94$ (to 3 s.f.)

(c)



Using Pythagoras' Theorem,

$(x + 13.8)^2 + 15.6^2 = 24.9^2$

$(x + 13.8)^2 = 376.65$

$x + 13.8 = \sqrt{376.65}$

$x = \sqrt{376.65} - 13.8$

$= 5.607$ (to 4 s.f.)

Using Pythagoras' Theorem,

$c^2 = 15.6^2 + 5.607^2$

$= 274.8$ (to 4 s.f.)

$c = \sqrt{274.8}$

$= 16.6$ (to 3 s.f.)

14. Using Pythagoras' Theorem,

$a^2 = 8^2 + 9^2$

$= 145$

$a = \sqrt{145}$

$= 12.0$ (to 3 s.f.)

Using Pythagoras' Theorem,

$b^2 = 16^2 + 9^2$

$= 337$

$b = \sqrt{337}$

$= 18.4$ (to 3 s.f.)

$\therefore a = 12.0, b = 18.4$

15. **(i)** Using Pythagoras' Theorem,

$QR^2 + 8.5^2 = 12.3^2$

$QR^2 = 12.3^2 - 8.5^2$

$= 79.04$

$QR = \sqrt{79.04}$

$= 8.89$ cm (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

$PS^2 + 12.3^2 = 17.8^2$

$PS^2 = 17.8^2 - 12.3^2$

$= 165.55$

$PS = \sqrt{165.55}$

$= 12.9$ cm (to 3 s.f.)

(iii) Area of trapezium $PQRS = \frac{1}{2}(8.5 + 17.8)\sqrt{79.04}$

$= 117$ cm^2 (to 3 s.f.)

16. Area of $\triangle ABC = \frac{1}{2} \times AB \times 14$

$180 = 7AB$

$AB = \frac{180}{7}$ cm

Using Pythagoras' Theorem,

$AC^2 = \left(\frac{180}{7}\right)^2 + 14^2$

$= 857.2$ (to 4 s.f.)

$AC = \sqrt{857.2}$

$= 29.3$ cm (to 3 s.f.)

17. Using Pythagoras' Theorem,

$BK^2 + 7^2 = 12^2$

$BK^2 = 12^2 - 7^2$

$= 95$

$BK = \sqrt{95}$

$= 9.746$ cm (to 4 s.f.)

$BC = 2(9.746)$

$= 19.49$ cm (to 4 s.f.)

Using Pythagoras' Theorem,

$(2x + 3)^2 = 19.49^2 + 8^2$

$= 444$

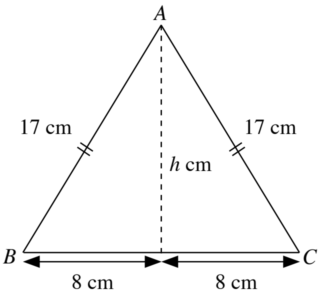
$2x + 3 = \sqrt{444}$

$= 21.07$ (to 4 s.f.)

$2x = 18.07$

$x = 9.04$ (to 3 s.f.)

18.



Using Pythagoras' Theorem,

$h^2 + 8^2 = 17^2$

$h^2 = 17^2 - 8^2$

$= 225$

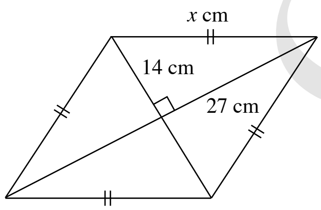
$h = \sqrt{225}$

$= 15$

Area of $\triangle ABC = \frac{1}{2}(16)(15)$

$= 120$ cm^2

19.



Using Pythagoras' Theorem,

$x^2 = 14^2 + 27^2$

$= 925$

$x = \sqrt{925}$

$= 30.41$ (to 4 s.f.)

$\therefore$ Perimeter $= 4(30.41)$

$= 122$ cm (to 3 s.f.)

20. Using Pythagoras' Theorem,

$PQ^2 = (28 - 11)^2 + (28 - 9)^2$

$= 17^2 + 19^2$

$= 650$

Area of $PQRS = PQ^2$

$= 650$ cm^2

21. **(i)** Using Pythagoras' Theorem,

$BD^2 = 12^2 + 5^2$

$= 169$

$BD = 13$ cm

Using Pythagoras' Theorem,

$(AD + 5)^2 + 12^2 = 15^2$

$(AD + 5)^2 = 15^2 - 12^2$

$= 81$

$AD + 5 = 9$

$AD = 4$ cm

(ii) Area of $\triangle ABC = \frac{1}{2}(12)(9)$

$= 54$ cm^2

(iii) Let the shortest distance from C to AB be h cm.

Area of $\triangle ABC = \frac{1}{2} \times 15 \times h$

$54 = \frac{15}{2}h$

$h = 54 \times \frac{2}{15}$

$= 7.2$ cm

22. Using Pythagoras' Theorem,

$XB^2 + 1.3^2 = 5^2$

$XB^2 = 5^2 - 1.3^2$

$= 23.31$

$XB = \sqrt{23.31}$

$= 4.828$ cm (to 4 s.f.)

$\therefore XY = 2(4.828)$

$= 9.66$ cm (to 3 s.f.)

23. $P(-2, -1)$, $T(6, 5)$

Using Pythagoras' Theorem,

$PT^2 = 8^2 + 6^2$

$= 100$

$PT = 10$

$\therefore$ The player has to run 10 units.

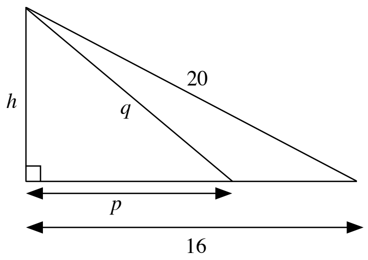
24. Let the height of the LCD screen be h inches.

Using Pythagoras' Theorem,

$$h^2 + 48.5^2 = 55^2$$
$$h^2 = 55^2 - 48.5^2$$
$$= 672.75$$
$$h = \sqrt{672.75}$$
$$= 25.9 \text{ (to 3 s.f.)}$$

Since $h > 24$, the box will not fit the LCD screen.

25.



Using Pythagoras' Theorem,

$$h^2 + 16^2 = 20^2$$
$$h^2 = 20^2 - 16^2$$
$$= 144$$

Using Pythagoras' Theorem,

$$p^2 + 144 = q^2$$

26. Using Pythagoras' Theorem,

$$(x + 2)^2 + x^2 = (x + 4)^2$$
$$x^2 + 4x + 4 + x^2 = x^2 + 8x + 16$$
$$x^2 - 4x - 12 = 0$$
$$(x - 6)(x + 2) = 0$$
$$x = 6 \quad \text{or} \quad x = -2 \text{ (rejected)}$$

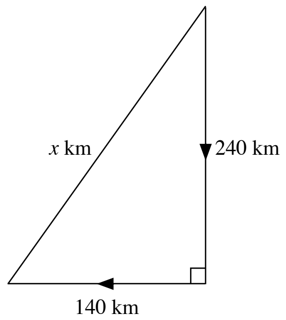
$$\text{Perimeter} = 2(x + 2 + x)$$
$$= 4x + 4$$
$$= 4(6) + 4$$
$$= 28 \text{ m}$$

27. Using Pythagoras' Theorem,

$$(2x)^2 + (4x - 1)^2 = (4x + 1)^2$$
$$4x^2 + 16x^2 - 8x + 1 = 16x^2 + 8x + 1$$
$$4x^2 - 16x = 0$$
$$4x(x - 4) = 0$$
$$x = 0 \text{ (rejected)} \quad \text{or} \quad x = 4$$

$$\text{Cross-sectional area of sandwich} = \frac{1}{2}(8)(15)$$
$$= 60 \text{ cm}^2$$

28.

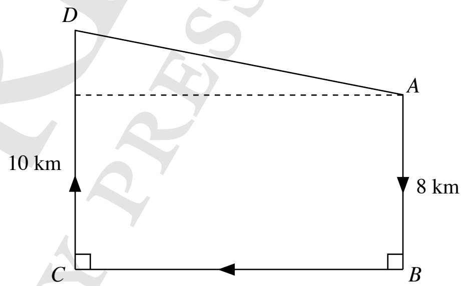


Using Pythagoras' Theorem,

$$x^2 = 140^2 + 240^2$$
$$= 77\ 200$$
$$x = \sqrt{77\ 200}$$
$$= 278 \text{ (to 3 s.f.)}$$

∴ The distance from the starting point is 278 km.

29.



$$AB = 40 \times \frac{12}{60}$$
$$= 8 \text{ km}$$
$$BC = 15 \text{ km}$$
$$CD = 60 \times \frac{10}{60}$$
$$= 10 \text{ km}$$

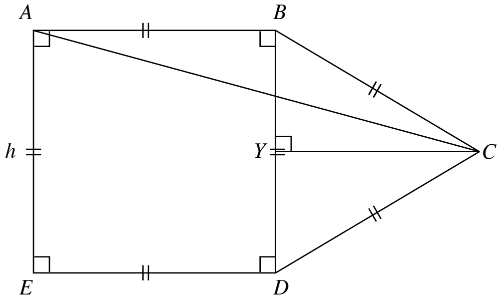
Using Pythagoras' Theorem,

$$DA^2 = 15^2 + 2^2$$
$$= 229$$
$$DA = \sqrt{229}$$
$$= 15.1 \text{ km (to 3 s.f.)}$$

∴ The shortest distance is 15.1 km.

Advanced

30.



Using Pythagoras' Theorem,

$$YC^2 + \left(\frac{h}{2}\right)^2 = h^2$$

$$YC^2 + \frac{h^2}{4} = h^2$$

$$YC^2 = h^2 - \frac{h^2}{4}$$

$$= \frac{3h^2}{4}$$

$$YC = \sqrt{\frac{3}{4}}\,h$$

$$= \frac{\sqrt{3}}{2}\,h$$

Using Pythagoras' Theorem,

$$AC^2 = \left(\frac{h}{2}\right)^2 + \left(h + \frac{\sqrt{3}}{2}h\right)^2$$

$$= 0.25h^2 + 3.482h^2$$

$$= 3.732h^2$$

$$AC = \sqrt{3.732h^2}$$

$= 1.93h$ units (to 3 s.f.)

31. Let the length of the diagonal of the base be x m.

Using Pythagoras' Theorem,

$$x^2 = 3^2 + 4^2$$

$$= 25$$

$$x = \sqrt{25}$$

$$= 5$$

Using Pythagoras' Theorem,

$$PQ^2 = 5^2 + 12^2$$

$$= 169$$

$$PQ = \sqrt{169}$$

$= 13$ m

32. (i) Let the radii of P, Q, and R be p, q and r respectively.

$$\text{Area of } P = \frac{1}{2}\pi p^2$$

$$578 = \frac{\pi}{2}p^2$$

$$p^2 = \frac{1156}{\pi}$$

$$\text{Area of } Q = \frac{1}{2}\pi q^2$$

$$128 = \frac{1}{2}\pi q^2$$

$$q^2 = \frac{256}{\pi}$$

$$\text{Area of } R = \frac{1}{2}\pi r^2$$

$$x = \frac{1}{2}\pi r^2$$

$$r^2 = \frac{2x}{\pi}$$

Using Pythagoras' Theorem,

$$(2q)^2 + (2r)^2 = (2p)^2$$

$$4q^2 + 4r^2 = 4p^2$$

$$q^2 + r^2 = p^2$$

$$\frac{256}{\pi} + \frac{2x}{\pi} = \frac{1156}{\pi}$$

$$2x = 1156 - 256$$

$$= 900$$

$$x = 450$$

(ii) Since $q^2 = \frac{256}{\pi}$ and $r^2 = \frac{900}{\pi}$,

then $q = \sqrt{\frac{256}{\pi}}$ and $r = \sqrt{\frac{900}{\pi}}$.

$$\therefore \frac{AB}{BC} = \frac{\sqrt{\frac{256}{\pi}}}{\sqrt{\frac{900}{\pi}}}$$

$$= \frac{\frac{16}{\sqrt{\pi}}}{\frac{30}{\sqrt{\pi}}}$$

$$= \frac{8}{15}$$

33. **(i)** At x-axis, $y = 0$

$$3x + 15 = 0$$
$$x = -5$$

At y-axis, $x = 0$

$$y + 15 = 0$$
$$y = -15$$

$\therefore$ The coordinates of A are $(-5, 0)$ and B are $(0, -15)$.

(ii) Using Pythagoras' Theorem,

$$AB^2 = 5^2 + 15^2$$
$$= 250$$
$$AB = \sqrt{250}$$
$$= 15.8 \text{ units (to 3 s.f.)}$$

$\therefore$ The length of the line joining A to B is 15.8 units.

34. **(i)** $BC = 23x - 2 - (3x - 2) - (5x + 1) - (6x - 7)$

$$= 23x - 3x - 5x - 6x - 2 + 2 - 1 + 7$$
$$= (9x + 6) \text{ cm}$$

(ii) Since $BC = 2AD$,

$$9x + 6 = 2(5x + 1)$$
$$9x + 6 = 10x + 2$$
$$x = 4$$

Perimeter of trapezium $= 23x - 2$

$$= 23(4) - 2$$
$$= 90 \text{ cm}$$

(iii) $BX + CY = BC - AD$

$$= 9(4) + 6 - [5(4) + 1]$$
$$= 42 - 21$$
$$= 21$$

Since $5BX = 2CY$, $\dfrac{BX}{CY} = \dfrac{2}{5}$

$$BX = \frac{21}{7} \times 2$$
$$= 6$$

$$AB = 3(4) - 2$$
$$= 10$$

Using Pythagoras' Theorem,

$$AX^2 = 10^2 - 6^2$$
$$= 64$$
$$AX = \sqrt{64}$$
$$= 8 \text{ cm}$$

Area of trapezium $= \dfrac{1}{2} \times 8 \times (21 + 42)$

$$= 252 \text{ cm}^2$$

Chapter 11 Trigonometric Ratios

Basic

1. **(a)** **(i)** AC

(ii) AB

(iii) BC

(b) **(i)** YZ

(ii) XZ

(iii) XY

2. **(a)** **(i)** $\sin X = \frac{4}{5}$

(ii) $\cos X = \frac{3}{5}$

(iii) $\tan X = 1\frac{1}{3}$

(iv) $\sin Y = \frac{3}{5}$

(v) $\cos Y = \frac{4}{5}$

(vi) $\tan Y = \frac{3}{4}$

(b) **(i)** $\sin X = \frac{12}{13}$

(ii) $\cos X = \frac{5}{13}$

(iii) $\tan X = 2\frac{2}{5}$

(iv) $\sin Y = \frac{5}{13}$

(v) $\cos Y = \frac{12}{13}$

(vi) $\tan Y = \frac{5}{12}$

(c) **(i)** $\sin X = \frac{4}{5}$

(ii) $\cos X = \frac{3}{5}$

(iii) $\tan X = 1\frac{1}{3}$

(iv) $\sin Y = \frac{3}{5}$

(v) $\cos Y = \frac{4}{5}$

(vi) $\tan Y = \frac{3}{4}$

(d) **(i)** $\sin X = \frac{24}{25}$

(ii) $\cos X = \frac{7}{25}$

(iii) $\tan X = 3\frac{3}{7}$

(iv) $\sin Y = \frac{7}{25}$

(v) $\cos Y = \frac{24}{25}$

(vi) $\tan Y = \frac{7}{24}$

3. **(a)** **(i)** $\sin X = \frac{b}{c}$

(ii) $\cos X = \frac{a}{c}$

(iii) $\tan X = \frac{b}{a}$

(iv) $\sin Y = \frac{a}{c}$

(v) $\cos Y = \frac{b}{c}$

(vi) $\tan Y = \frac{a}{b}$

(b) **(i)** $\sin X = \frac{q}{p}$

(ii) $\cos X = \frac{r}{p}$

(iii) $\tan X = \frac{q}{r}$

(iv) $\sin Y = \frac{r}{p}$

(v) $\cos Y = \frac{q}{p}$

(vi) $\tan Y = \frac{r}{q}$

4. **(a)** $\cos 27° + \cos 54° = 1.479$ (to 3 d.p.)

(b) $5 \cos 51° + 2 \sin 16° = 3.698$ (to 3 d.p.)

(c) $7 \tan 20° - 5 \sin 13° = 1.423$ (to 3 d.p.)

(d) $14 \sin 43° - 6 \cos 7° = 3.593$ (to 3 d.p.)

(e) $12 \cos 13° \times 12 \tan 49° = 161.407$ (to 3 d.p.)

(f) $9 \cos 41° - 4 \tan 12° = 5.942$ (to 3 d.p.)

5. **(a)** $\sin x = 0.4$

$x = 23.6°$ (to 1 d.p.)

(b) $\cos x = 0.4$

$x = 66.4°$ (to 1 d.p.)

(c) $\tan x = 0.3$

$x = 16.7°$ (to 1 d.p.)

(d) $\sin x = 0.45$

$x = 26.7°$ (to 1 d.p.)

(e) $\cos x = 0.74$

$x = 42.3°$ (to 1 d.p.)

(f) $\tan x = 1.34$

$x = 53.3°$ (to 1 d.p.)

(g) $\sin x = 0.453$

$x = 26.9°$ (to 1 d.p.)

(h) $\cos x = 0.973$

$x = 13.3°$ (to 1 d.p.)

(i) $\tan x = 0.354$

$x = 19.5°$ (to 1 d.p.)

(j) $\tan x = 1$

$x = 45°$

6. **(a)** $\sin 34° = \dfrac{a}{15}$

$a = 15 \sin 34°$

$= 8.39$ (to 3 s.f.)

$\cos 34° = \dfrac{b}{15}$

$b = 15 \cos 34°$

$= 12.4$ (to 3 s.f.)

$\therefore a = 8.39, b = 12.4$

(b) $\tan 64° = \dfrac{c}{12}$

$c = 12 \tan 34°$

$= 24.6$ (to 3 s.f.)

$\cos 64° = \dfrac{12}{d}$

$d = \dfrac{12}{\cos 64°}$

$= 27.4$ (to 3 s.f.)

$\therefore c = 24.6, d = 27.4$

(c) $\tan 51.7° = \dfrac{7.53}{e}$

$e = \dfrac{7.53}{\tan 51.7°}$

$= 5.95$ (to 3 s.f.)

$\sin 51.7° = \dfrac{7.53}{f}$

$f = \dfrac{7.53}{\sin 51.7°}$

$= 9.60$ (to 3 s.f.)

$\therefore e = 5.95, f = 9.60$

(d) $\cos 31.9° = \dfrac{71.6}{g}$

$g = \dfrac{71.6}{\cos 31.9°}$

$= 84.3$ (to 3 s.f.)

$\tan 31.9° = \dfrac{h}{71.6}$

$h = 71.6 \tan 31.9°$

$= 44.6$ (to 3 s.f.)

$\therefore g = 84.3, h = 44.6$

7. **(a)** $\tan a° = \dfrac{5.5}{7.6}$

$a° = 35.9°$ (to 1 d.p.)

$a = 35.9$

Using Pythagoras' Theorem,

$b^2 = 5.5^2 + 7.6^2$

$= 88.01$

$b = 9.38$ (to 3 s.f.)

$\therefore a = 35.9, b = 9.38$

(b) $\cos c° = \dfrac{24.3}{35.7}$

$c° = 47.1°$ (to 1 d.p.)

$c = 47.1$

Using Pythagoras' Theorem,

$24.3^2 + d^2 = 35.7^2$

$d^2 = 684$

$d = 26.2$ (to 3 s.f)

$\therefore c = 47.1, d = 26.2$

8. $\tan \angle QPR = \dfrac{32}{43}$

$\angle QPR = 36.7°$ (to 1 d.p.)

9. **(i)** $\sin 21.6° = \dfrac{SF}{86.5}$

$SF = 86.5 \sin 21.6°$

$= 31.8$ m (to 3 s.f.)

(ii) $\cos 21.6° = \dfrac{FH}{86.5}$

$FH = 86.5 \cos 21.6°$

$= 80.4$ m (to 3 s.f.)

10. $\tan 38° = \dfrac{45}{BE}$

$BE = \dfrac{45}{\tan 38°}$

$= 57.6$ m (to 3 s.f.)

$\therefore$ The distance between the enemy and the foot of the observatory is 57.6 m.

Intermediate

11. **(a)** $\dfrac{2 \sin 26°}{3 \cos 17°} = 0.306$ (to 3 d.p.)

(b) $\dfrac{(\tan 45°)^2}{\tan 10°} = 5.671$ (to 3 d.p.)

(c) $\dfrac{\sin 30° + \cos 40°}{\tan 50°} = 1.062$ (to 3 d.p.)

(d) $\dfrac{\cos 19°}{\tan 22° - \sin 58°} = -2.129$ (to 3 d.p.)

(e) $\dfrac{\sin 20° - \cos 61°}{\tan 47° \times \sin 91°} = -0.133$ (to 3 d.p.)

(f) $\dfrac{\cos 63° - \sin 2°}{\tan 54° \div \tan 3°} = 0.016$ (to 3 d.p.)

12. **(a)** $\tan 27.7° = \dfrac{18.1}{2a}$

$a = \dfrac{18.1}{2 \tan 27.7°}$

$= 17.2$ (to 3 s.f.)

$\sin 27.7° = \dfrac{18.1}{b}$

$b = \dfrac{18.1}{\sin 27.7°}$

$= 38.9$ (to 3 s.f.)

$\therefore a = 17.2, b = 38.9$

(b) $\sin 29° = \dfrac{c}{15.4}$

$c = 15.4 \sin 29°$

$= 7.47$ (to 3 s.f.)

$\sin 32° = \dfrac{d}{15.4}$

$d = 15.4 \sin 32°$

$= 8.16$ (to 3 s.f.)

$\cos 32° = \dfrac{e}{15.4}$

$e = 15.4 \cos 32°$

$= 13.1$ (to 3 s.f.)

13. **(i)** Area of $\triangle BCD = \dfrac{1}{2}(12)(AB)$

$45 = 6AB$

$AB = \dfrac{45}{6}$

$= 7.5$ cm

(ii) Using Pythagoras' Theorem,

$(AD + 12)^2 + 7.5^2 = 19^2$

$(AD + 12)^2 = 304.75$

$AD + 12 = \sqrt{304.75}$

$AD = \sqrt{304.75} - 12$

$= 5.46$ cm (to 3 s.f.)

(iii) $\tan \angle BDA = \dfrac{AB}{AD}$

$= \dfrac{7.5}{\sqrt{304.75} - 12}$

$= 1.374$ (to 4 s.f.)

$\angle BDA = 53.95°$ (to 2 d.p.)

$\therefore \angle BDC = 180° - 53.95°$

$= 126.0°$ (to 1 d.p.)

14. **(i)** Using Pythagoras' Theorem,

$AP^2 = 8^2 + 5^2$

$= 89$

$AP = \sqrt{89}$

$= 9.43$ cm (to 3 s.f.)

(ii) $\tan \angle APC = \dfrac{8}{5}$

$\angle APC = 57.99°$ (to 2 d.p.)

$\therefore \angle APQ = 180° - 57.99°$

$= 122.0°$ (to 1 d.p.)

(iii) $\angle BRP = 360° - 2(57.99°) - 90°$

$= 154.0°$ (to 1 d.p.)

15. **(i)** $\sin 65° = \dfrac{BQ}{7.6}$

$BQ = 7.6 \sin 65°$

$= 6.887$ (to 4 s.f.)

$= 6.89$ cm (to 3 s.f.)

(ii) Using Pythagoras' Theorem,

$PQ^2 + 6.887^2 = 8.7^2$

$PQ^2 = 28.24$ (to 4 s.f.)

$PQ = 5.314$ (to 4 s.f.)

$= 5.31$ cm (to 3 s.f.)

(iii) Using Pythagoras' Theorem,

$(AP + 5.314)^2 + 6.887^2 = 10.2^2$

$(AP + 5.314)^2 = 56.59$

$AP + 5.134 = 7.523$ (to 4 s.f.)

$AP = 2.21$ cm (to 3 s.f.)

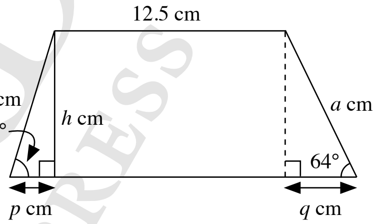
(iv) $\sin \angle BPQ = \dfrac{6.887}{8.7}$

$\angle BPQ = 52.34°$ (to 2 d.p.)

$\therefore \angle APB = 180° - 52.34°$

$= 127.7°$ (to 1 d.p.)

16. **(a)**



$\sin 73° = \dfrac{h}{11.7}$

$h = 11.7 \sin 73°$

$= 11.18$ (to 4 s.f.)

$\sin 64° = \dfrac{11.18}{a}$

$a = \dfrac{11.18}{\sin 64°}$

$= 12.4$ (to 3 s.f.)

$\cos 73° = \dfrac{p}{11.7}$

$p = 11.7 \cos 73°$

$= 3.420$ (to 4 s.f.)

$\tan 64° = \dfrac{11.18}{q}$

$q = \dfrac{11.18}{\tan 64°}$

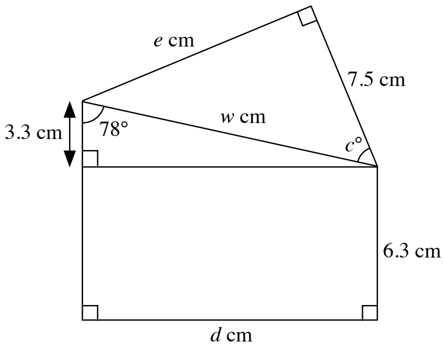
$= 5.457$ (to 4 s.f.)

$b = 3.420 + 5.457 + 12.5$

$= 21.4$ (to 3 s.f.)

$\therefore a = 12.4, b = 21.4$

(b)



$\tan 78° = \dfrac{d}{3.3}$

$d = 3.3 \tan 78°$

$= 15.5$ (to 3 s.f.)

$\cos 78° = \dfrac{3.3}{w}$

$w = \dfrac{3.3}{\cos 78°}$

$= 15.87$ (to 4 s.f.)

$\cos c° = \dfrac{7.5}{15.87}$

$c° = 61.80°$ (to 2 d.p.)

$c = 61.8$ (to 1 d.p.)

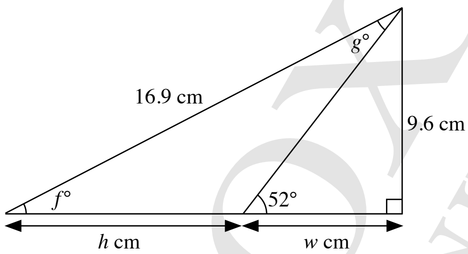
$\tan 61.80° = \dfrac{e}{7.5}$

$e = 7.5 \tan 61.80°$

$= 14.0$ (to 3 s.f.)

$\therefore c = 61.8, d = 15.5, e = 14.0$

(c)



$\sin f° = \dfrac{9.6}{16.9}$

$f° = 34.61°$ (to 2 d.p.)

$f = 34.6$ (to 1 d.p.)

$g° = 52° - 34.61°$

$= 17.4°$ (to 1 d.p.)

$g = 17.4$

$\tan 52° = \dfrac{9.6}{w}$

$w = \dfrac{9.6}{\tan 52°}$

$= 7.500$ (to 4 s.f.)

Using Pythagoras' Theorem,

$(h + 7.500)^2 + 9.6^2 = 16.9^2$

$(h + 7.500)^2 = 193.45$

$h + 7.500 = \sqrt{193.45}$

$h = \sqrt{193.45} - 7.500$

$= 6.41$ (to 3 s.f.)

$\therefore f = 34.6, g = 17.4, h = 6.41$

17. (i) Using Pythagoras' Theorem,

$QT^2 + 8.6^2 = 11.3^2$

$QT^2 = 53.73$

$QT = \sqrt{53.73}$

$= 7.330$ cm (to 4 s.f.)

Using Pythagoras' Theorem,

$9.82^2 + (10.2 + 7.330)^2 = PS^2$

$PS^2 = 403.3$

$PS = \sqrt{403.3}$

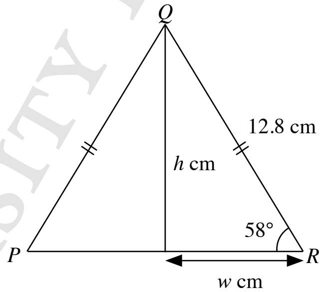
$= 20.08$ (to 4 s.f.)

$= 20.1$ cm (to 3 s.f.)

(ii) $\cos \angle SPQ = \dfrac{9.8}{20.08}$

$\angle SPQ = 60.8°$ (to 1 d.p.)

18.



$\sin 58° = \dfrac{h}{12.8}$

$h = 12.8 \sin 58°$

$= 10.85$ (to 4 s.f.)

$\cos 58° = \dfrac{w}{12.8}$

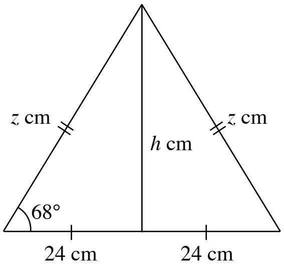
$w = 12.8 \cos 58°$

$= 6.782$ (to 4 s.f.)

$\therefore$ Area of $\triangle PQR = \dfrac{1}{2}(2 \times 6.782)(10.85)$

$= 73.6$ cm^2 (to 3 s.f.)

19.



$\cos 68° = \frac{24}{z}$

$z = \frac{24}{\cos 68°}$

$= 64.06$ (to 4 s.f.)

$\therefore$ Perimeter $= 48 + 2(64.06)$

$= 176$ cm (to 3 s.f.)

$\tan 68° = \frac{h}{24}$

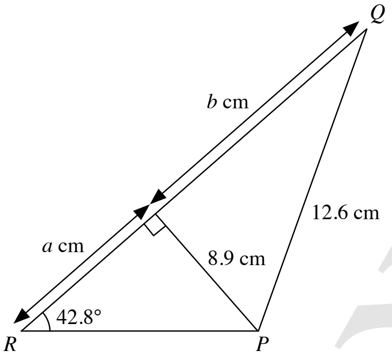
$h = 24 \tan 68°$

$= 59.40$ (to 4 s.f.)

$\therefore$ Area $= \frac{1}{2}(48)(59.40)$

$= 1430 \text{ cm}^2$ (to 3 s.f.)

20.



$\tan 42.8° = \frac{8.9}{a}$

$a = \frac{8.9}{\tan 42.8°}$

$= 9.611$ (to 4 s.f.)

Using Pythagoras' Theorem,

$b^2 + 8.9^2 = 12.6^2$

$b^2 = 79.55$

$b = \sqrt{79.55}$

$= 8.919$ (to 4 s.f.)

$\therefore$ Area of $\triangle PQR = \frac{1}{2}(9.611 + 8.919)(8.9)$

$= 82.5 \text{ cm}^2$ (to 3 s.f.)

21. Let the distance from the boat to the foot of the cliff be d m.

$\tan 26° = \frac{55}{d}$

$d = \frac{55}{\tan 26°}$

$= 113$ (to 3 s.f.)

$\therefore$ The distance from the boat to the foot of the cliff is 113 m.

22. (i) $\cos 54° = \frac{1.8}{PQ}$

$PQ = \frac{1.8}{\cos 54°}$

$= 3.062$ (to 4 s.f.)

$= 3.06$ m (to 3 s.f.)

(ii) $\tan 54° = \frac{QN}{1.8}$

$QN = 1.8 \tan 54°$

$= 2.477$ (to 4 s.f.)

$= 2.48$ m (to 3 s.f.)

(iii) $Q'N = 2.477 - 0.8$

$= 1.677$ (to 4 s.f.)

$= 1.68$ m (to 3 s.f.)

(iv) $\sin \angle NP'Q' = \frac{1.677}{3.062}$

$\angle NP'Q' = 33.2°$ (to 1 d.p.)

23. (i) $\sin 47° = \frac{KH}{240}$

$KH = 240 \sin 47°$

$= 176$ m (to 3 s.f.)

(ii) Assume that the string is taut.

24. (i) Using Pythagoras' Theorem,

$(BC + 5.2)^2 + 18.3^2 = 24^2$

$(BC + 5.2)^2 = 241.11$

$BC + 5.2 = \sqrt{241.11}$

$BC = \sqrt{241.11} - 5.2$

$= 10.32$ (to 4 s.f.)

$= 10.3$ m (to 3 s.f.)

(ii) $\tan \angle BMC = \frac{10.32}{18.3}$

$\angle BMC = 29.43°$ (to 2 d.p.)

$= 29.4°$ (to 1 d.p.)

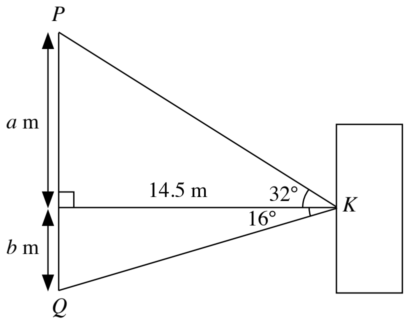
(iii) $\cos \angle AMC = \frac{18.3}{24}$

$\angle AMC = 40.31°$ (to 2 d.p.)

$\therefore \angle AMB = 40.31° - 29.43°$

$= 10.9°$ (to 1 d.p.)

25.



$\tan 32° = \dfrac{a}{14.5}$

$a = 14.5 \tan 32°$

$= 9.060$ (to 4 s.f.)

$\tan 16° = \dfrac{b}{14.5}$

$b = 14.5 \tan 16°$

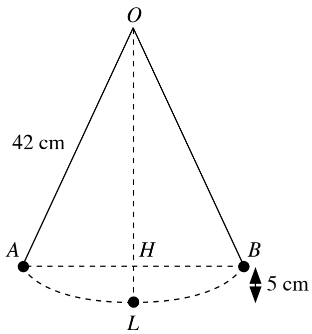
$= 4.157$ (to 4 s.f.)

$PQ = 9.060 + 4.157$

$= 13.2$

$\therefore$ The height of the monument is 13.2 m.

26.



$OH = 42 - 5$

$= 37$ cm

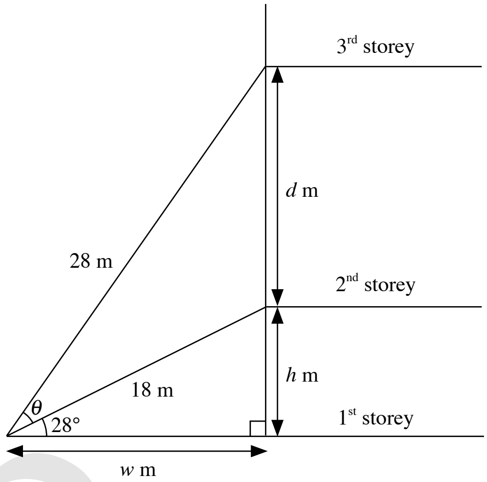
$\cos \angle AOL = \dfrac{37}{42}$

$\angle AOL = 28.24°$ (to 2 d.p.)

$\therefore \angle AOB = 2(28.24°)$

$= 56.5°$ (to 1 d.p.)

27. (i)



$\sin 28° = \dfrac{h}{18}$

$h = 18 \sin 28°$

$= 8.450$ (to 4 s.f.)

$= 8.45$ (to 3 s.f.)

$\therefore$ The height of the first storey is 8.45 m.

(ii) $\cos 28° = \dfrac{w}{18}$

$w = 18 \cos 28°$

$= 15.89$ (to 4 s.f.)

$= 15.9$ (to 3 s.f.)

Using Pythagoras' Theorem,

$(d + 8.450)^2 + 15.89^2 = 28^2$

$(d + 8.450)^2 = 531.4$

$d + 8.450 = \sqrt{531.4}$

$d = \sqrt{531.4} - 8.450$

$= 14.6$ (to 3 s.f.)

$\therefore$ The height of the second storey is 14.6 m.

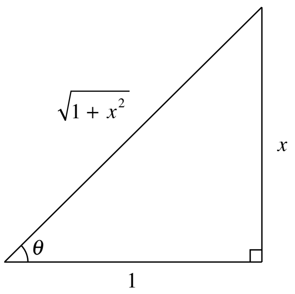
(iii) $\cos (\theta + 28°) = \dfrac{15.89}{28}$

$\theta + 28° = 55.41°$ (to 2 d.p.)

$\theta = 27.4°$ (to 1 d.p.)

Advanced

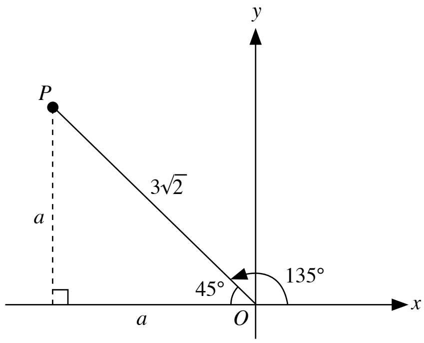
28.



(a) $2 \sin \theta = \dfrac{2x}{\sqrt{1 + x^2}}$

(b) $3 \cos \theta = \dfrac{3}{\sqrt{1 + x^2}}$

29.



Using Pythagoras' Theorem,

$a^2 + a^2 = (3\sqrt{2})^2$

$= 18$

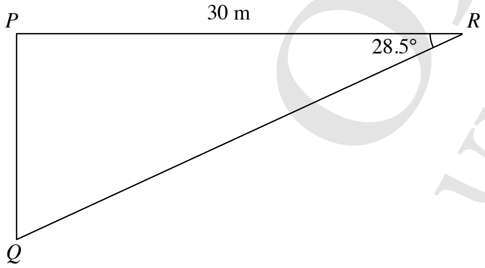
$2a^2 = 18$

$a^2 = 9$

$a = \pm 3$

$\therefore$ The coordinates of P are (–3, 3).

30.



$\tan 28.5° = \dfrac{PQ}{30}$

$PQ = 30 \tan 28.5°$

$= 16.3$ m

$\therefore$ The width of the river is 16.3 m.

31. $\tan 34° = \dfrac{TA}{AB}$

$AB = \dfrac{TA}{\tan 34°}$ —(1)

$\tan 26° = \dfrac{TA}{AB + 25}$

$AB \tan 26° + 25 \tan 26° = TA$ —(2)

Substitute (1) into (2):

$\left(\dfrac{\tan 26°}{\tan 34°}\right)TA + 25 \tan 26° = TA$

$TA - \left(\dfrac{\tan 26°}{\tan 34°}\right)TA = 25 \tan 26°$

$\left(1 - \dfrac{\tan 26°}{\tan 34°}\right)TA = 25 \tan 26°$

$TA = \dfrac{25 \tan 26°}{1 - \dfrac{\tan 26°}{\tan 34°}}$

$= 44.0$ m (to 3 s.f.)

$\therefore$ The height of the office tower is 44.0 m.

32. **(i)** $\tan 56° = \dfrac{PQ}{250}$

$PQ = 250 \tan 56°$

$= 370.6$ (to 4 s.f.)

$= 371$ m (to 3 s.f.)

$\therefore$ P is 371 m above the parade ground.

(ii) $\tan 46° = \dfrac{PQ}{250}$

$P'Q = 250 \tan 46°$

$= 258.8$ (to 4 s.f.)

$= 259$ m (to 3 s.f.)

$PP' = 370.6 - 258.8$

$= 111.7$ m (to 4 s.f.)

Speed of descent $= \dfrac{111.7}{45}$

$= 2.484$ m/s (to 4 s.f.)

Time taken to descend from P to Q

$= \dfrac{370.6}{2.484}$

$= 149$ s (to 3 s.f.)

Chapter 12 Volume and Surface Area of Pyramids, Cones and Spheres

Basic

1. (a) Volume of pyramid $= \frac{1}{3} \times 16^2 \times 27$
$= 2304 \text{ cm}^3$

(b) Volume of pyramid $= \frac{1}{3} \times \left(\frac{1}{2} \times 12 \times 9\right) \times 20$
$= 360 \text{ cm}^3$

(c) Volume of pyramid $= \frac{1}{3} \times 9 \times 5 \times 3$
$= 45 \text{ m}^3$

2. Volume of pyramid $= \frac{1}{3} \times 8 \times h$

$42 = \frac{8}{3}h$

$h = 15.75$

∴ The height of the figurine is 15.75 cm.

3. Volume of pyramid $= \frac{1}{3} \times 8 \times 3 \times h$

$86 = 8h$

$h = 10.75$

∴ The height of the pyramid is 10.75 m.

4. Volume of pyramid $= \frac{1}{3} \times \left(\frac{1}{2} \times 12 \times 5\right) \times h$

$160 = 10h$

$h = 16$

∴ The height of the pyramid is 16 m.

5. Total surface area $= 16^2 + 4 \times \frac{1}{2} \times 16 \times 17$
$= 800 \text{ m}^2$

6. $V = \frac{1}{3}\pi r^2 h$

(a) When $r = 8$ and $V = 320$,

$320 = \frac{1}{3}\pi(8)^2 h$

$h = \frac{960}{64\pi}$

$= 4.77$ (to 3 s.f.)

(b) When $r = 10.6$ and $V = 342.8$,

$342.8 = \frac{1}{3}\pi(10.6)^2 h$

$h = \frac{1028.4}{112.36\pi}$

$= 2.91$ (to 3 s.f.)

(c) When $h = 6$ and $V = 254$,

$254 = \frac{1}{3}\pi r^2(6)$

$r^2 = \frac{762}{6\pi}$

$r = \sqrt{\frac{762}{6\pi}}$

$= 6.36$ (to 3 s.f.)

(d) When $h = 11$ and $V = 695$,

$695 = \frac{1}{3}\pi r^2(11)$

$r^2 = \frac{2085}{11\pi}$

$r = \sqrt{\frac{2085}{11\pi}}$

$= 7.77$ (to 3 s.f.)

	Radius, r cm	Height, h cm	Volume, V cm^3
(a)	8	4.77	320
(b)	10.6	2.91	342.8
(c)	6.36	6	254
(d)	7.77	11	695

7. (a) Volume of cone $= \frac{1}{3}\pi(6)^2(8)$
$= 302 \text{ cm}^3$ (to 3 s.f.)

Total surface area of cone $= \pi(6)^2 + \pi(6)(10)$
$= 302 \text{ cm}^2$ (to 3 s.f.)

(b) Volume of cone $= \frac{1}{3}\pi(12)^2(28.8)$
$= 4340 \text{ cm}^3$ (to 3 s.f.)

Total surface area of cone $= \pi(12)^2 + \pi(12)(31.2)$
$= 1630 \text{ cm}^2$ (to 3 s.f.)

8. (a) Volume of sphere $= \frac{4}{3}\pi(5.8)^3$
$= 817 \text{ cm}^3$ (to 3 s.f.)

(b) Volume of sphere $= \frac{4}{3}\pi(12.6)^3$
$= 8380 \text{ m}^3$ (to 3 s.f.)

9. (a) Volume of sphere $= \frac{4}{3}\pi\left(\frac{24.2}{2}\right)^3$
$= 7420 \text{ cm}^3$ (to 3 s.f.)

(b) Volume of sphere $= \frac{4}{3}\pi\left(\frac{6.25}{2}\right)^3$
$= 128 \text{ mm}^3$ (to 3 s.f.)

10. **(a)** Volume of sphere = $\frac{4}{3}\pi r^3$

$34 = \frac{4}{3}\pi r^3$

$r^3 = \frac{51}{2\pi}$

$r = \sqrt[3]{\frac{51}{2\pi}}$

$= 2.009$ (to 4 s.f.)

$= 2.01$ cm (to 3 s.f.)

Surface area of sphere $= 4\pi(2.009)^2$

$= 50.8$ cm^2 (to 3 s.f.)

(b) Volume of sphere = $\frac{4}{3}\pi r^3$

$68.2 = \frac{4}{3}\pi r^3$

$r^3 = \frac{51.15}{\pi}$

$r = \sqrt[3]{\frac{51.15}{\pi}}$

$= 2.534$ (to 4 s.f.)

$= 2.53$ m (to 3 s.f.)

Surface area of sphere $= 4\pi(2.534)^2$

$= 80.7$ m^2 (to 3 s.f.)

11. Surface area of sphere $= 4\pi(8)^2$

$= 256\pi$ m^2

Cost of painting $= \frac{256\pi}{8} \times 8.5$

$= \$854.51$ (to 2 d.p.)

Intermediate

12. Let the height and the slant height of the pyramid be h cm and l cm respectively.

Total surface area of pyramid $= 8^2 + 4 \times \frac{1}{2}(8)l$

$144 = 64 + 16l$

$16l = 80$

$l = 5$

Using Pythagoras' Theorem,

$4^2 + h^2 = 5^2$

$16 + h^2 = 25$

$h^2 = 9$

$h = 3$

$\therefore$ Volume of pyramid $= \frac{1}{3} \times 8^2 \times 3$

$= 64$ cm^3

13. **(i)** Let the radius of the base be r m.

$2\pi r = 8.5$

$r = \frac{4.25}{\pi}$

$= 1.352$ (to 4 s.f)

Volume of rice $= \frac{1}{3}\pi(1.352)^2(1.2)$

$= 2.29$ (to 3 s.f.)

$= 2.3$ m^3 (to 2 s.f.)

(ii) Number of bags $= \frac{2.29}{0.5}$

$= 4.59$ (to 3 s.f.)

≈ 5

Assume that the space between the grains of rice is negligible.

14. Volume of crew cabin

$= \frac{1}{3}\pi\left(\frac{75}{2}\right)^2(92) - \frac{1}{3}\pi\left(\frac{27}{2}\right)^2(92 - 59)$

$= 129\ 000$ cm^3 (to 3 s.f.)

15. **(i)** Let the radius of the base be r cm.

$2\pi r = 88$

$r = \frac{44}{\pi}$

$= 14.00$ (to 4 s.f.)

Curved surface area of cone $= \pi\left(\frac{44}{\pi}\right)(15)$

$= 660$ cm^2

(ii) Total surface area of cone

$= 660 + \pi(14.00)^2$

$= 1276$ cm^2 (to the nearest integer)

16. **(i)** Curved surface area of cone $= \pi(x - 5)(x + 5)$

$75\pi = \pi(x^2 - 25)$

$75 = x^2 - 25$

$x^2 = 100$

$x = 10$

(ii) Base radius = 5 cm

Slant height = 15 cm

Height $= \sqrt{15^2 - 5^2}$

$= \sqrt{200}$

$\therefore$ Volume of cone $= \frac{1}{3}\pi(5)^2(\sqrt{200})$

$= 370$ cm^3 (to 3 s.f.)

17. (i) Volume of solid $= \frac{2}{3}\pi h^3 - \frac{2}{3}\pi\left(\frac{h}{2}\right)^3$

$= \frac{2}{3}\pi h^3 - \frac{1}{12}\pi h^3$

$= \frac{7}{12}\pi h^3$

(ii) Total surface area of solid

$= 2\pi h^2 + \left[\pi h^2 - \pi\left(\frac{h}{2}\right)^2\right] + 2\pi\left(\frac{h}{2}\right)^2$

$= 2\pi h^2 + \pi h^2 - \frac{1}{4}\pi h^2 + \frac{1}{2}\pi h^2$

$= \frac{13}{4}\pi h^2$

18. Volume of plastic $= \frac{4}{3}\pi(4)^3 - \frac{4}{3}\pi(3.6)^3$

$= 72.7 \text{ cm}^3$ (to 3 s.f.)

19. Volume of steel

$= 100 \times \left[\frac{4}{3}\pi\left(\frac{16}{2}\right)^3 - \frac{4}{3}\pi\left(\frac{16}{2} - 0.8\right)^3\right]$

$= 58\,100 \text{ cm}^3$ (to 3 s.f.)

20. Amount of space $= 6^3 - \frac{4}{3}\pi\left(\frac{6}{2}\right)^3$

$= 103 \text{ cm}^3$ (to 3 s.f.)

21. (a) Total surface area of hemisphere $= 2\pi r^2 + \pi r^2$

$374 = 3\pi r^2$

$r^2 = \frac{374}{3\pi}$

$r = \sqrt{\frac{374}{3\pi}}$

$= 6.3$ cm (to 1 d.p.)

Volume of hemisphere $= \frac{2}{3}\pi\left(\sqrt{\frac{374}{3\pi}}\right)^3$

$= 523.6 \text{ cm}^3$ (to 1 d.p.)

(b) Total surface area of hemisphere $= 2\pi r^2 + \pi r^2$

$1058.4 = 3\pi r^2$

$r^2 = \frac{352.8}{\pi}$

$r = \sqrt{\frac{352.8}{\pi}}$

$= 10.6$ m (to 1 d.p.)

Volume of hemisphere $= \frac{2}{3}\pi\left(\sqrt{\frac{352.8}{\pi}}\right)^3$

$= 2492.5 \text{ m}^3$ (to 1 d.p.)

22. (i) Volume of sphere $= \frac{4}{3}\pi\left(\frac{x+2}{2}\right)^3$

$972\pi = \frac{4}{3}\pi\left(\frac{x+2}{2}\right)^3$

$\left(\frac{x+2}{2}\right)^3 = 729$

$\frac{x+2}{2} = 9$

$x + 2 = 18$

$x = 16$

(ii) Surface area of sphere $= 4\pi\left(\frac{18}{2}\right)^2$

$= 1020 \text{ cm}^2$ (to 3 s.f.)

23. Volume of glass

= volume of prism + volume of pyramid

$= \left(\frac{1}{2} \times 3.6 \times 4.8\right)(6) + \frac{1}{3}\left(\frac{1}{2} \times 3.6 \times 4.8\right)(12)$

$= 86.4 \text{ m}^3$

24. Volume of hemisphere $= \frac{2}{3}\pi(4)^3$

$= \frac{128}{3}\pi \text{ cm}^3$

$\therefore$ Volume of model $= \frac{37}{4} \times \frac{128}{3}\pi$

$= 1240 \text{ cm}^3$ (to 3 s.f.)

25. (i) Capacity of container $= \frac{1}{3}\pi(21)^2(21)$

$= 9698 \text{ cm}^3$ (to 4 s.f.)

$= 9.70\ l$ (to 3 s.f.)

(ii) Mass of container $= 9698 \times 1.5$

$= 14\,540$ g (to 4 s.f.)

$= 15$ kg (to the nearest kg)

Advanced

26. (i) Volume of iron $= \frac{1}{3}\pi(1)^2(0.5)$

$= \frac{\pi}{6}$

$= 0.524 \text{ m}^3$ (to 3 s.f.)

Volume of lead $= \pi(2)^2(3) - \frac{\pi}{6}$

$= 12\pi - \frac{\pi}{6}$

$= \frac{71\pi}{6}$

$= 37.2 \text{ m}^3$ (to 3 s.f.)

(ii) Let the denisty of lead be ρ g/m^3.

Original mass of cylinder $= \pi(2)^2(3)\rho$

$= 12\pi\rho\ g$

New mass of cylinder $= \dfrac{\pi}{6}\left(\dfrac{2}{3}\rho\right) + \dfrac{71\pi}{6}(\rho)$

$= \dfrac{\pi}{9}\rho + \dfrac{71\pi}{6}\rho$

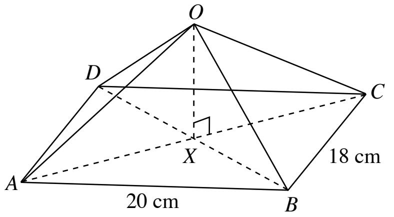
$= \dfrac{215\pi}{18}\rho\ g$

$\therefore$ Percentage reduction in mass

$$= \frac{12\pi\rho - \dfrac{215\pi}{18}\rho}{12\pi\rho} \times 100\%$$

$$= \frac{25}{54}\ \%$$

27.



Using Pythagoras' Theorem,

$AC^2 = 20^2 + 18^2$

$= 724$

$AC = \sqrt{724}$ cm

$\tan 50° = \dfrac{AX}{OX}$

$OX = \dfrac{AX}{\tan 50°}$

$= \dfrac{\frac{1}{2}\sqrt{724}}{\tan 50°}$

$\therefore$ Volume of pyramid $= \dfrac{1}{3}(20 \times 18)\left(\dfrac{\frac{1}{2}\sqrt{724}}{\tan 50°}\right)$

$= 1350$ cm^3 (to 3 s.f.)

New Trend

28. (a) Using Pythagoras' Theorem,

$h^2 + 8^2 = 17^2$

$h^2 = 225$

$h = \sqrt{225}$

$= 15$

$\therefore$ The height of the cone is 15 cm. (shown)

(b) Volume of solid

= volume of cone + volume of hemisphere

$= \dfrac{1}{3}\pi(8)^2(15) + \dfrac{1}{2}\left[\dfrac{4}{3}\pi(8)^3\right]$

$= 2080$ cm^3 (to 3 s.f.)

29. Total surface area of solid $= \dfrac{1}{2}(4\pi x)^2 + 2\pi x(3x) + \pi x^2$

$= 2\pi x^2 + 6\pi x^2 + \pi x^2$

$= 9\pi x^2$

Total surface area of solid = 2 × surface area of cone

$9\pi x^2 = 2(\pi x l + \pi x^2)$

$7\pi x^2 = 2\pi x l$

$l = \dfrac{7\pi x^2}{2\pi x}$

$= \dfrac{7x}{2}$

30. (i) Let the height of the pyramid be h cm.

Using Pythagoras' Theorem,

$h^2 + 15^2 = 39^2$

$h^2 = 1296$

$h = \sqrt{1296}$

$= 36$

Volume of solid $= (30)(30)(70) + \dfrac{1}{3}(30)^2(36)$

$= 73\ 800$ cm^3

(ii) Volume of spherical candle $= \dfrac{1}{10} \times 73\ 800$

$\dfrac{4}{3}\pi r^3 = 7380$

$r^3 = \dfrac{7380 \times 3}{4\pi}$

$r = \sqrt[3]{\dfrac{7380 \times 3}{4\pi}}$

$= 12.078$ cm (to 5 s.f.)

$= 12.1$ cm (to 3 s.f.)

(shown)

(iii) Volume of cuboid

$= 4(12.078) \times 2(12.078) \times 2(12.078)$

$= 28\ 191$ cm^3 (to 5 s.f.)

Volume of empty space $= 28\ 191 - 2(7380)$

$= 13\ 400$ cm^3 (to 3 s.f.)

31. Total surface area $= \pi(4r)^2 + 2(2\pi r)(3r) + \frac{1}{2}[4\pi(4r)^2]$

$= 16\pi r^2 + 12\pi r^2 + 32\pi r^2$

$= 60\pi r^2$ cm^2

32. **(i)** Using Pythagoras' Theorem,

$x^2 = (15 - 9)^2 + 16^2$

$x^2 = 292$

$x = \sqrt{292}$

$= 17.088$ (to 5 s.f.)

$= 17.09$ cm (to 4 s.f.) (shown)

(ii) Let the slant height of the cone with radius 9 cm be l cm.

Using Pythagoras' Theorem,

$l^2 = (40 - 16)^2 + 9^2$

$l^2 = 657$

$l = \sqrt{657}$

$= 25.63$ cm (to 2 d.p.)

Total surface area of vase

$= \pi(15)(17.088 + 25.63) - \pi(9)(25.63) + \pi(15)^2$

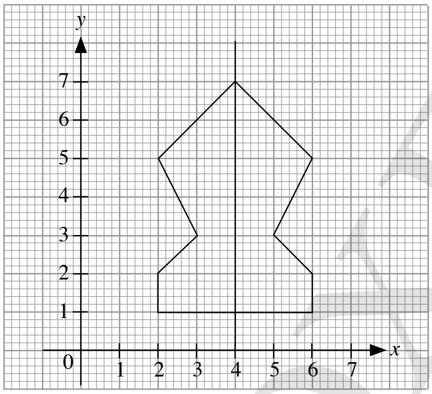
$= 1995$ cm^2 (to the nearest whole number)

Chapter 13 Symmetry

Basic

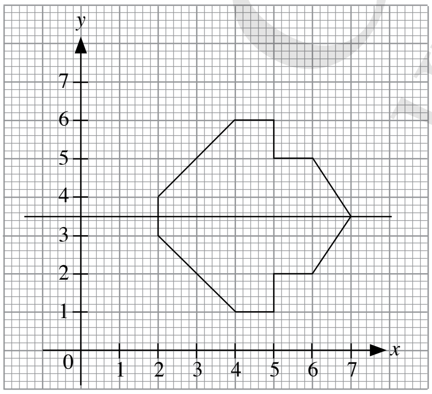
1. **(a)** **(i)** The figure has 1 line of symmetry.
   (ii) The figure has rotational symmetry of order 1 i.e. no rotational symmetry.
   (b) **(i)** The figure has 1 line of symmetry.
   (ii) The figure has rotational symmetry of order 1 i.e. no rotational symmetry.
   (c) **(i)** The figure has 2 lines of symmetry.
   (ii) The figure has rotational symmetry of order 2.
   (d) **(i)** The figure has 0 lines of symmetry, i.e. no line symmetry.
   (ii) The figure has rotational symmetry of order 3.
   (e) **(i)** The figure has 1 line of symmetry.
   (ii) The figure has rotational symmetry of order 1 i.e. no rotational symmetry.
   (f) **(i)** The figure has 4 lines of symmetry.
   (ii) The figure has rotational symmetry of order 4.
   (g) **(i)** The figure has 1 line of symmetry.
   (ii) The figure has rotational symmetry of order 1 i.e. no rotational symmetry.

2. **(a)**



Line of symmetry: $x = 4$

(b)



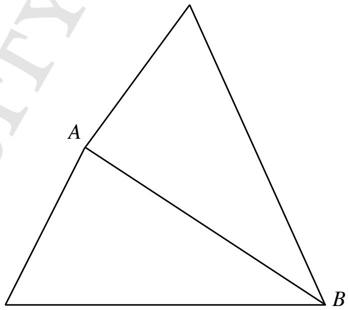
Line of symmetry: $y = 3.5$

3. **(a)** The figure has rotational symmetry of order 1 i.e. no rotational symmetry.
   (b) The figure has rotational symmetry of order 5.
   (c) The figure has rotational symmetry of order 2.
   (d) The figure has rotational symmetry of order 1 i.e. no rotational symmetry.
   (e) The figure has rotational symmetry of order 4.
   (f) The figure has rotational symmetry of order 8.
   (g) The figure has rotational symmetry of order 2.

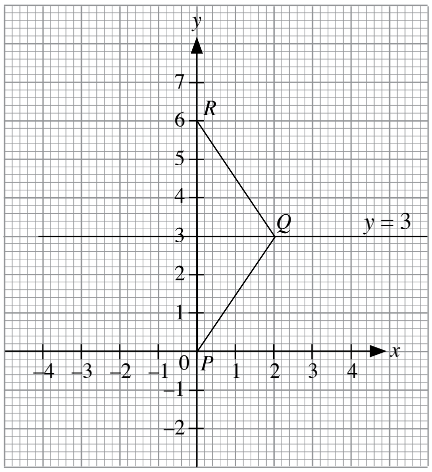
4. **(i)** The letters with line symmetry are O, E, H and I.
   (ii) The letters with rotational symmetry are O, S, H and I.

5. **(a)** False
   (b) False
   (c) True
   (d) True
   (e) False
   (f) True
   (g) True
   (h) False
   (i) True
   (j) False
   (k) False
   (l) False

6. **(a)** An equilateral triangle has 3 lines of symmetry.
   (b)



7. **(i)**



   (ii) The equation of the line of symmetry is $y = 3$.

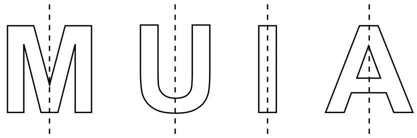
8. **(a)** The letters with a vertical line of symmetry are M, U, I and A.

(b) The letters with horizontal line of symmetry are I and C.

(c) The letter I has two lines of symmetry.

(d) The letters S and L are not symmetrical.

(e)



9. **(a)** The figure has rotational symmetry of order 4.

(b) The figure has rotational symmetry of order 3.

(c) The figure has rotational symmetry of order 5.

(d) The figure has infinite rotational symmetry.

(e) The figure has rotational symmetry of order 8.

(f) The figure has infinite rotational symmetry.

(g) The figure has rotational symmetry of order 2.

10. **(i)** There are infinite planes of symmetry.

(ii) There is 1 axis of rotational symmetry.

(iii) The pencil has infinite rotational symmetry.

Revision Test C1

1. Using Pythagoras' Theorem,

$AB^2 + BC^2 = AC^2$

$5^2 + BC^2 = 13^2$

$BC^2 = 13^2 - 5^2$

$= 144$

$BC = 12$ cm

Area of $\triangle ABC = \frac{1}{2}(5)(12)$

$= 30 \text{ cm}^2$

2. Using Pythagoras' Theorem,

$PR^2 = PQ^2 + QR^2$

$= 3^2 + 3^2$

$= 18$

Using Pythagoras' Theorem,

$RS^2 = PR^2 + PS^2$

$l = 18 + 3^2$

$= 27$

3. **(i)** $\tan 62° = \frac{AB}{46}$

$AB = 46 \tan 62°$

$= 86.5$ m (to 3 s.f.)

$\therefore$ Height of building is 86.5 m

(ii) $\tan 64° = \frac{AB + BC}{46}$

$AB + BC = 46 \tan 64°$

$BC = 46 \tan 64° - 46 \tan 62°$

$= 7.80$ m (to 3 s.f.)

$\therefore$ Height of flag pole is 7.80 m

4. Volume of sphere $= \frac{4}{3}\pi(13.5)^3$

Volume of cone $= \frac{1}{3}\pi(4.5)^2(6)$

$\therefore$ Number of cones $= \dfrac{\frac{4}{3}\pi(13.5)^3}{\frac{1}{3}\pi(4.5)^2(6)}$

$= 81$

5. **(a)** **(i)** Using Pythagoras' Theorem,

$PS^2 + SR^2 = PR^2$

$PS^2 + 5^2 = 13^2$

$PS^2 = 13^2 - 5^2$

$= 144$

$PS = 12$ cm

(ii) Using Pythagoras' Theorem,

$PQ^2 = PS^2 + QS^2$

$= 144 + 9^2$

$= 225$

$PQ = 15$ cm

(iii) Area of $\triangle PQS = \frac{1}{2}(9)(12)$

$= 54 \text{ cm}^2$

(b) $\frac{1}{2}(13)(QT) = \frac{1}{2}(14)(12)$

$QT = \frac{14 \times 12}{13}$

$= 12\frac{12}{13}$ cm (shown)

6. **(a)** **(i)** $\sin 55° = \frac{PT}{6}$

$PT = 6 \sin 55°$

$= 4.91$ cm (to 3 s.f.)

(ii) $\cos 55° = \frac{UR}{6}$

$UR = 6 \cos 55°$

$= 3.44$ cm (to 3 s.f.)

(iii) $\tan \angle PST = \frac{6 \sin 55°}{3}$

$\angle PST = 58.6°$ (to 1 d.p.)

(iv) $\cos \angle PST = \frac{3}{PS}$

$PS = \frac{3}{\cos \angle PST}$

$= 5.76$ cm (to 3 s.f.)

(b) Using Pythagoras' Theorem,

$TR^2 + PT^2 = PR^2$

$TR^2 + (6 \sin 55°)^2 = 12.0^2$

$TR^2 = 12.0^2 - (6 \sin 55°)^2$

$TR = 10.95$ cm (to 4 s.f.)

$TU = TR - UR$

$= 10.95 - 6 \cos 55°$

$= 7.509$ cm (to 4 s.f.)

Area of $PQUT = (7.509)(6 \sin 55°)$

$= 36.9 \text{ cm}^2$ (to 3 s.f.)

7. **(i)** In $\triangle ABC$,

$\cos \theta = \frac{12}{24}$

$= \frac{1}{2}$

$\theta = 60°$

Using Pythagoras' Theorem,

$AC^2 + 12^2 = 24^2$

$AC^2 = 432$

$AC = \sqrt{432}$ cm

In $\triangle ACD$,

$\sin 60° = \frac{AD}{\sqrt{432}}$

$AD = \sqrt{432} \sin 60°$

$= 18$ cm

In $\triangle ADE$,

$\cos 60° = \dfrac{ED}{18}$

$ED = 18 \cos 60°$

$= 9$ cm

(ii) $\tan 60° = \dfrac{AE}{9}$

$AE = 9 \tan 60°$

$= 15.6$ cm (to 3 s.f.)

8. (i) $\dfrac{x}{x+5} = \dfrac{8}{8+6}$

$= \dfrac{8}{14}$

$14x = 8x + 40$

$6x = 40$

$x = 6\dfrac{2}{3}$

$\dfrac{y}{4} = \dfrac{8+6}{8}$

$y = \dfrac{14}{8} \times 4$

$= 7$

$\therefore x = 6\dfrac{2}{3}, y = 7$

(ii) $AP^2 + PQ^2 = \left(11\dfrac{2}{3}\right)^2 + 7^2$

$= 185\dfrac{1}{9}$

$AQ^2 = 14^2$

$= 196$

Since $AP^2 + PQ^2 \neq AQ^2$,

$\triangle APQ$ is not a right-angled triangle.

9. (i) Volume $= \dfrac{2}{3}\pi(5)^3 + \pi(5)^2(15) + \dfrac{1}{3}\pi(5)^2(12)$

$= \dfrac{1675}{3}\pi$

$= 1750$ cm^3 (to 3 s.f.)

(ii) Let the slant height of the cone be l cm.

Using Pythagoras' Theorem,

$l = \sqrt{12^2 + 5^2}$

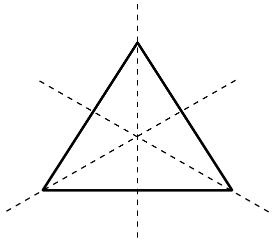
$= 13$

Cost $= 1.4[2\pi(5)^2 + 2\pi(5)(15) + \pi(5)(13)]$

$= 371\pi$

$= \$1165.53$ (to 2 d.p.)

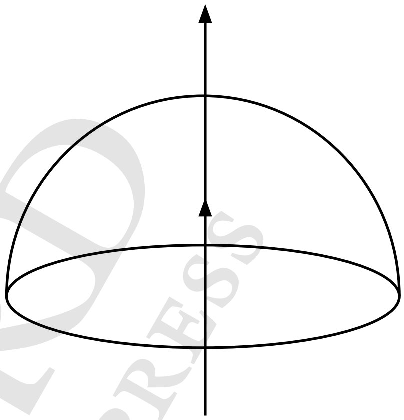
10.



Lines of symmetry: 3

Order of rotational symmetry: 3

11.



A hemisphere has only one axis of rotational symmetry.

Revision Test C2

1. $2 \tan \theta + 3 \cos \theta$

$= 2\left(\frac{3}{4}\right) + 3\left(\frac{4}{5}\right)$

$= 3\frac{9}{10}$

2. **(i)** Using Pythagoras' Theorem,

$x^2 + 23.4^2 = 32.7^2$

$x^2 = 32.7^2 - 23.4^2$

$= 521.73$

$x = 22.84$ m (to 4 s.f.)

Perimeter $= 2(23.4 + 22.84)$

$= 92.5$ m (to 3 s.f.)

(ii) Area $= (23.4)(22.84)$

$= 534$ m^2 (to 3 s.f.)

3. Using Pythagoras' Theorem,

$x^2 + x^2 = 34.2^2$

$2x^2 = 34.2^2$

$x^2 = 584.82$

$x = 24.18$ cm (to 4 s.f.)

$\therefore$ Length of ribbon $= 4(24.18)$

$= 96.7$ cm (to 3 s.f.)

4. **(i)** Using Pythagoras' Theorem,

$PN^2 + NR^2 = PR^2$

$10^2 + NR^2 = 26^2$

$NR^2 = 26^2 - 10^2$

$= 576$

$NR = 24$ cm

(ii) $\sin \angle QRP = \frac{10}{26}$

$= \frac{5}{13}$

$\angle QRP = 22.6°$ (to 1 d.p.)

(iii) $\cos 34° = \frac{10}{PQ}$

$PQ = \frac{10}{\cos 34°}$

$= 12.1$ cm (to 3 s.f.)

(iv) $\tan 34° = \frac{NQ}{10}$

$NQ = 10 \tan 34°$

$= 6.75$ cm (to 3 s.f.)

(v) Area of $\triangle PQR = \frac{1}{2}(10 \tan 34° + 24)(10)$

$= 154$ cm^2

5. $\tan 34° = \frac{123}{d_1}$

$d_1 = \frac{123}{\tan 34°}$

$\tan 49° = \frac{123}{d_2}$

$d_2 = \frac{123}{\tan 49°}$

$\therefore d = d_1 + d_2$

$= \frac{123}{\tan 34°} + \frac{123}{\tan 49°}$

$= 289$ m (to 3 s.f.)

6. **(i)** Using Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$

$= 11^2 + 15^2$

$= 346$

$AC = \sqrt{346}$ cm

$\frac{1}{2}\left(\sqrt{346}\right)(KB) = \frac{1}{2}(11)(15)$

$KB = \frac{11 \times 15}{\sqrt{346}}$

$= 8.87$ cm (to 3 s.f.)

(ii) $\cos \angle KBC = \frac{KB}{15}$

$= \frac{8.870}{15}$

$\angle KBC = 53.7°$ (to 1 d.p.)

7. **(i)** Total volume $= 100 \times \frac{4}{3}\pi(1.2)^3 + 2000$

$= 2720$ cm^3 (to 3 s.f.)

(ii) Number of cups $= \frac{2724}{\pi(4)^2(8)}$

$= 6.77$

≈ 7 (round up to the nearest integer)

8. **(i)** Volume $= \frac{1}{3}(1.8 \times 1.6)(1.1)$

$= 1.1$ m^3 (to the nearest 0.1 m^3)

(ii) Using Pythagoras' Theorem,

$VB^2 = 1.1^2 + 1.6^2$

$= 3.77$

$VB = \sqrt{3.77}$ m

Using Pythagoras' Theorem,

$AC^2 = 1.6^2 + 1.8^2$

$= 5.8$

Using Pythagoras' Theorem,

$VC^2 = 5.8 + 1.1^2$

$= 7.01$

$VC = \sqrt{7.01}$ m

Using Pythagoras' Theorem,

$VD^2 = 1.1^2 + 1.8^2$

$\quad = 4.45$

$VD = \sqrt{4.45}$ m

$\therefore$ Sum of lengths $= \sqrt{3.77} + \sqrt{7.01} + \sqrt{4.45}$

$= 6.70$ m (to 3 s.f.)

9. **(i)** Capacity $= \frac{1}{3}\pi(3.5)^2(2.1) + \pi(3.5)^2(4.9) + (1)(1)(1.5)$

$+ \frac{1}{3}(1)^2(0.9)$

$= 217$ m^3 (to 3 s.f.)

(ii) Total area $= \pi(3.5)\sqrt{3.5^2 + 2.1^2} + 2\pi(3.5)(4.9)$

$+ \pi(3.5)^2 - (1)^2 + 4(1)(1.5)$

$+ 4\left(\frac{1}{2}\right)(1)\sqrt{0.5^2 + 0.9^2}$

$= 198$ m^2 (to 3 s.f.)

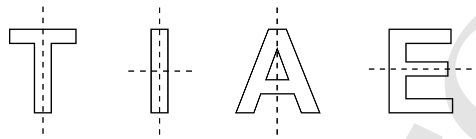
10. **(a)** T, I and A

(b) I and E

(c) I

(d) R, S, N, G, L

(e)



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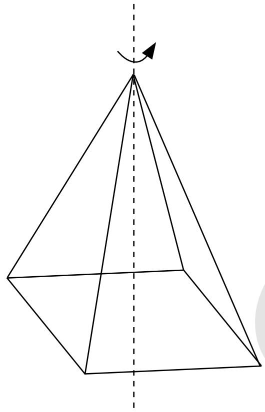


A square pyramid has only one axis of rotational symmetry and rotational symmetry of order 4.

Chapter 14 Sets

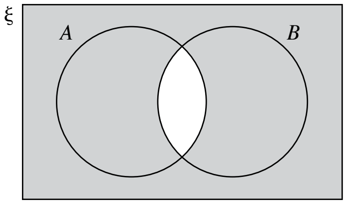
Basic

1. **(a)** Yes, because it is clear if a pupil has no siblings.
(b) No, because a bag may be considered nice by some but not to others.
(c) No, because a singer may be considered attractive to some, but not others.
(d) No, because a song may be well-liked by some, but not others.
(e) Yes, because it is clear whether a teacher teaches Art.
(f) No, because a move may be considered funny to some, but not others.

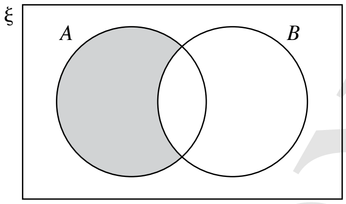
2. **(a)** $A \cup B' = \{a, b, c, x, y, m, n\}$
(b) $A' \cap B' = \{m, n\}$
(c) $A \cap B' = \{a, b, c\}$

3. **(a)** $A \cup B' = \{1, 2, 3, 4, 5, 7, 8\}$
(b) $A' \cap B' = \{4, 8\}$
(c) $A \cap B' = \{1, 2, 7\}$

4. **(a)** $A' \cup B'$



(b) $A \cap B'$

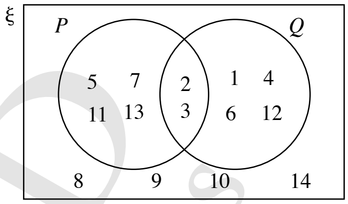


5. **(a)** T
(b) T
(c) T
(d) F
(e) F
(f) T
(g) T
(h) T
(i) F
(j) F

6. **(a)** T
(b) T
(c) F
(d) T
(e) T
(f) F
(g) T
(h) F

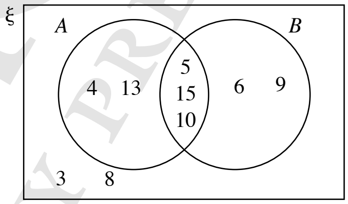
7. $\xi = \{x : x \text{ is an integer}, 1 \leqslant x \leqslant 14\} = \{1, 2, 3, \ldots, 13, 14\}$
$P = \{x : x \text{ is a prime number}\} = \{2, 3, 5, 7, 11, 13\}$
$Q = \{x : x \text{ is a factor of } 12\} = \{1, 2, 3, 4, 6, 12\}$

(a)



(b) **(i)** $P \cup Q' = \{2, 3, 5, 7, 8, 9, 10, 11, 13, 14\}$
(ii) $P' \cap Q' = \{8, 9, 10, 14\}$

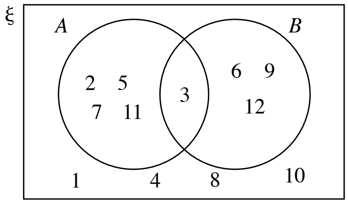
8. **(a)**



(b) **(i)** $(A \cap B)' = \{3, 4, 6, 8, 9, 13\}$
(ii) $A' \cap B = \{6, 9\}$

9. $\xi = \{x : x \text{ is an integer}, 1 \leqslant x \leqslant 12\} = \{1, 2, 3, \ldots, 11, 12\}$
$A = \{x : x \text{ is a prime number}\} = \{2, 3, 5, 7, 11\}$
$B = \{x : x \text{ is a multiple of } 3\} = \{3, 6, 9, 12\}$

(i)



(ii) $A \cap B' = \{2, 5, 7, 11\}$

10. **(a)** $B \cap A'$
(b) B'

Intermediate

11. **(a)** $A \cap B = \{e, x\}$

(b) $A \cup C' = \{a, b, e, d, x, h, m, y, z\}$

(c) $B \cup A' = \{e, x, h, m, n, k, y, z\}$

(d) $B' \cap C' = \{a, b, y, z\}$

(e) $A \cap B \cup C = \{e, x, d, k, n\}$

12. ξ = {polygons}

A = {quadrilaterals}

B = {regular polygons}

(a) square or rhombus

(b) rectangle or parallelogram

13. $\xi = \{x : x \text{ is an integer}, 12 \leqslant x \leqslant 39\}$

$= \{12, 13, 14, \ldots, 37, 38, 39\}$

$A = \{x : x \text{ is a multiple of } 5\} = \{15, 20, 25, 30, 35\}$

$B = \{x : x \text{ is a perfect square}\} = \{16, 25, 36\}$

$C = \{x : x \text{ is odd}\} = \{13, 15, 17, \ldots, 35, 37, 39\}$

(a) $A \cap B = \{25\}$

(b) $A \cap C = \{15, 25, 35\}$

(c) $B \cup C = \{13, 15, 16, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 36, 37, 39\}$

14. $\xi = \{x : x \text{ is an integer}\}$

$A = \{x : x > 4\}$

$B = \{x : -1 < x \leqslant 10\}$

$C = \{x : x < 8\}$

(a) $A \cap B = \{x : 4 < x \leqslant 10\}$

(b) $B \cap C = \{x : -1 < x < 8\}$

(c) $A' \cap B = \{x : -1 < x \leqslant 4\}$

(d) $A' \cap C = \{x : x \leqslant 4\}$

15. $\xi = \{x : x \text{ is an integer}, 0 \leqslant x < 25\}$

$= \{0, 1, 2, 3, \ldots, 23, 24\}$

$B = \{x : x \text{ is divisible by } 5\} = \{0, 5, 10, 15, 20\}$

$C = \{x : x \text{ is prime and } x \leqslant 19\}$

$= \{2, 3, 5, 7, 11, 13, 17, 19\}$

16. $\xi = \{x : x \text{ is an integer}, 0 < x \leqslant 13\}$

$= \{1, 2, 3, \ldots, 11, 12, 13\}$

$A = \{x : 2x > 9\}$

$B = \{x : (x - 2)(x - 5) = 0\}$

$C = \{x : x \text{ is prime}\}$

(a) $C = \{5, 6, 7, 8, 9, 10, 11, 12, 13\}$

(b) $C = \{2, 5\}$

(c) $C = \{1, 3, 5, 7, 11, 13\}$

$A \cap C = \{5, 7, 11, 13\}$

17. c

18. $\xi = \{x : x \text{ is whole number and } x \leqslant 20\}$

$A = \{2, 4, 6, 8, 10, 12\}$

$B = \{1, 4, 9, 16\}$

(a) $A \cap B' = \{2, 6, 8, 10, 12\}$

(b) $A' \cap B = \{1, 9, 16\}$

(c) $A' \cap B' = \{3, 5, 7, 11, 13, 14, 15, 17, 18, 19, 20\}$

(d) $A' \cup B' = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

19. $\xi = \{(x, y) : x \text{ and } y \text{ are integers}\}$

$P = \{(x, y) : 0 < x \leqslant 3 \text{ and } 0 \leqslant y < 6\}$

$Q = \{(x, y) : 2 \leqslant x < 8 \text{ and } 5 \leqslant y \leqslant 9\}$

$P \cap Q = \{(x, y) : 2 \leqslant x \leqslant 3 \text{ and } 5 \leqslant y < 6\}$

$x = 2, 3$ and $y = 5$

$\therefore P \cap Q = \{(2, 5), (3, 5)\}$

20. $\xi = \{a, b, c, d, e, f, g\}$

$A = \{a, c, f, g\}$

$B = \{a, c, g\}$

$C = \{b, c, e, f\}$

(i) $(A \cap B)' = \{b, d, e, f\}$

(ii) $A \cup C' = \{a, c, d, f, g\}$

21. ξ = {all triangles}

A = {isosceles triangles}

B = {equilateral triangles}

C = {right-angled triangles}

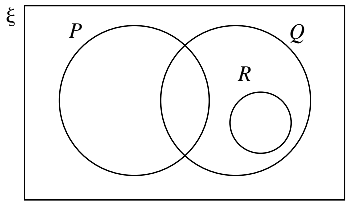
(a) $A \cup B = A$

(b) $B \cap C = \varnothing$

(c) $A \cap B = B$

22. **(a)** $A = B$

(b)



23. $\xi = \{x : x \text{ is an integer}\}$

$A = \{x : 20 < x \leqslant 32\}$

$B = \{x : 24 \leqslant x \leqslant 37\}$

(a) $A \cap B = \{x : 24 \leqslant x \leqslant 32\}$

$= \{24, 25, 26, 27, 28, 29, 30, 31, 32\}$

(b) $A \cup B = \{x : 20 < x \leqslant 37\}$

$= \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37\}$

24. $\xi = \{x : x \text{ is an integer}, 4 \leqslant x \leqslant 22\} = \{4, 5, 6, \ldots, 22\}$

$A = \{x : x \text{ is a multiple of } 5\} = \{5, 10, 15, 20\}$

$B = \{x : x \text{ is a prime number}\} = \{5, 7, 11, 13, 17, 19\}$

$C = \{x : x \text{ is a factor of } 30\} = \{5, 6, 10, 15\}$

(a) $A \cup C = \{5, 6, 10, 15, 20\}$

(b) $B \cap C = \{5\}$

25. $\xi = \{6, 8, 10, 12, 13, 14, 15, 16, 18, 20, 21\}$
$A = \{x : x \text{ is a multiple of } 3\} = \{6, 12, 15, 18, 21\}$
$B = \{x : 2x < 33\} = \{6, 8, 10, 12, 13, 14, 15, 16\}$
$A \cup B = \{6, 8, 10, 12, 13, 14, 15, 16, 18, 21\}$

26. $\xi = \{x : x \text{ is a natural number}, 2 \leqslant x \leqslant 15\}$
$= \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
$A = \{x : x \text{ is a multiple of } 3\} = \{3, 6, 9, 12, 15\}$
$B = \{x : x \text{ is even}\} = \{2, 4, 6, 8, 10, 12, 14\}$
$A' \cap B = \{2, 4, 8, 10, 14\}$

27. $\xi = \{x : x \text{ is a positive integer}\}$
$A = \{x : 7 < 3x < 28\} = \{3, 4, 5, 6, 7, 8, 9\}$
$B = \{x : 3 < 2x + 1 < 25\} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$
$C = \{x : 1 < \frac{x}{2} \leqslant 9\}$
$= \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$

28. **(a)** $A' \cap B = B$
(b) $A \cup B' = B'$

29. $\xi = \{x : x \text{ is a positive integer and } 20 \leqslant x \leqslant 90\}$
$A = \{x : x \text{ is a multiple of } 3\}$
$= \{21, 24, 27, 30, 33, 36, ..., 90\}$
$B = \{x : x \text{ is a perfect square}\} = \{25, 36, 49, 64, 81\}$
$C = \{x : \text{unit digit of } x \text{ is } 1\} = \{21, 31, 41, 51, 61, 71, 81\}$
(i) $A \cap B = \{36, 81\}$
(ii) $A \cap C = \{21, 51, 81\}$

30. $\xi = \{x : x \text{ is a positive integer and } 0 \leqslant x \leqslant 24\}$
$A = \{x : x \text{ is a prime number}\}$
$= \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
$B = \{x : 12 < 3x < 37\} = \{5, 6, 7, 8, 9, 10, 11, 12\}$
$A \cap B = \{5, 7, 11\}$

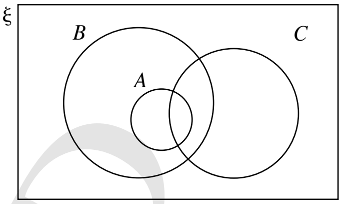
31. **(a)** $P \cup Q = P$
(b) $Q \cap P' = \varnothing$

32. **(a)** $A \cap B = A$
(b) $A \cup B = B$

33. $\xi = \{\text{integers}\}$
$A = \{\text{factors of } 4\} = \{1, 2, 4\}$
$B = \{\text{factors of } 6\} = \{1, 2, 3, 6\}$
$C = \{\text{factors of } 12\} = \{1, 2, 3, 4, 6, 12\}$
$D = \{\text{factors of } 9\} = \{1, 3, 9\}$
(a) $A \cup B = \{1, 2, 3, 4, 6\}$
(b) $B \cap C = \{1, 2, 3, 6\}$
(c) $C \cap D = \{1, 3\}$

Advanced

34. $\xi = \{\text{polygons}\}$
$A = \{\text{polygons with all sides equal}\}$
$B = \{\text{polygons with all angles equal}\}$
$C = \{\text{triangles}\}$
$D = \{\text{quadrilaterals}\}$
(a) $A \cap C$ = equilateral triangle
(b) $A \cap D$ = rhombus
(c) $B \cap D$ = square or rectangle

35.


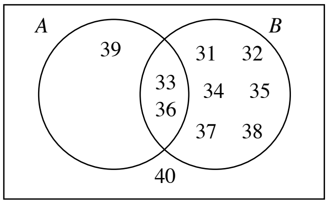
36. $\xi = \{x : x \text{ is an integer less than } 22\}$
$A = \{x : x \text{ is a prime number less than } 20\}$
$= \{2, 3, 5, 7, 11, 13, 17, 19\}$
$B = \{x : a < x < b\}$
For $A \cap B = \varnothing$,
$8 < x < 10$ or $14 < x < 16$
$\therefore a = 8, b = 10$ or $a = 14, b = 16$.

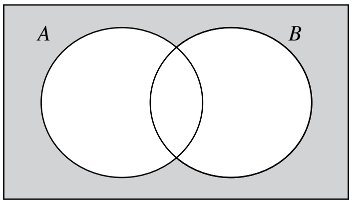
37. $A = \{(x, y) : x + y = 4\}$
$B = \{(x, y) : x = 2\}$
$C = \{(x, y) : y = 2x\}$
(a) $A \cap B = \{(x, y) : x = 2, y = 2\} = \{(2, 2)\}$
(b) $B \cap C = \{(x, y) : x = 2, y = 4\} = \{(2, 4)\}$
(c) $A \cap C = \{(x, y) : x + y = 4, y = 2x\}$
$= \left\{(x, y) : x = 1\frac{1}{3}, y = 2\right\}$
$= \left\{\left(1\frac{1}{3}, 2\frac{2}{3}\right)\right\}$

New Trend

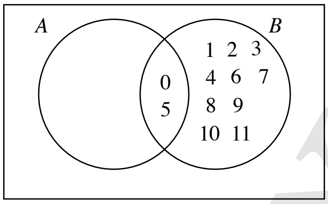
38. $\xi = \{x : x \text{ is an integer}, 30 < x \leqslant 40\}$
$= \{31, 32, 33, \ldots, 39, 40\}$
$A = \{x : x \text{ is a multiple of } 3\} = \{33, 36, 39\}$
$B = \{x : 2x - 4 < 73\} = \{31, 32, 33, 34, 35, 36, 37, 38\}$

(i) $A' \cap B = \{31, 32, 34, 35, 37, 38\}$

(ii) ξ


(iii) ξ


39. $\xi = \{x : x \text{ is an integer}, 0 \leqslant x < 12\}$
$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$
$A = \{x : x(x - 5) = 0\} = \{0, 5\}$
$B = \{x : \frac{1}{3}x - 1 < 3\frac{1}{3}\}$
$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

(a) ξ


(b) **(i)** $A \cap B = \{0, 5\}$
(ii) $A \cup B' = \{0, 5\}$

40. $\xi = \{1, 2, 3, 4, 5, \ldots, 19\}$

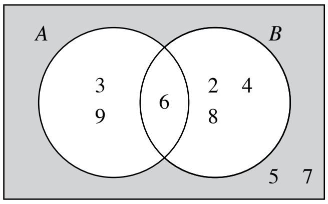
(i) $A = \{x : x \text{ is prime}\} = \{2, 3, 5, 7, 11, 13, 17, 19\}$

(ii) $C = \{x : x \text{ is a factor of } 12\} = \{1, 2, 3, 4, 6, 12\}$

(iii) $B = \{3, 6, 9, 12, 15, 18\}$
$C' = \{5, 7, 8, 9, 10, 11\}$
$\therefore B \cap C' = \{9\}$

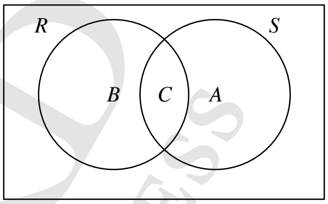
(iv) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 17, 19\}$
$\therefore (A \cup C)' = \{8, 9, 10, 14, 15, 16, 18\}$

41. $\xi = \{x : x \text{ is a positive integer}, 5 < 3x \leqslant 28\}$
$= \{2, 3, 4, 5, 6, 7, 8, 9\}$
$A = \{x : x \text{ is a multiple of } 3\} = \{3, 6, 9\}$
$B = \{x : x \text{ is divisible by } 2\} = \{2, 4, 6, 8\}$

(i) ξ


(ii) $(A \cup B)' = \{5, 7\}$
$n(A \cup B)' = 2$

(iii) $A \cap B' = \{3, 9\}$

42. ξ


Chapter 15 Probability of Single Events

Basic

1. **(a)** $\{A_1, A_2, C, E, H, I, M_1, M_2, S, T_1, T_2\}$

(b) **(i)** Probability of obtaining the letter 'A' = $\frac{2}{11}$

(ii) Probability of obtaining the letter 'H' = $\frac{1}{11}$

(iii) Probability of obtaining a vowel = $\frac{4}{11}$

2. **(a)** $\{HH, HT, TH, TT\}$

(b) **(i)** Probability of obtaining two tails = $\frac{1}{4}$

(ii) Probability of obtaining a head and a tail = $\frac{2}{4}$

$= \frac{1}{2}$

3. **(i)** Probability of getting an odd number = $\frac{3}{6}$

$= \frac{1}{2}$

(ii) Probability of getting a number less than 4 = $\frac{3}{6}$

$= \frac{1}{2}$

(iii) Probability of getting a '5' or a '6' = $\frac{2}{6}$

$= \frac{1}{3}$

(iv) Probability of getting a number which is not '6' = $\frac{5}{6}$

4. **(i)** Probability of drawing a number that is a multiple of 3 = $\frac{5}{8}$

(ii) Probability of drawing a prime number = $\frac{2}{8}$

$= \frac{1}{4}$

(iii) Probability of drawing a number whose digits have a sum that is divisible by 2 = $\frac{3}{8}$

5. **(i)** Probability of drawing a King = $\frac{4}{52}$

$= \frac{1}{13}$

(ii) Probability of drawing the King of diamonds = $\frac{1}{52}$

(iii) Probability of drawing a heart = $\frac{13}{52}$

$= \frac{1}{4}$

(iv) Probability of drawing a picture card = $\frac{12}{52}$

$= \frac{3}{13}$

6. **(i)** Number of white pearls = 50 – 24 – 15 = 11

Probability of selecting a white pearl = $\frac{11}{50}$

(ii) Probability that the pearl selected is not green

$= \frac{24 + 11}{50}$

$= \frac{35}{50}$

$= \frac{7}{10}$

(iii) Probability of selecting a pink pearl = 0

7. **(i)** Probability that the month is December = $\frac{1}{12}$

(ii) Probability that the month begins with the letter J

$= \frac{3}{12}$

$= \frac{1}{4}$

(iii) Probability that the month has exactly 30 days

$= \frac{4}{12}$

$= \frac{1}{3}$

8. **(a)** **(i)** Probability that the customer wins \$88 cash

$= \frac{1}{8}$

(ii) Probability that the customer wins a \$10 shopping voucher = $\frac{3}{8}$

(iii) Probability that the customer wins a packet of dried scallops = 0

(b) A pair of movie tickets and a can of abalone

9. **(i)** Angle corresponding to the sector representing beans

= 360° – 150° – 90° – 50°

= 70°

Probability that the student prefers beans = $\frac{70°}{360°}$

$= \frac{7}{36}$

(ii) Probability that the student prefers broccoli or carrots

$= \frac{90° + 50°}{360°}$

$= \frac{140°}{360°}$

$= \frac{7}{18}$

10. (i) Probability that a bag selected has a mass of exactly $1 \text{ kg} = 1 - \frac{1}{40} - \frac{1}{160}$

$= \frac{31}{32}$

(ii) Number of bags each with a mass of less than 1 kg

$= \frac{1}{160} \times 8000$

$= 50$

Intermediate

11. (i) Number of cards remaining = 13

Probability of drawing the Jack of diamonds = $\frac{1}{13}$

(ii) Probability of drawing a King, a Queen or a Jack

$= \frac{12}{13}$

(iii) Probability of drawing the ace of hearts or the King of hearts = $\frac{2}{13}$

(iv) Probability of drawing a joker = 0

12. (i) Number of slots = 37

Probability that the ball lands in the slot numbered 13

$= \frac{1}{37}$

(ii) Prime numbers from 0 to 37: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

Probability that the ball lands in the slot numbered with a prime number = $\frac{11}{37}$

(iii) Probability that the ball lands in the slot numbered with a number less than 19 = $\frac{19}{37}$

(iv) Probability that the ball lands in the slot numbered with an odd number = $\frac{18}{37}$

13. (i) Number of two-digit numbers = 90

Two-digit numbers greater than 87: 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99

Probability that the number generated is greater than 87 = $\frac{12}{90}$

$= \frac{2}{15}$

(ii) Two-digit numbers less than 23: 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22

Probability that the number generated is less than 23

$= \frac{13}{90}$

(iii) Two-digit numbers divisible by 4: 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96

Probability that the number generated is divisible by 4

$= \frac{22}{90}$

$= \frac{11}{45}$

(iv) Number of two-digit numbers between 55 and 72 inclusive = 18

Probability that the number is between 55 and 72 inclusive = $\frac{18}{90}$

$= \frac{1}{5}$

14. (i) Number of cards = 16

Probability of selecting a vowel = $\frac{7}{16}$

(ii) Probability of selecting a letter which appears in the word 'SCIENCE' = $\frac{6}{16}$

$= \frac{3}{8}$

(iii) Probability of selecting a letter which appears in the word 'SMART' = $\frac{7}{16}$

(iv) Probability of selecting a letter which appears in the word 'DUG' = 0

15. (a) Number of cards = 11

(i) Probability that the card shows the letter 'P'

$= \frac{1}{11}$

(ii) Probability that the card shows the letter 'E'

$= \frac{3}{11}$

(iii) Probability that the card shows a vowel or a consonant = 1

(b) Number of cards = 10

(i) Probability that the card shows the letter 'P'

$= \frac{1}{10}$

(ii) Probability that the card shows the letter 'E'

$= \frac{2}{10}$

$= \frac{1}{5}$

(iii) Probability that the card shows a vowel = $\frac{3}{10}$

16. **(a)** Number of students = 210

(i) Probability of selecting a Secondary 1 student

$= \frac{22 + 38}{210}$

$= \frac{60}{210}$

$= \frac{2}{7}$

(ii) Probability of selecting a girl

$= \frac{38 + 25 + 35 + 22}{210}$

$= \frac{120}{210}$

$= \frac{4}{7}$

(iii) Probability of selecting an upper secondary

student $= \frac{25 + 35 + 24 + 22}{210}$

$= \frac{106}{210}$

$= \frac{53}{105}$

(iv) Probability of selecting a Secondary 2 student

who is a boy $= \frac{19}{210}$

(b) **(i)** Probability of selecting a Secondary 3 student

who is a girl $= \frac{38}{215}$

(ii) Probability of selecting a Secondary 2 student or

a Secondary 4 student $= \frac{21 + 25 + 24 + 22}{215}$

$= \frac{92}{215}$

17. Probability that it is labelled Gold $= 1 - \frac{1}{5} - \frac{1}{4}$

$= \frac{11}{20}$

Total number of boxes $= 55 \div \frac{11}{20}$

$= 100$

18. **(i)** Number of medical staff $= \frac{1}{5} \times 30 - 2$

$= 4$

(ii) Number of footballers $= 30 - 4 - 2$

$= 24$

Number of midfielders $= \frac{3}{8} \times 24$

$= 9$

Number of goalkeepers $= \frac{1}{3} \times 9$

$= 3$

Number of forwards $= 24 - 3 - 7 - 9$

$= 5$

Probability of selecting a forward from the contingent

$= \frac{5}{30}$

$= \frac{1}{6}$

19. **(a)** **(i)** Probability of selecting a vowel $= \frac{1}{7}$

(ii) Probability of selecting a card that bears the letter

C $= \frac{3}{7}$

(b) $\frac{3}{7 + x} = \frac{1}{7}$

$21 = 7 + x$

$x = 14$

20. **(a)** **(i)** Probability that the mark is less than 44

$= \frac{8}{15}$

(ii) Probability that the mark is not a prime number

$= \frac{14}{15}$

(iii) Probability that the mark is divisible by 11

$= \frac{3}{15}$

$= \frac{1}{5}$

(b) Probability that the student obtained the badge

$= \frac{9}{15}$

$= \frac{3}{5}$

(c) Probability that the mark was 39 $= \frac{2}{6}$

$= \frac{1}{3}$

21. (i) Experimental probability of obtaining a '1' $= \frac{2}{20}$

$= \frac{1}{10}$

Experimental probability of obtaining a '2' $= \frac{4}{20}$

$= \frac{1}{5}$

Experimental probability of obtaining a '3' $= \frac{4}{20}$

$= \frac{1}{5}$

Experimental probability of obtaining a '4' $= \frac{3}{20}$

Experimental probability of obtaining a '5' $= \frac{4}{20}$

$= \frac{1}{5}$

Experimental probability of obtaining a '6' $= \frac{3}{20}$

(ii) No. As the number of rolls increases, the experimental probability of an outcome occurring tends towards the theoretical probability of the outcome happening i.e. $\frac{1}{6}$.

22. (i) $\frac{x}{35 + x} = \frac{1}{6}$

$6x = 35 + x$

$5x = 35$

$x = 7$

(ii) Probability of selecting a sports car $= \frac{35 + 5}{35 + 7 + 5}$

$= \frac{40}{47}$

23. $\frac{12 + x + 2}{36 + 12 + 2x + x + 2} = 0.3$

$\frac{x + 14}{3x + 50} = 0.3$

$x + 14 = 0.9x + 15$

$0.1x = 1$

$x = 10$

24. (i) $\frac{x}{18 + x} = \frac{3}{5}$

$5x = 54 + 3x$

$2x = 54$

$x = 27$

(ii) Probability of selecting a pink sweet

$= \frac{15}{18 + 27 + 10 + 15}$

$= \frac{15}{70}$

$= \frac{3}{14}$

Advanced

25. (i) Number of elements of $S = 50$

Integers in S that are not divisible by 2 or 3: 1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, 41, 43, 47, 49

Probability that the element is not divisible by 2 or 3

$= \frac{17}{50}$

(ii) Number of elements that contain the digit '2' at least once: 2, 12, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 42

Probability that the element contains the digit '2' at least once $= \frac{14}{50}$

$= \frac{7}{25}$

26. (i)

	Smoke	Do not smoke	Total
Male	18	42	60
Female	8	32	40
Total	26	74	100

(ii) Probability that a randomly selected smoker is male

$= \frac{18}{26}$

$= \frac{9}{13}$

(iii) The respondents of this online survey may not be a good representation of the country's population.

New Trend

27. (a) Probability of selecting a red chip $= \frac{10}{24}$

$= \frac{5}{12}$

(b) Let x be the number of extra green chips added,

$\frac{6+x}{24+x} = \frac{1}{3}$

$18 + 3x = 24 + x$

$2x = 6$

$x = 3$

$\therefore$ 3 green chips must be placed in the bag so that the probability of choosing a green chip would be $\frac{1}{3}$.

Chapter 16 Statistical Diagrams

Basic

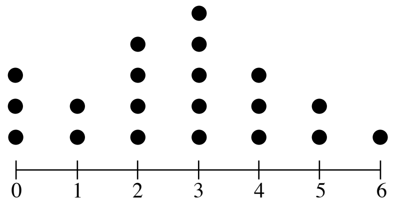
1. **(i)** Most common length = 7 cm

(ii) Length of longest fish = 9 cm

(iii) Percentage of fish which have lengths of more than 6 cm $= \frac{10}{20} \times 100\%$

$= 50\%$

2. **(i)**



(ii) Most common number of universities = 3

(iii) Probability that the student has not applied to a university $= \frac{3}{20}$

3. **(i)** Total number of people = 29

(ii) Most common duration = 20 minutes

(iii) Percentage of people who take less than half an hour

$= \frac{17}{29} \times 100\%$

$= 58.6\%$ (to 3 s.f.)

4.

Stem	Leaf
1	5 6 8
2	1 4 7 8 9
3	0 5

Key: 1 | 5 means 15 ohms

5. **(i)** Take two points on the line and draw dotted lines to form the right-angled triangle.

Vertical change (or rise) $= 7.6 - 3.8$

$= 3.8$

Horizontal change (or run) $= 6 - 2$

$= 4$

Since the line slopes downwards from the left to the right, its gradient is negative.

Gradient $= -\frac{\text{rise}}{\text{run}}$

$= -\frac{3.8}{4}$

$= -0.950$ (to 3 s.f.)

(ii) y-intercept = 9.5

$\therefore$ The equation of the line of best fit is $y = -0.950x + 9.5$.

(iii) Extrapolating the line of best fit, we see that 1900 people will visit the gallery 8 years after its opening.

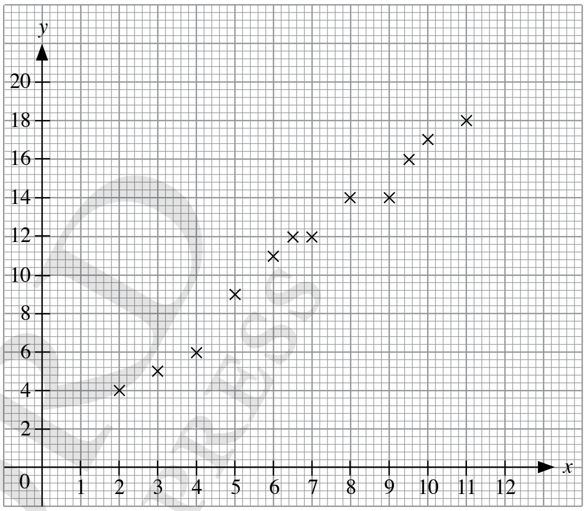
(iv) It would be unreliable since year 8 lies outside of the range between year 0 and year 6.

6. **(a)** The data shows strong, negative correlation.

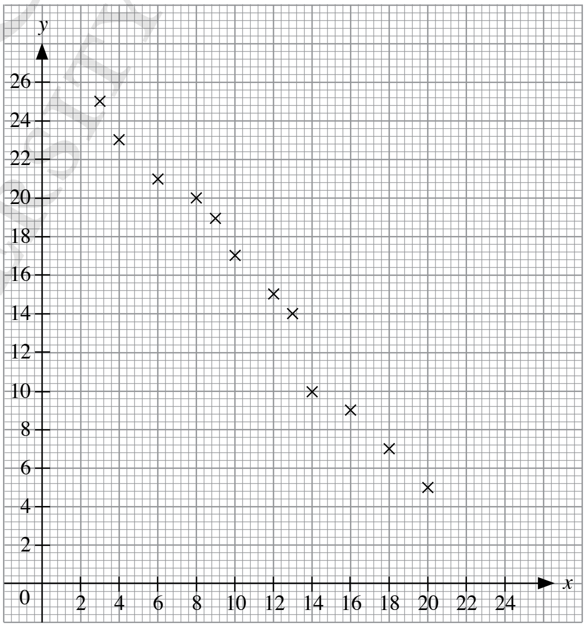
(b) The data shows strong, positive correlation.

(c) The data shows no correlation.

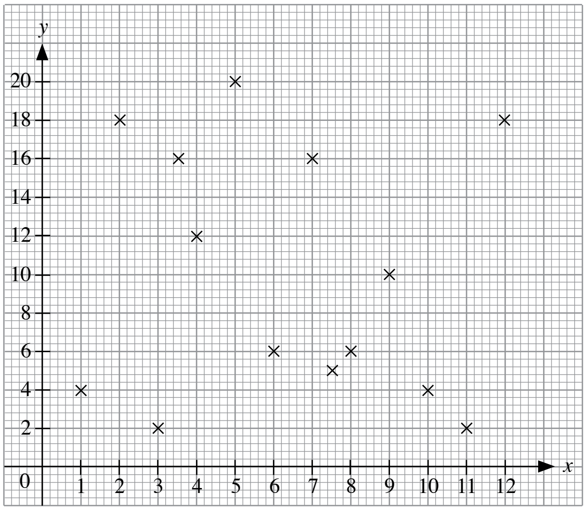
7. **(a)** The data shows strong, positive correlation.



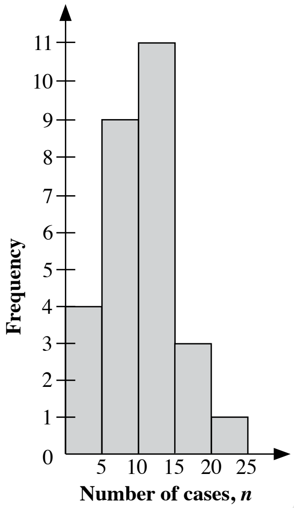
(b) The data shows strong, negative correlation.



(c) The data shows no correlation.



8. **(i)**



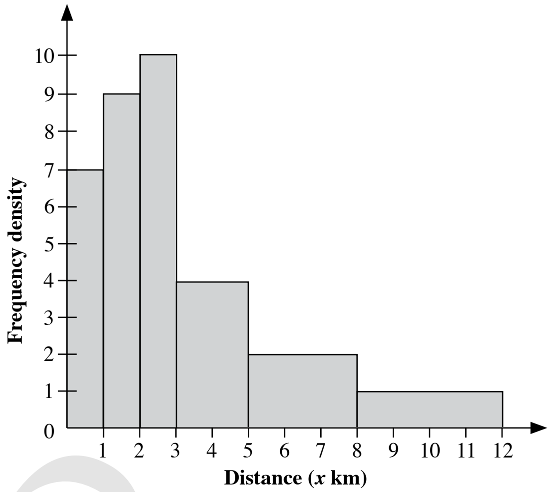
(ii) Number of days = 3 + 1
= 4

9. **(a)** The 4th day had the greatest number of employees report sick. 35 workers reported sick.

(b) The 10th day had the least number of employees report sick. 13 workers reported sick.

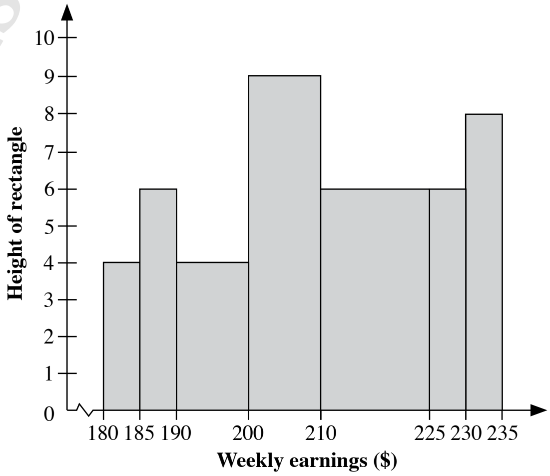
(c) The number of employees who reported sick was more than 30 on the 4th and 8th day.

10.



11. Since the class intervals are unequal, the histogram is to be drawn using either height of rectangle or frequency density.

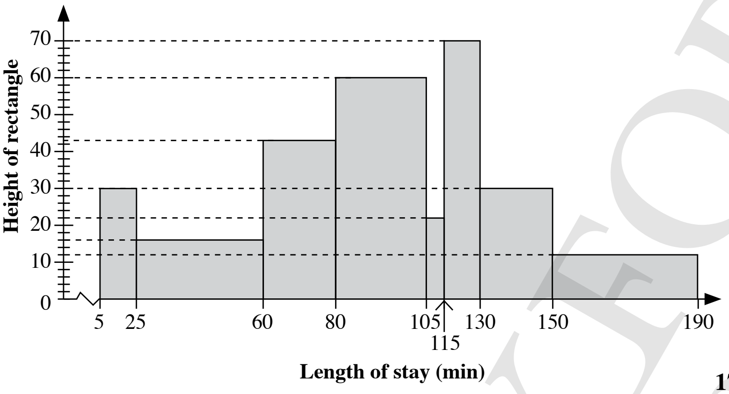
Weekly earnings ($)	Class width		Frequency	Rectangle's height
$180 \leqslant x < 185$	5	1 × standard	4	$4 \div 1 = 4$
$185 \leqslant x < 190$	5	1 × standard	6	$6 \div 1 = 6$
$190 \leqslant x < 200$	10	2 × standard	8	$8 \div 2 = 4$
$200 \leqslant x < 210$	10	2 × standard	18	$18 \div 2 = 9$
$210 \leqslant x < 225$	15	3 × standard	18	$18 \div 3 = 6$
$225 \leqslant x < 230$	5	1 × standard	6	$6 \div 1 = 6$
$230 \leqslant x < 235$	5	1 × standard	8	$8 \div 1 = 8$



12. **(a)** Total number of cars
$= 60 + 56 + 86 + 150 + 60 + 105 + 60 + 48$
$= 625$

(b) Since the class intervals are unequal, the histogram is to be drawn using either height of rectangle or frequency density.

Class interval	Class width		Frequency	Rectangle's height
5 – 24	20	2 × standard	60	60 ÷ 2 = 30
25 – 59	35	3.5 × standard	56	56 ÷ 3.5 = 16
60 – 79	20	2 × standard	86	86 ÷ 3 = 43
80 – 104	25	2.5 × standard	150	150 ÷ 2.5 = 60
105 – 114	10	1 × standard	60	60 ÷ 1 = 22
115 – 129	15	1.5 × standard	105	105 ÷ 1.5 = 70
130 – 149	20	2 × standard	60	60 ÷ 2 = 30
150 – 189	40	4 × standard	48	48 ÷ 4 = 12



Intermediate

13. **(i)** Total number of children who participated in the survey = 23

(ii) Greatest number of children in a family = 6 + 1
= 7

(iii) Average number of children in a family

$$= \frac{5 \times 1 + 8 \times 2 + 4 \times 3 + 3 \times 4 + 2 \times 5 + 1 \times 7}{23}$$

$$= \frac{62}{23}$$

$= 2.70$ (to 3 s.f.)

(iv) Number of children with fewer than 2 siblings = 13

$$\frac{13}{23 + k} = \frac{13}{25}$$

$$23 + k = 25$$

$$k = 2$$

14. **(i)** Fraction of people in Group 1 $= \frac{2}{12}$

$= \frac{1}{6}$

Fraction of people in Group 2 $= \frac{11}{12}$

(ii) Group 1 consists of healthy human beings because a large proportion of the people do not have to undergo a blood test.

15. **(i)** Total number of boys = 56

(ii) Most common mass = 63 kg

(iii) Number of boys who have to gain mass = 18
Number of boys who have to lose mass = 16
∴ Ratio is 18 : 16 = 9 : 8

16. **(i)**

Stem	Leaf
1	0 3 4
1	7 8
2	1 2 4
2	5 6 6 6 7 9
3	3
3	5 6 6
4	0 4

Key: 1 | 0 means 10

(ii) The most common number of smartphone applications downloaded last month is 26.

(iii) Percentage of people $= \frac{9}{20} \times 100\%$
$= 45\%$

17. **(i)** Publishing House *A*: 46 hours
Publishing House *B*: 48 hours

(ii) Publishing House *A*: $\frac{10}{17} \times 100\% = 58.8\%$ (to 3 s.f.)

Publishing House *B*: $\frac{10}{18} \times 100\% = 55.6\%$ (to 3 s.f.)

18. **(i)**

Leaves for Factory A	Stem	Leaves for Factory B
	43	3 7
7 2	44	5 6 6 9
8 7 4 0	55	3 7 7
5 5 4 0	66	1 5 5 9 9
8 7 3 3 2 1	77	2 3 5 6 6
8 3 3 0	88	2

Key: 43 | 3 means 433 hours

(ii) Factory A produces longer-lasting light bulbs as there are more light bulbs with durations of more than 770 hours.

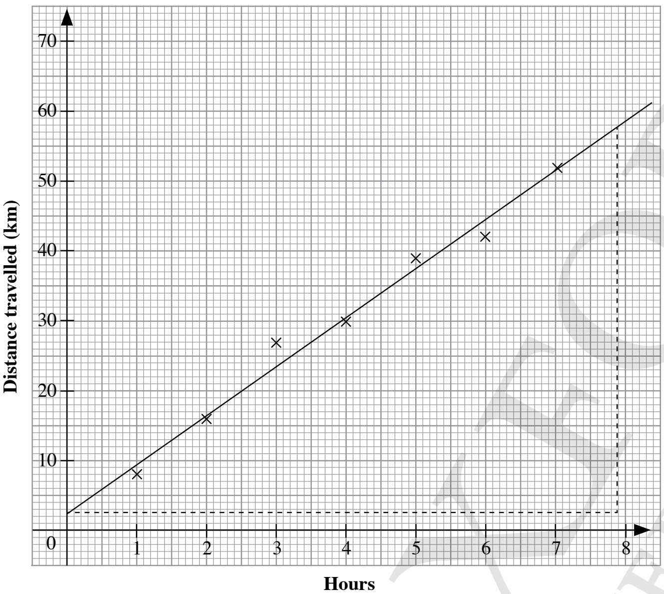
19. (i)

Leaves for test scores before the remedial	Stem	Leaves for test scores after the remedial
5 3	3	9
8 6 6	4	0 6
7 6 3	5	2 3 7 9
8 6 1 0	6	1 3 8
9 8 7 7 5 4 2	7	0 2 2 5 7 9
0	8	0 0 2 9

Key: 3 | 9 means 39

(ii) Yes, it is effective because the test scores after the remedial are generally higher than those before the remedial.

20. (a)



(b) The line of best fit is drawn passing through as many points as possible and as close as possible to all the other points.

(c) Using the line of best fit on the scatter diagram, the hiker travels 48 km in 6.5 hours.

(d) Take two points on the line and draw dotted lines to form the right-angled triangle.

Vertical change (or rise) = 58 – 2.5

= 55.5

Horizontal change (or run) = 7.9 – 0

= 7.9

Since the line slopes upwards from the left to the right, its gradient is positive.

$$\text{Gradient} = \frac{55.5}{7.9}$$

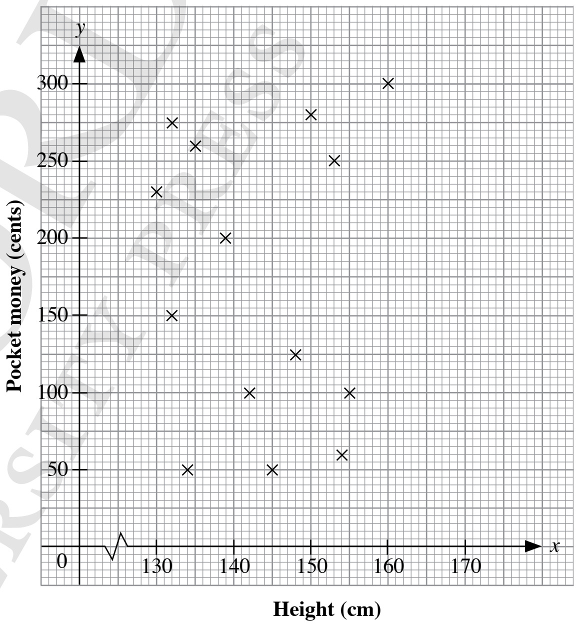
= 7 (to nearest whole number)

y-intercept = 2.5

$\therefore$ The equation of the line of best fit is $y = 7x + 2.5$.

(e) The data displays strong, positive correlation.

21. (i)



(ii) The data displays no correlation.

(iii) Since there is no correlation between the heights of pupils and the amount of pocket money they receive, we cannot use the graph to predict the amount of pocket money that a child of height 147 cm will receive.

22. (i)

Age of patient, x years	Frequency
$10 \leqslant x < 20$	85
$20 \leqslant x < 30$	117
$30 \leqslant x < 40$	38
$40 \leqslant x < 50$	24
$50 \leqslant x < 60$	18
$60 \leqslant x < 70$	16
Total frequency	300

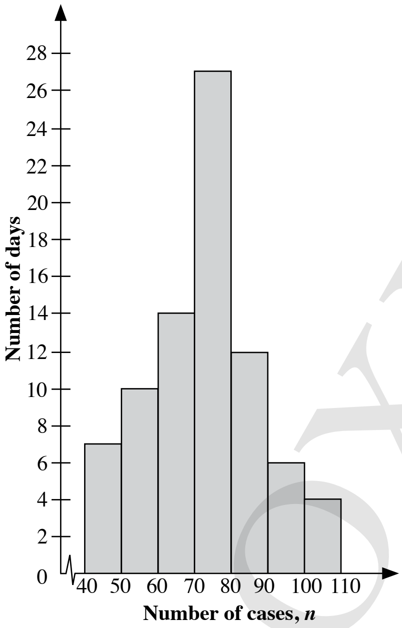
(ii) Percentage of patients who are at least 50 years old

$= \frac{18 + 16}{300} \times 100\%$

$= 11.3\%$ (to 3 s.f.)

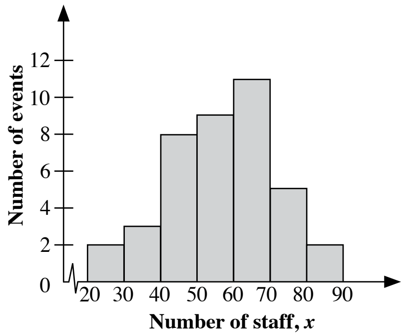
(iii) No. The actual ages of the patients in the interval $20 \leqslant x < 30$ are not known, so it is incorrect for Priya to assume that al the patients in this interval are $\frac{20 + 30}{2} = 25$ years old.

23. (i)



(ii) No, the most number of cases occur in the interval $70 \leqslant n < 80$, but it is not correct to take the mid-value of this interval.

24. (i)



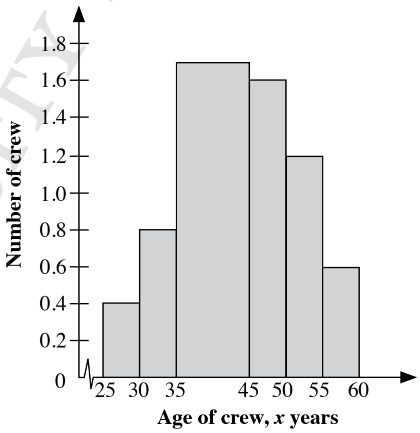
(ii) Number of events $= 45\% \times 40$

$= 18$

$\therefore p = 60$

25. (i)

Age of crew, x years	Frequency	Frequency density
$25 \leqslant x < 30$	2	0.4
$30 \leqslant x < 35$	4	0.8
$35 \leqslant x < 45$	17	1.7
$45 \leqslant x < 50$	8	1.6
$50 \leqslant x < 55$	6	1.2
$55 \leqslant x < 60$	3	0.6



(ii) Number of crew $= 0.85 \times 40$

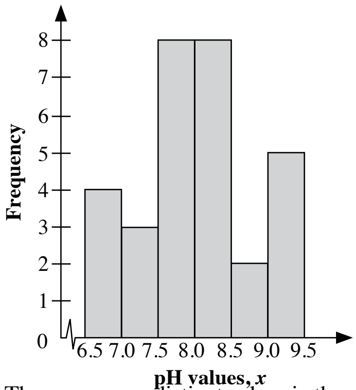
$= 34$

$\therefore p = 35$

26. (i)

pH values, x	Tally	Frequency
$6.5 \leqslant x < 7.0$	////	4
$7.0 \leqslant x < 7.5$	///	3
$7.5 \leqslant x < 8.0$	~~////~~ ///	8
$8.0 \leqslant x < 8.5$	~~////~~ ///	8
$8.5 \leqslant x < 9.0$	//	2
$9.0 \leqslant x < 9.5$	~~////~~	5
Total frequency		30

(ii)



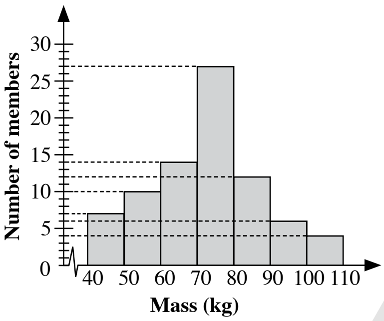
(iii) There are many distinct values in the set of data. Using a histogram for grouped data would be more suitable.

(iv) Percentage of the types which are alkaline

$= \frac{26}{30} \times 100\%$

$= 86.7\%$ (to 3 s.f.)

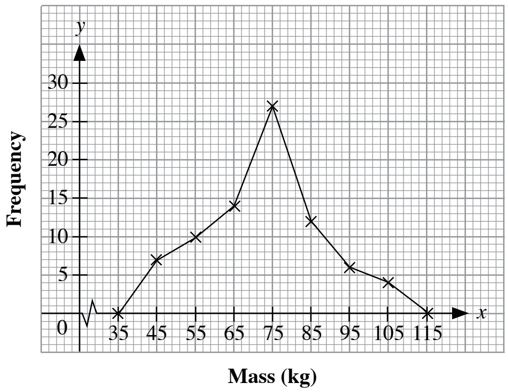
27. (a)



(b)

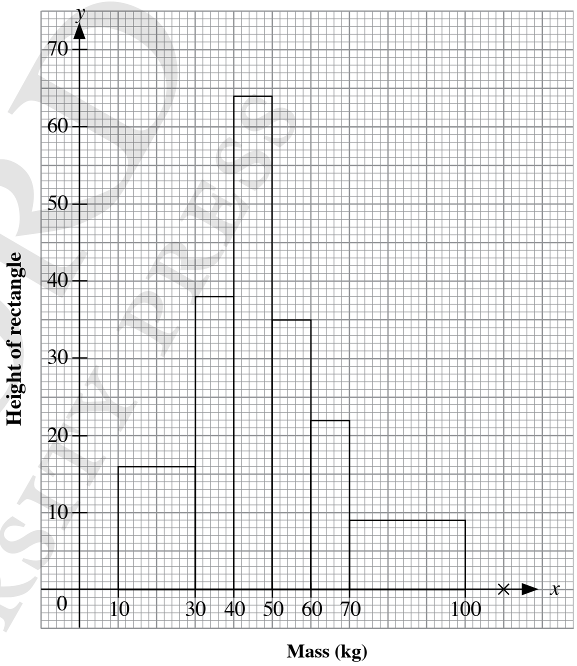
Mass, (x kg)	Mid-value	Frequency
$40 < x \leqslant 50$	45	7
$50 < x \leqslant 60$	55	10
$60 < x \leqslant 70$	65	14
$70 < x \leqslant 80$	75	27
$80 < x \leqslant 90$	85	12
$90 < x \leqslant 100$	95	6
$100 < x \leqslant 110$	105	4

The points to be plotted are (35, 0), (45, 7), (55, 10), (65, 14), (75, 27), (85, 12), (95, 6), (105, 4) and (115, 0).

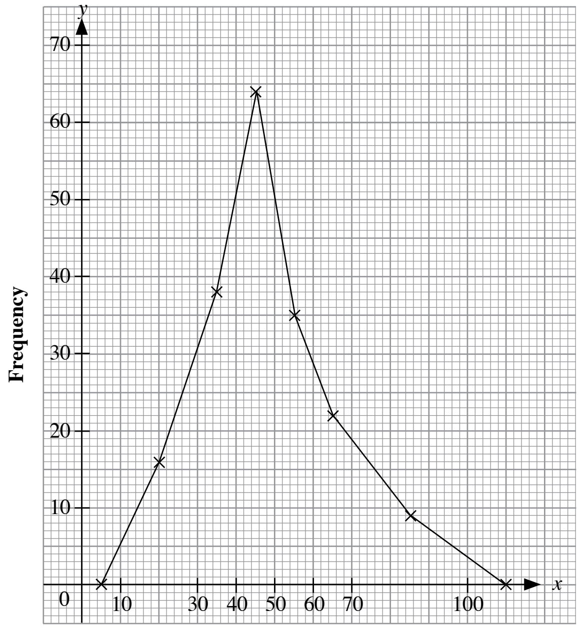


28. (a) Since the class intervals are unequal, the histogram is to be drawn using either height of rectangle or frequency density.

Class interval	Class width		Frequency	Rectangle's height
10 – 29	20	2 × standard	32	32 ÷ 2 = 16
30 – 39	10	1 × standard	38	38 ÷ 1 = 38
40 – 49	10	2 × standard	64	64 ÷ 1 = 64
50 – 59	10	2 × standard	35	35 ÷ 1 = 35
60 – 69	10	1 × standard	22	22 ÷ 1 = 22
70 – 99	30	3 × standard	9	9 ÷ 3 = 3



(b) The points to be plotted are (5, 0), (20, 16), (35, 38), (45, 64), (55, 35), (65, 22), (85, 9) and (105, 0).



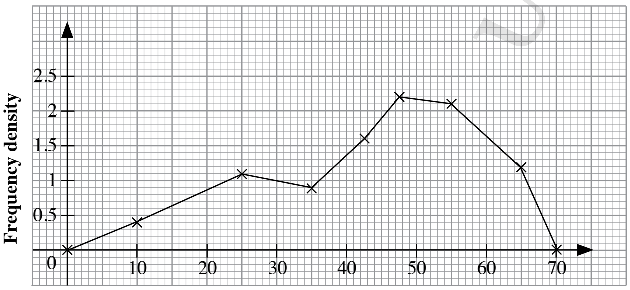
Mass (kg)

29. (a)

Height, (x cm)	Number of plants
$0 < x \leqslant 20$	$0.4 \times 20 = 8$
$20 < x \leqslant 30$	11
$30 < x \leqslant 40$	9
$40 < x \leqslant 45$	8
$45 < x \leqslant 50$	$2.2 \times 5 = 11$
$50 < x \leqslant 60$	$2.1 \times 10 = 21$
$60 < x \leqslant 70$	$1.2 \times 10 = 12$

(b) Number of plants $= 8 + 11 + 9 + 8 + 11 + 21 + 12$
$= 80$

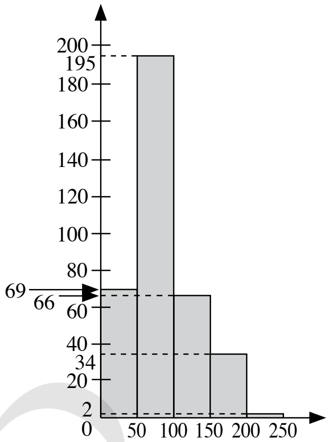
(c) The points to be plotted are (0, 0), (10, 0.4), (25, 1.1), (35, 0.9), (42.5, 1.6), (47.5, 2.2), (55, 2.1), (65, 1.2) and (70, 0).



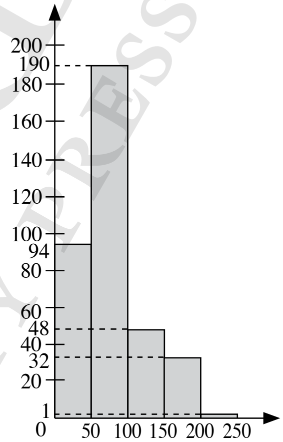
Height (x cm)

Advanced

30. (i)



PSI value, x, in 2012



PSI value, x, in 2013

(ii) The measures taken have been effective in improving the air quality as the PSI values in 2013 are generally lower than those in 2012.

Chapter 17 Averages of Statistical Data

Basic

1. (a) 11, 11, 12, 13, 16

$$\text{Mean} = \frac{11 + 11 + 12 + 13 + 16}{5}$$
$$= 12.6$$

Median = 12

Mode = 11

(b) 11, 12, 18, 18, 20, 20, 20, 24, 29, 41

$$\text{Mean} = \frac{11 + 12 + 18 + 18 + 20 + 20 + 20 + 24 + 29 + 41}{10}$$
$$= 21.3$$
$$\text{Median} = \frac{20 + 20}{2}$$
$$= 20$$

Mode = 20

(c) 10.5, 12.6, 12.6, 13.5, 14.3, 15.3, 16.0, 16.4

$$\text{Mean} = \frac{10.5 + 12.6 + 12.6 + 13.5 + 14.3 + 15.3 + 16.0 + 16.4}{8}$$
$$= 13.9$$
$$\text{Median} = \frac{13.5 + 14.3}{2}$$
$$= 13.9$$

Mode = 12.6

(d) 7, 8.1, 8.1, 8.1, 9.4, 9.4, 9.6, 10.4, 10.5, 11, 11.7

$$\text{Mean} = \frac{7 + 8.1 + 8.1 + 8.1 + 9.4 + 9.4 + 9.6 + 10.4 + 10.5 + 11 + 11.7}{11}$$
$$= 9.39 \text{ (to 3 s.f.)}$$

Median = 9.4

Mode = 8.1

2. 35, 36, 38, 38, 38, 39, 39, 40, 42, 43, 45, 45, 45, 45, 47

(i) $$\text{Mean} = \frac{35 + 36 + 38 + 38 + 38 + 39 + 39 + 40 + 42 + 43 + 45 + 45 + 45 + 45 + 47}{15}$$
$$= 41$$

(ii) Mode = 45

(iii) Median = 40

3. $$\text{Mean} = \frac{3 + 7 + 13 + 14 + 16 + 19 + 20 + x}{8}$$
$$= \frac{92 + x}{8}$$
$$\text{Median} = \frac{14 + 16}{2}$$
$$= 15$$

Since mean = median,

$$\frac{92 + x}{8} = 15$$
$$92 + x = 120$$
$$x = 28$$

4. (i) Total number of seeds $= 100 \times 5$

$= 500$

(ii) Number of seeds that germinated

$= 30 \times 1 + 25 \times 2 + 20 \times 3 + 10 \times 4 + 5 \times 5$

$= 205$

$$\text{Fraction of seeds that germinated} = \frac{205}{500}$$
$$= \frac{41}{100}$$

(iii) $$\text{Mean} = \frac{10 \times 0 + 30 \times 1 + 25 \times 2 + 20 \times 3 + 10 \times 4 + 5 \times 5}{100}$$
$$= 2.05$$

Median = 2

Mode = 1

5. (i)

Number of countries	Frequency
0	7
1	9
2	7
3	4
4	2
5	1
Total frequency	30

(ii) $$\text{Mean} = \frac{7 \times 0 + 9 \times 1 + 7 \times 2 + 4 \times 3 + 2 \times 4 + 1 \times 5}{30}$$
$$= 1.6$$

Median = 1

Mode = 1

Intermediate

6. Let the eighth number be x.

1, 2, 2, 4, x, 7, 8, 13

$$\text{Median} = \frac{4 + x}{2}$$
$$4.5 = \frac{4 + x}{2}$$
$$9 = 4 + x$$
$$x = 5$$

∴ The eighth number is 5.

Mode = 2

7. Sum of the set of 12 nunbers $= 12 \times 5$
$= 60$

Sum of the set of 8 numbers $= 8a$

$$\text{Mean of combined set of 20 numbers} = \frac{60 + 8a}{20}$$

$$8 = \frac{60 + 8a}{20}$$

$$160 = 60 + 8a$$

$$8a = 100$$

$$a = \frac{100}{8} = 12.5$$

8. **(a)** **(i)** Modal profit = \$3 million

(ii) Median profit = \$2 million

(b) Mean profit

$$= \frac{2 \times 0 + 6 \times 1 + 8 \times 2 + 10 \times 3 + 4 \times 4}{30}$$

= \$2.27 million (to 3 s.f.)

∴ Raj is incorrect.

9. **(a)** **(i)** $12 + 9 + x + 6 + y + 7 = 49$

$x + y + 34 = 49$

$x + y = 15$ (shown)

(ii) $$\text{Mean} = \frac{12 \times 1 + 9 \times 2 + x \times 3 + 6 \times 4 + y \times 5 + 7 \times 6}{49}$$

$$3\frac{2}{49} = \frac{96 + 3x + 5y}{49}$$

$149 = 96 + 3x + 5y$

$3x + 5y = 53$ (shown)

(iii) $x + y = 15$ —(1)

$3x + 5y = 53$ —(2)

$(1) \times 3$: $3x + 3y = 45$ —(3)

$(2) - (3)$: $2y = 8$

$y = 4$

Substitute $y = 4$ into (1):

$x + 4 = 15$

$x = 11$

∴ $x = 11, y = 4$

(b) **(i)** Mode = 1

(ii) Median = 3

(c) Let the number shown on the die be n.

$$\text{Mean} = \frac{12 \times 1 + 9 \times 2 + 11 \times 3 + 6 \times 4 + 4 \times 5 + 7 \times 6 + n}{50}$$

$$3 = \frac{149 + n}{50}$$

$150 = 149 + n$

$n = 1$

∴ The number shown on the die is 1.

10. Initial sum of eye pressure $= 30 \times 12.4$
= 372 mm Hg

New sum of eye pressure $= 30 \times 12.6$
= 378 mm Hg

∴ Nora's actual eye presssure = 8 + (378 – 372)
= 14 mm Hg

11. 62.0, 62.0, 62.6, 63.1, 63.7, 64.2, 64.3, 64.7, 65.1, 65.2, 65.2, 65.2, 65.5, 65.9, 66.8, 67.1, 67.4, 68.2

(a) **(i)** $$\text{Mean} = \frac{62.0 + 62.0 + 62.6 + 63.1 + 63.7 + 64.2 + 64.3 + 64.7 + 65.1 + 65.2 + 65.2 + 65.2 + 65.5 + 65.9 + 66.8 + 67.1 + 67.4 + 68.2}{18}$$

= 64.9 s

(ii) Mode = 65.2 s

(iii) $$\text{Median} = \frac{65.1 + 65.2}{2}$$

= 65.15 s

(b) $$\text{Percentage} = \frac{\frac{100}{62.0}}{\frac{100}{68.2}} \times 100\%$$

= 110%

12. 1.6, 1.7, 1.8, 1.8, 1.8, 1.8, 1.9, 1.9, 1.9, 2.0, 2.0

(a) **(i)** Modal height = 1.8 m

(ii) Median height = 1.8 m

(iii) $$\text{Mean height} = \frac{1.6 + 1.7 + 1.8 + 1.8 + 1.8 + 1.8 + 1.9 + 1.9 + 1.9 + 2.0 + 2.0}{11}$$

$$= \frac{20.2}{11}$$

= 1.84 m (to 3 s.f.)

(b) Sum of heights of the first 11 boys = 20.2 m

Sum of heights of the 12 boys $= 12 \times 1.85$
= 22.2 m

∴ Height of the 12^{th} boy = 22.2 – 20.2
= 2.0 m

13. **(a)** $1 + 4 + 8 + x + 9 + y + 2 = 40$

$x + y + 24 = 40$

$x + y = 16$ —(1)

$$\frac{1 \times 0 + 4 \times 1 + 8 \times 2 + x \times 3 + 9 \times 4 + y \times 5 + 2 \times 6}{40} = 3.2$$

$$\frac{3x + 5y + 68}{40} = 3.2$$

$3x + 5y + 68 = 128$

$3x + 5y = 60$ —(2)

$(1) \times 3$: $3x + 3y = 48$ —(3)

$(2) - (3)$: $2y = 12$

$y = 6$

Substitute $y = 6$ into (1):

$x + 6 = 16$

$x = 10$

∴ $x = 10, y = 6$

(b) (i) Largest possible value of $x = 8$

(ii) Mean number of fillings

$$= \frac{1 \times 0 + 4 \times 1 + 8 \times 2 + 8 \times 3 + 9 \times 4 + 8 \times 5 + 2 \times 6}{40}$$

$= 3.3$

14. (i) Total number of pages $= 1 + 3 + 10 + 7 + 4 + 3 + 2$
$= 30$

(ii) Number of pages with fewer than 3 errors
$= 1 + 3 + 10$
$= 14$

Percentage of pages with fewer than 3 errors

$= \frac{14}{30} \times 100\%$

$= 46.7\%$ (to 3 s.f.)

(iii) Mode = 2

(iv) Mean $= \frac{1 \times 0 + 3 \times 1 + 10 \times 2 + 7 \times 3 + 4 \times 4 + 3 \times 5 + 2 \times 6}{30}$

$= 2.9$

15. (i) $p = 7, q = 4, r = 4, s = 3, t = 1$

(ii) Mean $= \frac{7 \times 0 + 6 \times 1 + 4 \times 2 + 5 \times 3 + 4 \times 4 + 3 \times 5 + 1 \times 6}{30}$

$= 2.2$

Median = 2

Mode = 0

(iii) Percentage of students who consume at least 5 servings of fruit and vegetables on a typical weekday

$= \frac{4}{30} \times 100\%$

$= 13.3\%$ (to 3 s.f.)

∴ Most of the students do not consume at least 5 servings of fruit and vegetables.

16. (i) Total number of days $= 3 + 5 + 8 + 7 + 10 + 6 + 1$
$= 40$

(ii) Mean number of security cameras sold

$$= \frac{3 \times 32 + 5 \times 57 + 8 \times 82 + 7 \times 107 + 10 \times 132 + 6 \times 157 + 1 \times 182}{40}$$

$= 105.75$

(iii) Median = 107

Mode = 132

∴ The median gives a better comparison.

17. (i) Modal number of emergency calls received in December = 49

(ii) Median number of emergency calls received in October = 26

Median number of emergency calls received in December = 37

Mean number of emergency calls received in October

$= \frac{4 + 4 + 4 + \ldots + 41 + 44 + 45}{31}$

$= 23.4$ (to 3 s.f.)

Mean number of emergency calls received in December $= \frac{8 + 10 + 13 + \ldots + 49 + 49 + 49}{31}$

$= 34.1$ (to 3 s.f.)

(iii) More emergency calls were received per day in December than in October.

18. (i) Mean mass $\approx \frac{32 \times 20 + 38 \times 35 + 64 \times 45 + 35 \times 55 + 22 \times 65 + 9 \times 85}{200}$

$= 44.85$ kg

(ii) Probability that the steel bar requires another transportation vehicle $= \frac{9}{200}$

19. (a) Mean amount of medical claims

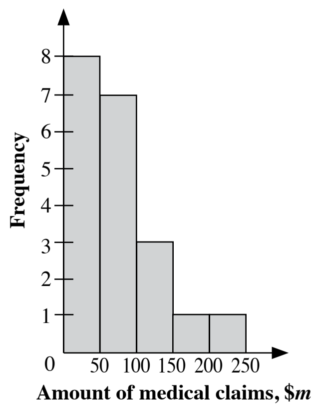
$= \frac{150 + 44 + 225 + \ldots + 77 + 55 + 136}{20}$

$= \$70.40$

(b)

Amount of medical claims, $\$m$	Frequency
$0 \leqslant m < 50$	8
$50 \leqslant m < 100$	7
$100 \leqslant m < 150$	3
$150 \leqslant m < 200$	1
$200 \leqslant m < 250$	1
Total frequency	20

(c) (i)



(ii) Estimate for the mean amount of medical claims

$$= \frac{8 \times 25 + 7 \times 75 + 3 \times 125 + 1 \times 175 + 1 \times 225}{20}$$

$= \$75$

(d) There is a difference of \$4.60 in the answers in **(a)** and **(c)(ii)**. The mean amount calculated in **(a)** is the exact value as it is based on the individual values, but the mean amount calculated in **(c)(ii)** is an estimate as the mid-values of each interval are used.

20. Total number of vehicles along Section $A = 50$

The median average speed along Section A lies in the interval $60 \leqslant v < 70$.

Total number of vehicles along Section $B = 49$

The median average speed along Section B lies in the interval $70 \leqslant v < 80$.

As the actual data in these intervals is not known, it is incorrect for Ethan to obtain the median average speed along Section A by taking $\frac{60 + 70}{2} = 65$ km/h or to obtain the median average speed along Section B by taking $\frac{70 + 80}{2} = 75$ km/h.

Advanced

21. Total mass of the children

$= 15 + 15 + 11 + 13 + 9 + 20 + 15 + a + 13 + 18$

$= 129 + a$

Mean mass of the children $= \frac{129 + a}{10}$

Arrangement of the masses without a:

9, 11, 13, 13, 15, 15, 15, 18, 20

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

X X X X Y Z Z Z Z Z

Case 1: a lies at one of the points labelled X.

Median $= 14$

$\frac{129 + a}{10} = 14 - 0.4$

$129 + a = 136$

$a = 7$

Case 2: a lies at the point labelled Y.

Median $= \frac{a + 15}{2}$

$\frac{129 + a}{10} = \frac{a + 15}{2} - 0.4$

$129 + a = 5a + 75 - 4$

$4a = 58$

$a = 14.5$

Case 3: a lies at one of the points labelled Z.

Median $= 15$

$\frac{129 + a}{10} = 15 - 0.4$

$129 + a = 146$

$a = 17$

$\therefore a = 7$ or $a = 14.5$ or $a = 17$

New Trend

22. Let the numbers be x, y, 60 and 60, such that $x < y$.

Since the median is 56,

$\frac{y + 60}{2} = 56$

$y + 60 = 112$

$y = 52$

Since the mean is 54,

$\frac{x + 52 + 60 + 60}{4} = 54$

$x + 172 = 216$

$x = 44$

$\therefore$ The four numbers are 44, 52, 60 and 60.

23. **(a)** Difference $= 100 - (-210)$

$= 310°C$

(b) Mean boiling points $= \frac{2856 + 100 + (-195.79)}{3}$

$= \frac{2760.21}{3}$

$= 920.07°C$ (to 2 d.p.)

Mean melting points $= \frac{1064.18 + 0 + (-210)}{3}$

$= \frac{854.18}{3}$

$= 284.73°C$ (to 2 d.p.)

Revision Test D1

1. **(a)** $A \cup B' = \{2, 4, 6, 8, 10, 12, 14\}$

(b) $A' \cap B' = \{10, 12\}$

(c) $A \cap B' = \{8, 14\}$

2. **(i)** Most common number of pencils = 7

(ii) $$\text{Mean} = \frac{2 \times 2 + 3 \times 3 + 5 \times 4 + 4 \times 5 + 4 \times 6 + 6 \times 7 + 4 \times 8 + 1 \times 9 + 1 \times 10}{30}$$

$= 5.67$ (to 3 s.f.)

(iii) $$\text{Probability} = \frac{20}{30} = \frac{2}{3}$$

3. **(i)** $$\text{Mean} = \frac{7 + 8 + 12 + \ldots + 74}{40}$$

$= 33$ minutes

(ii) Fraction of students $= \frac{11}{40}$

(iii) No. The median time is $\frac{29 + 32}{2} = 30.5$ minutes.

4. **(i)**

Stem	Leaf
1	0 0 0 0 0 1 1 1 1 1 1 1
1	2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3
1	4 4 4 4 4 5 5 5 5 5 5
1	6 6 6 6 6 6 7 7
1	9

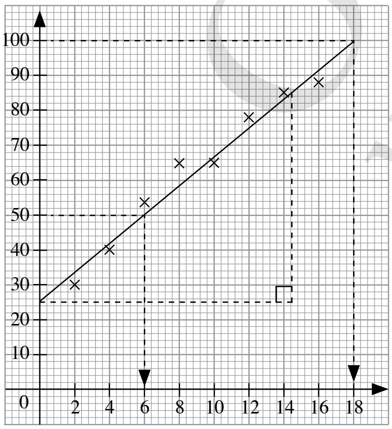
Key: 1 | 0 means 10 points

Note that a frequency table would be a more appropriate statistical diagram as compared to a stem-and-leaf diagram.

(ii) Mode = 13

Median = 13

5. **(a)**



(c) **(i)** 6 hours

(ii) 18 hours

(d) Since 100 marks lies outside of the range, the result obtained in **(c)(ii)** is not reliable.

(e) Take two points on the line and draw dotted lines to form the right-angled triangle.

Vertical change (or rise) $= 85 - 25$
$= 60$

Horizontal change (or run) $= 14.4 - 0$
$= 14.4$

Since the line slopes upwards from the left to the right, its gradient is positive.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{60}{14.4} = 4.17 \text{ (to 3 s.f.)}$$

$\therefore$ The equation of the line of best fit is $y = 4.17x + 25$.

(f) The data displays a strong, positive correlation.

6. Total height of the 9 players $= 9 \times 1.8$
$= 16.2$ m

$$\text{Mean height of 3 reserve players} = \frac{16.2 - 6 \times 1.82}{3}$$

$= 1.76$ m

7. **(a)** $$\text{Mean} = \frac{5 \times 1 + 8 \times 2 + 5 \times 3 + x \times 4 + 4 \times 5 + 2 \times 6}{5 + 8 + 5 + x + 4 + 2}$$

$$3.5 = \frac{68 + 4x}{24 + x}$$

$84 + 3.5x = 68 + 4x$

$0.5x = 16$

$x = 32$

(b) $x = 3, 4, 5, 6, 7, 8, 9, 10, 11$

(c) $x = 7$

(d) $$\text{Median} = \frac{1 + 2}{2} = 1.5$$

(e) We do not know the exact number of books read by the last student in the category '$\geqslant 6$'

8. **(i)** $$\frac{x + 2}{x + 2 + y + 2x + 3y} = \frac{1}{5}$$

$5x + 10 = 3x + 4y + 2$

$2x - 4y = -8$

$x = 2y - 4$ —(1)

$$\frac{2x + 3y}{3x + 4y + 2} = \frac{24}{35}$$

$70x + 105y = 72x + 96y + 48$

$2x - 9y = -48$ —(2)

(ii) Substitute (1) into (2):

$2(2y - 4) - 9y = -48$

$4y - 8 - 9y = -48$

$5y = 40$

$y = 8$

Substitute $y = 8$ into (1):

$x = 2(8) - 4$

$= 12$

$\therefore x = 12, y = 8$

(iii) Probability $= \dfrac{2(12) + 3(8)}{3(12) + 4(8) + 2 + 2}$

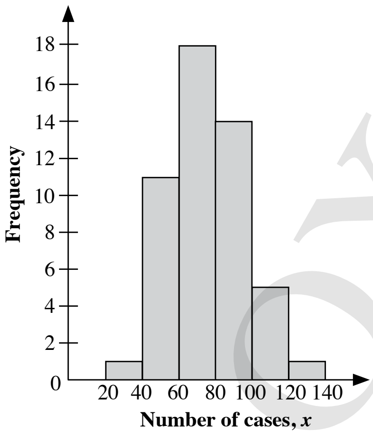
$= \dfrac{48}{72}$

$= \dfrac{2}{3}$

9. (i)

Number of cases, x	Frequency
$20 \leqslant x < 40$	1
$40 \leqslant x < 60$	11
$60 \leqslant x < 80$	18
$80 \leqslant x < 100$	14
$100 \leqslant x < 120$	5
$120 \leqslant x < 140$	1
Total frequency	60

(ii)



(iii) Median $= \dfrac{72 + 74}{2}$

$= 73$

Mode = 48

(iv) Estimate for the mean

$= \dfrac{1 \times 30 + 11 \times 50 + 18 \times 70 + 14 \times 90 + 5 \times 110 + 1 \times 130}{50}$

$= 75.6$

(v) There is a difference of 0.5 in the actual mean and the estimated mean. The mean calculated in **(iv)** is an estimate as the mid-values of each interval were used.

10. (i) Estimate for the mean profit

$= \dfrac{6 \times 2.5 + 11 \times 7.5 + 18 \times 12.5 + 12 \times 17.5 + 3 \times 22.5}{50}$

= \$12 million

(ii) Profit = \$120 000 × 125

= \$15 million

Percentage of number of years $= \dfrac{15}{50} \times 100\%$

$= 30\%$

Revision Test D2

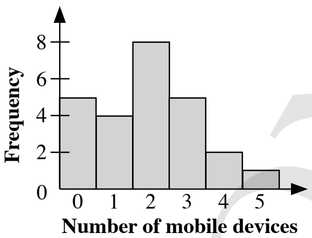
1. **(a)** $A \cap B' = \{8, 10, 12, 14, 16, 20\}$
   (b) $A' \cap B = \{3, 5, 11, 13\}$
   (c) $A' \cap B' = \{0, 1, 7, 9, 15, 17, 19\}$
   (d) $A' \cup B' = \{0, 1, 3, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20\}$

2. **(i)** Probability that is divisible by 2 $= \frac{5}{10}$
   $= \frac{1}{2}$
   (ii) Probability that it is divisible by 5 $= \frac{2}{10}$
   $= \frac{1}{5}$
   (iii) Probability that is divisible by 4 $= \frac{2}{10}$
   $= \frac{1}{5}$

3. **(a)**

Number of mobile devices	Frequency
0	5
1	4
2	8
3	5
4	2
5	1
Total frequency	25

(b)



(c) **(i)** Median number of mobile devices owned = 2
(ii) Modal number of mobile devices owned = 2
(iii) Mean number of mobile devices owned

$$= \frac{5 \times 0 + 4 \times 1 + 8 \times 2 + 5 \times 3 + 2 \times 4 + 1 \times 5}{25}$$

$= 1.92$

(d) **(i)** Probability that he owns 2 mobile devices $= \frac{8}{25}$
(ii) Probability that he owns at least 3 mobile devices

$= \frac{5 + 2 + 1}{25}$

$= \frac{8}{25}$

4. **(i)** Total number of students = 57
   (ii) Mass of lightest school bag = 3.0 kg
   (iii) Most common mass = 6.3 kg
   (iv) Percentage of bags that were considered 'overweight'
   $= \frac{21}{57} \times 100\%$
   $= 36.8\%$ (to 3 s.f.)

5. **(i)**

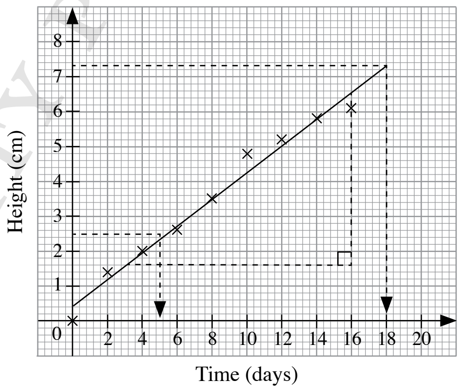
Stem	Leaf
0	4 4 4 4 4 4 4 4 4
0	5 5 5 5 5 5 5
0	6 6 6 6 6 6 6 6 6
0	7 7 7 7 7
0	8 8 8 8 8 8
0	9 9 9 9

Key: 0 | 4 means 4

Note that a frequency table would be a more appropriate statistical diagram as compared to a stem-and-leaf diagram.

(ii) Percentage of patients $= \frac{10}{40} \times 100\%$
$= 25\%$

6. **(a)**



(c) **(i)** 5 days
(ii) 18 days

(d) The result obtained in **(c)(ii)** is not reliable since the height of 7.2 cm lies outside of the range.

(e) Take two points on the line and draw dotted lines to form the right-angled triangle.

Vertical change (or rise) = 6.6 – 1.6
= 5

Horizontal change (or run) = 16 – 3
= 13

Since the line slopes upwards from the left to the right, its gradient is positive.

$\text{Gradient} = \frac{\text{rise}}{\text{run}}$

$= \frac{5}{13}$

= 0.385 (to 3 s.f.)

∴ The equation of the line of best fit is
$y = 0.385x + 0.4$

(f) The data displays a strong, positive correlation.

7. (i) Modal score = 78

(ii) Number of points
= 9 × 79 – (78 + 85 + 64 + 97 + 68 + 78 + 73 + 77)
= 91

8. (a) Modal class is $48 < x \leqslant 52$

(b) Estimate of the mean number of hours worked

$= \frac{8 \times 42 + 11 \times 46 + 14 \times 50 + 9 \times 54}{42}$

= 48.3 h

(c) (i) Probability that he worked more than 52 hours

$= \frac{9}{42}$

$= \frac{3}{14}$

(ii) Probability that he worked not more than 44 hours

$= \frac{8}{42}$

$= \frac{4}{21}$

9. (a) $x = 13$

(b) $x = 21$

(c) Mean number of fish caught

$= \frac{4 \times 1 + 14 \times 2 + 7 \times 3 + 21 \times 4 + 3 \times 5}{4 + 14 + 7 + 21 + 3}$

= 3.10 (to 3 s.f.)

10. (i) $x + 6y - 9 + 2x + 4y - 4 + 3x + 2y + 2 + 5x + 2y + 9 + 9x + 4y - 3 = 26 \times 5$

$20x + 18y = 135$ —(1)

(ii) $8x + 5y + 2 + 5x + 7y + 6x + 4y + 7 + x + 6y - 3 = 39 \times 4$

$20x + 22y = 150$ —(2)

$10x + 11y = 75$

(iii) (2) – (1): $4y = 15$

$y = 3\frac{3}{4}$

Substitute $y = 3\frac{3}{4}$ into (1):

$20x + 18\left(3\frac{3}{4}\right) = 135$

$20x = 67\frac{1}{2}$

$x = 3\frac{3}{8}$

$\therefore x = 3\frac{3}{8}, y = 3\frac{3}{4}$

(iv) $\text{Mean} = \frac{5 \times 26 + 4 \times 39}{9}$

$= 31\frac{7}{9}$

(v) When $x = 3\frac{3}{8}$, $y = 3\frac{3}{4}$,

$x + 6y - 9 = 5\frac{5}{8}$

$2x + 4y - 4 = 17\frac{3}{4}$

$3x + 2y + 2 = 19\frac{5}{8}$

$5x + 2y + 9 = 33\frac{3}{8}$

$9x + 4y - 3 = 42\frac{3}{8}$

$8x + 5y + 2 = 47\frac{3}{4}$

$5x + 7y = 43\frac{1}{8}$

$6x + 4y + 7 = 42\frac{1}{4}$

$x + 6y - 3 = 22\frac{7}{8}$

∴ Probability that the number is greater than 30 = $\frac{5}{9}$

End-of-Year Examination Specimen Paper A

Part I

1. $5x + 3y = 23 \quad —(1)$

$7y - x = -35 \quad —(2)$

From (2),

$x = 7y + 35 \quad —(3)$

Substitute (3) into (1):

$5(7y + 35) + 3y = 23$

$35y + 175 + 3y = 23$

$38y = -152$

$y = -4$

Substitute $y = -4$ into (3):

$x = 7(-4) + 35$

$= 7$

$\therefore x = 7, y = -4$

2. **(i)** $a^2 - b^2 = (a + b)(a - b)$

(ii) $20x = 402^2 - 398^2$

$= (402 + 398)(402 - 398)$

$= (800)(4)$

$= 3200$

$x = 160$

3. $2x^4y - 18x^2y^3$

$= 2x^2y(x^2 - 9y^2)$

$= 2x^2y(x + 3y)(x - 3y)$

4. $\dfrac{6x - 3}{2x + 7} = \dfrac{3x - 2}{x + 5}$

$(6x - 3)(x + 5) = (3x - 2)(2x + 7)$

$6x^2 + 30x - 3x - 15 = 6x^2 + 21x - 4x - 14$

$10x = 1$

$x = \dfrac{1}{10}$

5. **(i)** $y = \dfrac{5x + 3}{x - 5}$

$xy - 5y = 5x + 3$

$xy - 5x = 3 + 5y$

$x(y - 5) = 3 + 5y$

$x = \dfrac{3 + 5y}{y - 5}$

(ii) When $y = 1$,

$x = \dfrac{3 + 5(1)}{1 - 5}$

$= -2$

6. $\dfrac{p + 5q}{2p - q} = \dfrac{9}{5}$

$5p + 25q = 18p - 9q$

$13p = 34q$

$\dfrac{p}{q} = \dfrac{34}{13}$

$\dfrac{p^2}{q^2} = \dfrac{1156}{169}$

$\dfrac{338p^2}{q^2} = 2312$

7. **(i)** $y = k(x + 1)^2$

When $x = 1$,

$y = k(1 + 1)^2$

$= 4k$

When $x = 2$,

$y = k(2 + 1)^2$

$= 9k$

$9k - 4k = 20$

$5k = 20$

$k = 4$

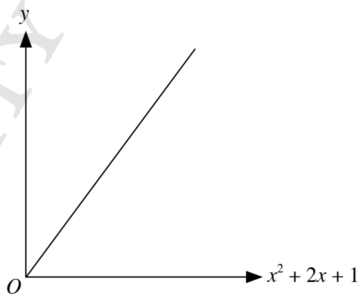
$\therefore y = 4(x + 1)^2$

(ii) When $x = 3$,

$y = 4(3 + 1)^2$

$= 64$

(iii)



8. **(i)** $A(4, 0)$

(ii) $x = \dfrac{0 + 4}{2}$

$= 2$

$\therefore$ Equation of line of symmetry is $x = 2$

(iii) When $x = 2$,

$y = 2(2 - 4)$

$= -4$

$\therefore$ Minimum value of y is -4 when $x = 2$

9. Let the length of the rhombus be x cm.

Using Pythagoras' Theorem,

$x^2 = 5^2 + 12^2$

$= 169$

$x = 13$

$\therefore$ Perimeter $= 4(13)$

$= 52$ cm

10. (i) $k = \frac{9}{6}$

$= 1.5$

(ii) $\frac{BP + 8}{8} = 1.5$

$BP + 8 = 12$

$BP = 4$ cm

(iii) $\frac{AC}{AC + 5} = \frac{6}{9}$

$9AC = 6AC + 30$

$3AC = 30$

$AC = 10$ cm

11. 1 cm represents 0.5 km

16 cm represents 8 km

0.6 km is represented by 1 cm

8 km is represented by $13\frac{1}{3}$ cm

12. Volume of pyramid = $\frac{1}{3}(15w)(18)$

$826 = 90w$

$w = 9.18$ (to 3 s.f.)

13. $\frac{1}{3}\pi(6)^2(3) = \frac{4}{3}\pi r^3$

$r^3 = 27$

$r = 3$

14. (i) Probability that the player does note win anything

$= \frac{9}{20}$

(ii) Probability that the player wins either a key chain or a can of soft drink = $\frac{11}{20}$

(iii) Probability that the player wins a soft toy = 0

15. (i) Estimate for the mean height

$= \frac{8 \times 125 + 13 \times 135 + 12 \times 145 + 7 \times 155}{40}$

$= 139.5$ cm

(ii) Fraction of plants = $1 - \frac{8}{40}$

$= \frac{4}{5}$

16. Let the translation vector T be $\begin{pmatrix} a \\ b \end{pmatrix}$.

$\begin{pmatrix} 7 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$\therefore$ The column vector representing the translation vector T is $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

$\begin{pmatrix} -4 \\ 2 \end{pmatrix} = Q + \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$Q = \begin{pmatrix} -4 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ 1 \end{pmatrix}$

$\therefore$ The coordinates of Q are $(-8, 1)$.

17. The object shown is a regular hexagon.

A regular hexagon has a rotational symmetry of order 6.

Part II

Section A

1. $\frac{\frac{3}{x} - \frac{x}{3}}{\frac{1}{x} - \frac{1}{3}} = \frac{9 - x^2}{3 - x}$

$= \frac{(3 + x)(3 - x)}{3 - x}$

$= 3 + x$

2. (i) $x^2 + y^2 = 2xy + 64$

(ii) $x^2 - 2xy + y^2 = 64$

$(x - y)^2 = 64$

$x - y = \pm 8$

$\therefore$ Difference is 8

3. $c = at^3 + \frac{b}{t^2}$

When $t = 1$, $c = 74$,

$74 = a(1)^3 + \frac{b}{1^2}$

$a + b = 74$ —(1)

When $t = 2$, $c = 34$,

$34 = a(2)^3 + \frac{b}{2^2}$

$8a + \frac{1}{4}b = 34$

$32a + b = 136$ —(2)

(2) – (1): $31a = 62$

$a = 2$

Substitute $a = 2$ into (1):

$2 + b = 74$

$b = 72$

$\therefore c = 2t^3 + \frac{72}{t^2}$

When $t = 3$,

$c = 2(3)^3 + \frac{72}{3^2}$

$= 62$

4. $f(x) = 4x - 6$

$f\left(2\frac{1}{8}\right) = 4\left(2\frac{1}{8}\right) - 6$

$= 2\frac{1}{2}$

$f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right) - 6$

$= -8$

5. **(i)** $\tan \angle ACB = \frac{34}{43}$

$\angle ACB = 38.3°$ (to 1 d.p.)

(ii) Using Pythagoras' Theorem,

$AC^2 = 34^2 + 43^2$

$= 3005$

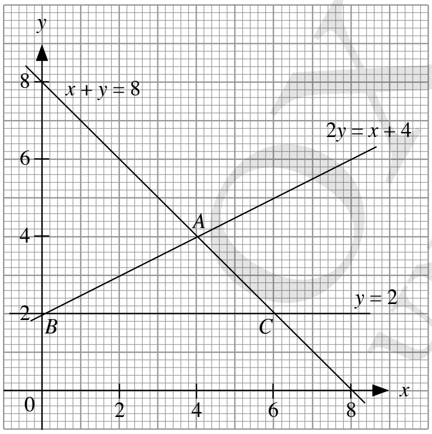
$AC = \sqrt{3005}$ cm

$AS = (\sqrt{3005} - \sqrt{250})$ cm

$\therefore$ Area of $APQS = (\sqrt{3005} - \sqrt{250})^2$

$= 1520$ cm^2 (to 3 s.f.)

6. **(a)**



(b) $A(4, 4)$, $B(0, 2)$

(c) $C(6, 2)$

(d) Area of $\triangle ABC = \frac{1}{2}(6)(2)$

$= 6$ units^2

Section B

7. **(a)** $n = 3$

(b) **(i)** $F = \frac{k}{\sqrt[3]{R}}$

When $R = 125$, $F = 4$,

$4 = \frac{k}{\sqrt[3]{125}}$

$= \frac{k}{5}$

$k = 20$

$\therefore F = \frac{20}{\sqrt[3]{R}}$

(ii) When $R = 512$,

$F = \frac{20}{\sqrt[3]{512}}$

$= 2.5$

8. **(a)** $\triangle QLM$ and $\triangle MRQ$

(b) **(i)** Using Pythagoras' Theorem,

$RS^2 = 7^2 + 5^2$

$= 74$

$RS = \sqrt{74}$

$= 8.60$ cm (to 3 s.f.)

(ii) $\sin 50° = \frac{5}{PQ}$

$PQ = \frac{5}{\sin 50°}$

$= 6.53$ cm (to 3 s.f.)

(iii) $\tan 50° = \frac{5}{PL}$

$PL = \frac{5}{\tan 50°}$

$\therefore PS = \frac{5}{\tan 50°} + 4 + 7$

$= 15.2$ cm (to 3 s.f.)

(iv) $\tan \angle MSR = \frac{5}{7}$

$\angle MSR = 35.5°$ (to 1 d.p.)

(v) Area of $PQRS = \frac{1}{2}\left(\frac{5}{\tan 50°} + 4 + 7 + 4\right)(5)$

$= 48.0$ cm^2 (to 3 s.f.)

9. **(i)**

Stem	Leaf
2	5 8
3	6 9
4	1 1 1 2 3 4 6 6 9
5	
6	1 3
7	2 3 5 8
8	9

Key: 2 | 5 means 25

(ii) Modal number of points = 41

(iii) Median number of points $= \dfrac{44 + 46}{2}$

$= 45$

(iv) Mean number of points $= \dfrac{25 + 28 + \ldots + 89}{20}$

$= 51.6$

(v) Fraction of clubs $= \dfrac{5}{20}$

$= \dfrac{1}{4}$

(vi) Percentage of clubs $= \dfrac{2}{20} \times 100\%$

$= 10\%$

10. (a) When $x = -1$, $y = a$,

$a = (-1 + 2)(-1 - 4)$

$= -5$

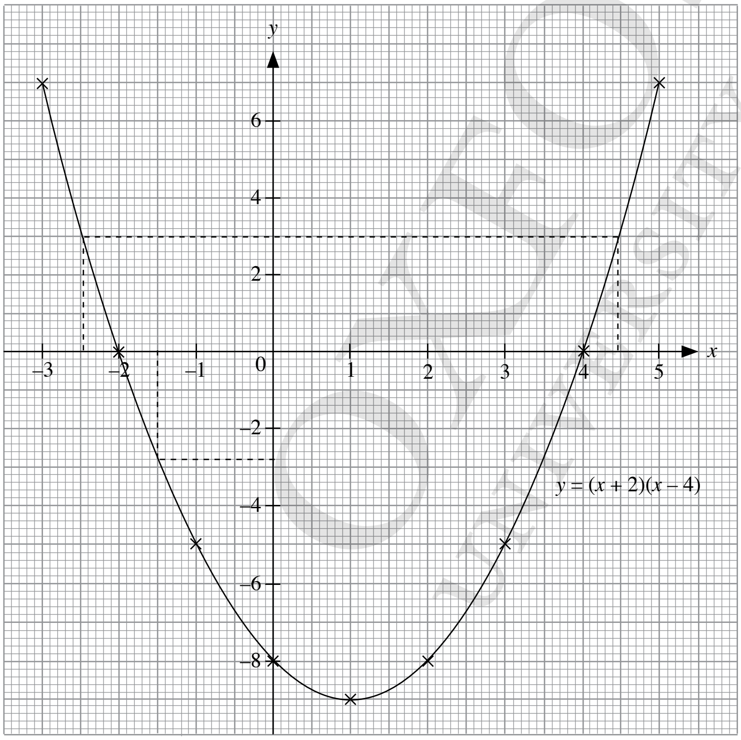
When $x = 3$, $y = b$,

$b = (3 + 2)(3 - 4)$

$= -5$

$\therefore a = -5, b = -5$

(b)



(c) (i) When $x = 1\frac{1}{2}$, $y = -2.8$

(ii) Least value of $y = -9$

(iii) When $y = 3$, $x = 4.45$ or $x = -2.45$

End-of-Year Examination Specimen Paper B

Part I

1. $y^2 - x^2 = (y + x)(y - x)$
$= (-4)(-8)$
$= 32$

2. **(a)** $(x + 7)(x - 3) - 5(x - 3)$
$= x^2 - 3x + 7x - 21 - 5x + 15$
$= x^2 - x - 6$

(b) $4y^2 - 3(y - 2)(y + 3) - 7$
$= 4y^2 - 3(y^2 + 3y - 2y - 6) - 7$
$= 4y^2 - 3y^2 - 3y + 18 - 7$
$= y^2 - 3y + 11$

3. **(a)** $18x^2 - 3x - 6$
$= 3(6x^2 - x - 2)$
$= 3(2x + 1)(3x - 2)$

(b) $2x^2 - xy - 15y^2$
$= (x - 3y)(2x + 5y)$

4. $$6x - 4 = \frac{2}{x}$$
$$6x^2 - 4x = 2$$
$$6x^2 - 4x - 2 = 0$$
$$3x^2 - 2x - 1 = 0$$
$$(x - 1)(3x + 1) = 0$$
$$x = 1 \quad \text{or} \quad x = -\frac{1}{3}$$

5. **(a)** $$\frac{2}{3x} - \frac{x - 3}{10x^2}$$
$$= \frac{20x - 3(x - 3)}{30x^2}$$
$$= \frac{20x - 3x + 9}{30x^2}$$
$$= \frac{17x + 9}{30x^2}$$

(b) $$\frac{2}{3x - y} - \frac{5}{2y - 6x}$$
$$= \frac{2}{3x - y} + \frac{5}{6x - 2y}$$
$$= \frac{4 + 5}{6x - 2y}$$
$$= \frac{9}{6x - 2y}$$

6. $x(y + 2) - 3(y + 2) = 0$
$(x - 3)(y + 2) = 0$
$x = 3 \quad \text{or} \quad y = -2$

7. $$x = \frac{3y^2 - 1}{(2y + 1)(2y - 1)}$$
$$x(4y^2 - 1) = 3y^2 - 1$$
$$4xy^2 - x = 3y^2 - 1$$
$$4xy^2 - 3y^2 = x - 1$$
$$y^2(4x - 3) = x - 1$$
$$y^2 = \frac{x - 1}{4x - 3}$$
$$y = \pm\sqrt{\frac{x - 1}{4x - 3}}$$

8. $x - \frac{y}{3} = 4\frac{1}{3}$ —(1)
$0.5x - 0.25y = 2$ —(2)
$(1) \times 3$: $3x - y = 13$ —(3)
$(2) \times 4$: $2x - y = 8$ —(4)
$(3) - (4)$: $x = 5$
Substitute $x = 5$ into (4):
$2(5) - y = 8$
$10 - y = 8$
$y = 2$
$\therefore x = 5, y = 2$

9. $$\frac{2x + 3}{2x + 3 + x} = \frac{5}{7}$$
$$14x + 21 = 15x + 15$$
$$x = 6$$
$\therefore$ There are 6 \$50-vouchers.

10. $f(x) = 13 - 4x$
$f(-2) = 13 - 4(-2)$
$= 21$

11. $$\frac{DE}{8} = \frac{6 + 8}{6}$$
$$DE = \frac{14}{6} \times 8$$
$$= 18\frac{2}{3} \text{ cm}$$

12. **(i)** $\frac{3}{5}$ h = 36 minutes

Percentage of students $= \frac{9}{36} \times 100\%$
$= 25\%$

(ii) Modal time taken = 29 minutes

Angle $= \frac{4}{36} \times 360°$
$= 40°$

13. 2.0, 2.5, 2.5, 3.0, 3.5, 3.5, 3.5, 4.0, 4.0, 4.0, 4.5, 4.5, 4.5, 4.5, 4.5

(i) Mode = 4.5 kg

(ii) Median = 4.0 kg

(iii) Mean

$$= \frac{2.0 + 2(2.5) + 3.0 + 3(3.5) + 3(4.0) + 5(4.5)}{15}$$

= 3.67 kg (to 3 s.f.)

14. **(i)** 0.25 km is represented by 1 cm

4.5 km is represented by 18 cm

(ii) 1 cm^2 represents 0.0625 km^2

40 cm^2 represents 2.5 km^2

(iii) 0.5 km is represented by 1 cm

0.25 km^2 is represented by 1 cm^2

2.5 km^2 is represented by 10 cm^2

15. Using Pythagoras' Theorem,

$PQ^2 = 25^2 + 34^2$

$= 1781$

$PQ = 42.2$ m (to 3 s.f.)

16. Estimate for the mean lifespan

$$= \frac{45 \times 10 + 28 \times 30 + 19 \times 50 + 6 \times 70 + 2 \times 90}{100}$$

= 28.4 days

17. **(i)** $(A \cap B)' = \{o, p, q, r, s, u, v\}$

(ii) $A \cup C' = \{o, p, q, r, s, t, u\}$

Part II

Section A

1. Using Pythagoras' Theorem,

$h^2 + 2.4^2 = 8.5^2$

$h^2 = 8.5^2 - 2.4^2$

$= 66.49$

$h = \sqrt{66.49}$

$= 8.15$ m (to 3 s.f.)

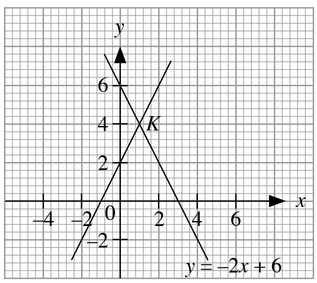
$\therefore$ The ladder reaches 8.15 m up the wall.

2. **(a)** **(i)** Gradient $= \frac{4}{2}$

$= 2$

(ii) $y = 2x + 2$

(b)



(c) $K(1, 4)$

3. **(i)** $h = kt^2$

When $t = 5$, $h = 200$,

$200 = k(5)^2$

$= 25k$

$k = 8$

$\therefore h = 8t^2$

When $t = 7$,

$h = 8(7)^2$

$= 392$

$\therefore$ It falls 392 m in 7 seconds.

(ii) When $h = 1250$,

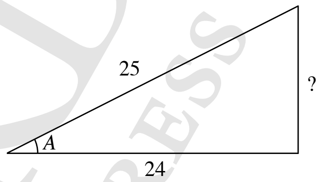
$1250 = 8t^2$

$t^2 = 156.25$

$t = \pm 12.5$

$\therefore$ It takes 12.5 seconds.

4.



(a) $2 \cos A = 2\left(\frac{24}{25}\right)$

$= 1\frac{23}{25}$

(b) $\tan (90° - A) = \frac{24}{7}$

$= 3\frac{3}{7}$

(c) $2 \cos (90° - A) + 4 \tan A = 2\left(\frac{7}{25}\right) + 4\left(\frac{7}{24}\right)$

$= 1\frac{109}{150}$

5. **(i)** Total volume $= \frac{1}{3}\pi(7)^2(18) + \frac{2}{3}\pi(7)^3$

$= 522\frac{2}{3}\pi \text{ cm}^3$

(ii) Area to be painted pink

$= 2\pi(7)^2$

$= 98\pi \text{ cm}^2$

Area to be painted brown

$= \pi(7)(\sqrt{18^2 + 7^2})$

$= 7\sqrt{373}\,\pi \text{ cm}^2$

Area to be painted pink : Area to be painted brown

$= 98\pi : 7\sqrt{373}\,\pi$

$= 1 : 1.4$ (to 1 d.p.)

$\therefore n = 1.4$

Section B

6. **(i)** $C = a + \frac{b}{n}$

When $n = 300$, $C = 8.5$,

$$8.5 = a + \frac{b}{300}$$

$300a + b = 2550$ —(1)

When $n = 700$, $C = 4.5$,

$$4.5 = a + \frac{b}{700}$$

$700a + b = 3150$ —(2)

(ii) (2) – (1): $400a = 600$

$$a = 1\frac{1}{2}$$

Substitute $a = 1\frac{1}{2}$ into (1):

$$300\left(1\frac{1}{2}\right) + b = 2550$$

$$450 + b = 2550$$

$$b = 2100$$

$\therefore a = 1\frac{1}{2}$, $b = 2100$

(iii) $C = 1\frac{1}{2} + \frac{2100}{n}$

When $n = 200$,

$$C = 1\frac{1}{2} + \frac{2100}{200}$$

$$= 12$$

$\therefore$ The cost of each book is \$12.

(iv) When $C = 5.7$,

$$5.7 = 1\frac{1}{2} + \frac{2100}{n}$$

$$\frac{2100}{n} = 4.2$$

$$n = 500$$

$\therefore$ 500 copies are printed.

7. **(a)** **(i)** Mode = 2

(ii) Median = 3

(iii) Mean = $\frac{7 \times 1 + 9 \times 2 + 6 \times 3 + 4 \times 4 + 5 \times 5 + 8 \times 6}{39}$

$= \frac{132}{39}$

$= 3.38$ (to 3 s.f.)

(b) Number shown = $40 \times 3.45 - 132$

$= 6$

(c) Number shown = 6

8. **(i)** Total surface area

$= (10)(10) + 4 \times \frac{1}{2}(10)(\sqrt{13^2 - 5^2})$

$= 340$ cm^2

(ii) Let the height of the pyramid be h cm.

Using Pythagoras' Theorem,

$h^2 + 5^2 = 12^2$

$h^2 = 12^2 - 5^2$

$= 119$

$h = \sqrt{119}$ cm

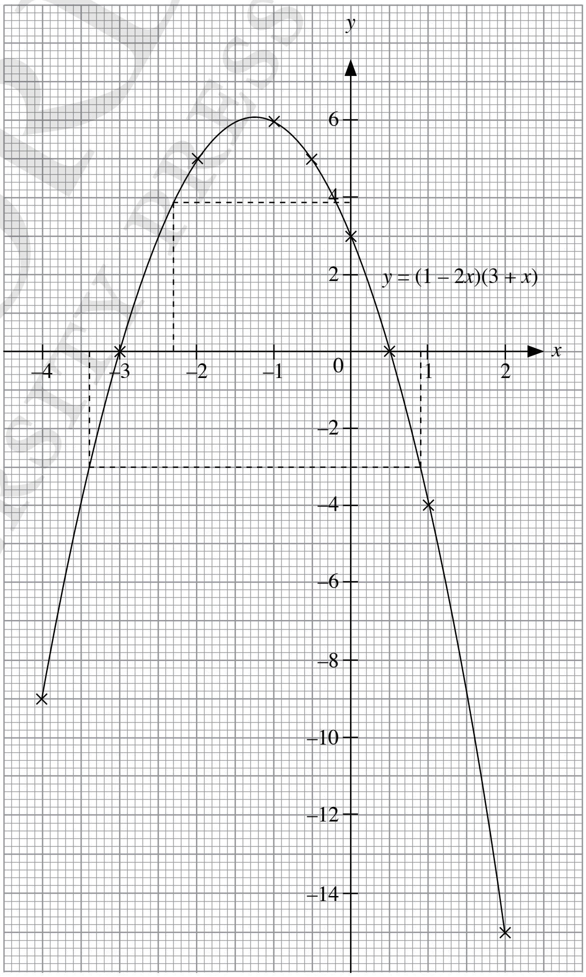
$\therefore$ Volume = $\frac{1}{3}(10)(10)(\sqrt{119})$

$= 364$ cm^3 (to 3 s.f.)

9. **(a)**

x	–4	–3	–2	–1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y	–9	0	5	6	5	3	0	–4	–15

(b)



(c) **(i)** Greatest value of $y = 6.1$

(ii) When $x = -2.3$, $y = 3.9$

(iii) When $y = -3$, $x = 0.9$ or $x = -3.4$

NOTES