7th EDITION

NEW SYLLABUS MATHEMATICS WORKBOOK FULL SOLUTIONS

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ANSWERS

Chapter 1 Primes, Highest Common Factor and Lowest Common Multiple

Basic

1. **(a)** 101 is an odd number, so it is not divisible by 2. Since the sum of the digits $1 + 0 + 1 = 2$ is not divisible by 3 (divisibility test for 3), then 101 is not divisible by 3. The last digit of 101 is neither 0 nor 5, so 101 is not divisible by 5.
A calculator may be used to test whether 101 is divisible by prime numbers more than 5. Since 101 is not divisible by any prime numbers less than 101, 101 is a prime number.

(b) 357 is an odd number, so it is not divisible by 2. Since the sum of the digits $3 + 5 + 7 = 15$ which is divisible by 3, therefore 357 is divisible by 3 (divisibility test for 3).
$\therefore$ 357 is a composite number.

(c) 411 is an odd number, so it is not divisible by 2. Since the sum of the digits $4 + 1 + 1 = 6$ which is divisible by 3, therefore 411 is divisible by 3 (divisibility test for 3).
$\therefore$ 411 is a composite number.

(d) 1223 is an odd number, so it is not divisible by 2. Since the sum of the digits $1 + 2 + 2 + 3 = 8$ which is not divisible by 3, then 1223 is not divisible by 3. The last digit of 1223 is neither 0 nor 5, so 1223 is not divisible by 5.
A calculator may be used to test whether 1223 is divisible by prime numbers more than 5. Since 1223 is not divisible by any prime numbers less than 1223, 1223 is a prime number.

(e) 1555 is an odd number, so it is not divisible by 2. Since the sum of the digits $1 + 5 + 5 + 5 = 16$ which is not divisible by 3, so 1555 is not divisible by 3. The last digit of 1555 is 5, so 1555 is divisible by 5.
$\therefore$ 1555 is a composite number.

(f) 3127 is an odd number, so it is not divisible by 2. Since the sum of the digits $3 + 1 + 2 + 7 = 13$, then 3127 is not divisible by 3. A calculator may be used to test whether 3127 is divisible by prime numbers more than 3 and 3127 is divisible by 53, which is a prime number.
$\therefore$ 3127 is a composite number.

2. The prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.
Sum of prime numbers less than 30
$= 2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 + 29$
$= 129$

3. The two prime numbers between 20 and 30 are 23 and 29.
Difference of the two prime numbers $= 29 - 23 = 6$.

4. **(a)** Divide 315 by the smallest prime factor and continue the process until we obtain 1.

3	315
3	105
5	35
7	7
	1

$315 = 3 \times 3 \times 5 \times 7 = 3^2 \times 5 \times 7$

(b)

2	8008
2	4004
2	2002
7	1001
11	143
13	13
	1

$8008 = 2 \times 2 \times 2 \times 7 \times 11 \times 13$
$= 2^3 \times 7 \times 11 \times 13$

(c)

2	61 200
2	30 600
2	15 300
2	7650
3	3825
3	1275
5	425
5	85
17	17
	1

$61\ 200 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 17$
$= 2^4 \times 3^2 \times 5^2 \times 17$

(d)

2	58 752
2	29 376
2	14 688
2	7344
2	3672
2	1836
2	918
3	459
3	153
3	51
17	17
	1

$58\ 752 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 17$
$= 2^7 \times 3^3 \times 17$

(e)

2	117 800
2	58 900
2	29 450
5	14 725
5	2945
19	589
31	31
	1

$117\ 800 = 2 \times 2 \times 2 \times 5 \times 5 \times 19 \times 31$
$= 2^3 \times 5^2 \times 19 \times 31$

5. **(a)** $2025 = 3 \times 3 \times 3 \times 3 \times 5 \times 5$
$= (3 \times 3 \times 5) \times (3 \times 3 \times 5)$
$= (3 \times 3 \times 5)^2$
$\therefore \sqrt{2025} = 3 \times 3 \times 5 = 45$

(b) $2304 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$= (2 \times 2 \times 2 \times 2 \times 3) \times (2 \times 2 \times 2 \times 2 \times 3)$
$= (2 \times 2 \times 2 \times 2 \times 3)^2$
$\therefore \sqrt{2304} = 2 \times 2 \times 2 \times 2 \times 3 = 48$

(c) $3969 = 3 \times 3 \times 3 \times 3 \times 7 \times 7$
$= (3 \times 3 \times 7) \times (3 \times 3 \times 7)$
$= (3 \times 3 \times 7)^2$
$\therefore \sqrt{3969} = 3 \times 3 \times 7 = 63$

(d) $7056 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7$
$= 2^4 \times 3^2 \times 7^2 = (2^2 \times 3 \times 7)^2$
$\therefore \sqrt{7056} = 2^2 \times 3 \times 7 = 84$

(e) $5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 2^3 \times 3^6 = (2 \times 3^2)^3$
$\sqrt[3]{5832} = 2 \times 3^2 = 18$

(f) $9261 = 3 \times 3 \times 3 \times 7 \times 7 \times 7$
$= 3^3 \times 7^3 = (3 \times 7)^3$
$\sqrt[3]{9261} = 3 \times 7 = 21$

(g) $17\ 576 = 2 \times 2 \times 2 \times 13 \times 13 \times 13$
$= (2 \times 13) \times (2 \times 13) \times (2 \times 13)$
$= (2 \times 13)^3$
$\sqrt[3]{17\ 576} = 2 \times 13 = 26$

(h) $39\ 304 = 2 \times 2 \times 2 \times 17 \times 17 \times 17$
$= (2 \times 17) \times (2 \times 17) \times (2 \times 17)$
$= (2 \times 17)^3$
$\sqrt[3]{39\ 304} = 2 \times 17 = 34$

6. $3136 = 2^6 \times 7^2$
$\therefore \sqrt{3136} = \sqrt{2^6 \times 7^2}$
$= 2^3 \times 7$
$= 56$

7. $59\ 319 = 3^3 \times 13^3$
$\therefore \sqrt[3]{59\ 319} = \sqrt[3]{3^3 \times 13^3}$
$= 3 \times 13$
$= 39$

8. **(a)** We observe that 48 is close to 49 which is a perfect square. Thus $\sqrt{48} \approx \sqrt{49} = 7$.

(b) We observe that 626 is close to 625 which is a perfect square. Thus $\sqrt{626} \approx \sqrt{625} = 25$.

(c) 65 is close to 64 which is a perfect cube. Thus $\sqrt[3]{65} \approx \sqrt[3]{64} = 4$.

(d) 998 is close to 1000 which is a perfect cube. Thus $\sqrt[3]{998} \approx \sqrt[3]{1000} = 10$.

(e) We observe that 99 is close to 100 which is a perfect square and 28 is close to 27 which is a perfect cube. Thus $\sqrt{99} - \sqrt[3]{28} \approx \sqrt{100} - \sqrt[3]{27} = 7$.

(f) We observe that 19 is close to 20 and 10 004 is close to 10 000 which is a perfect square. Thus $19^2 \times \sqrt{10\ 004} \approx 20^2 \times \sqrt{10\ 000} = 400 \times 100$
$= 40\ 000$.

(g) We observe that 11 is close to 10 and 7999 is close to 8000 which is a perfect cube. Thus $11^3 + \sqrt[3]{7999} \approx 10^3 + \sqrt[3]{8000} = 1000 + 20 = 1020$.

9. **(a)** $69^3 + 126^2 - \sqrt{71\ 289} \times \sqrt[3]{912\ 673} = 318\ 486$

(b) $\dfrac{\sqrt[3]{12\ 167} \times 57^2 - 56^3}{\sqrt{153\ 664}} = -257.3699$ (to 4 d.p.)

(c) $\dfrac{\left(\sqrt{576} + \sqrt{961} - \sqrt[3]{12\ 167}\right)}{\sqrt[3]{4096}} = 2$

(d) $\dfrac{18^3}{\sqrt{5184}} + \dfrac{16^2 - \sqrt[3]{75\ 357}}{22^3 - 103^2 - \sqrt[3]{753\ 571}}$
$= 76.8892$ (to 4 d.p.)

10. **(a)**
$16 = \boxed{2} \times \boxed{2} \times \boxed{2} \times 2$
$24 = \boxed{2} \times \boxed{2} \times \boxed{2} \times 3$
$\downarrow$ 2 2 2

HCF of 16 and 24 = $2 \times 2 \times 2 = 8$

(b)
$45 = \boxed{3} \times \boxed{3} \times 5$
$63 = \boxed{3} \times \boxed{3} \times 7$
$\downarrow$ 3 3

HCF of 45 and 63 = $3 \times 3 = 9$

(c)
$56 = \boxed{2} \times 2 \times 2 \times \boxed{7}$
$70 = \boxed{2} \times 5 \times \boxed{7}$
$\downarrow$ 2 7

HCF of 56 and 70 = $2 \times 7 = 14$

(d)
$90 = \boxed{2} \times \boxed{3} \times \boxed{3} \times 5$
$126 = \boxed{2} \times \boxed{3} \times \boxed{3} \times 7$
$\downarrow$ 2 3 3

HCF of 90 and 126 = $2 \times 3 \times 3 = 18$

(e)
$1008 = \boxed{2} \times \boxed{2} \times \boxed{2} \times 2 \times 3 \times 3 \times \boxed{7}$
$1960 = \boxed{2} \times \boxed{2} \times \boxed{2} \times 5 \times \boxed{7} \times 7$
$\downarrow$ 2 2 2 7

HCF of 1008 and 1960 = $2 \times 2 \times 2 \times 7 = 56$

(f)
$1080 = \boxed{2} \times \boxed{2} \times \boxed{2} \times \boxed{3} \times \boxed{3} \times 3 \times 5$
$1584 = \boxed{2} \times \boxed{2} \times \boxed{2} \times 2 \times \boxed{3} \times \boxed{3} \times 11$
$\downarrow$ 2 2 2 3 3

HCF of 1080 and 1584 = $2 \times 2 \times 2 \times 3 \times 3 = 72$

(g)
$42 = \boxed{2} \times \boxed{3} \times 7$
$66 = \boxed{2} \times \boxed{3} \times 11$
$78 = \boxed{2} \times \boxed{3} \times 13$
$\downarrow$ 2 3

HCF of 42, 66 and 78 = $2 \times 3 = 6$

(h)
$132 = \boxed{2} \times \boxed{2} \times \boxed{3} \times 11$
$156 = \boxed{2} \times \boxed{2} \times \boxed{3} \times 13$
$180 = \boxed{2} \times \boxed{2} \times \boxed{3} \times 3 \times 5$
$\downarrow$ 2 2 3

HCF of 132, 156 and 180 = $2 \times 2 \times 3 = 12$

(i)
$84 = \boxed{2} \times 2 \times 3 \times \boxed{7}$
$98 = \boxed{2} \times \boxed{7} \times 7$
$112 = \boxed{2} \times 2 \times 2 \times 2 \times \boxed{7}$
$\downarrow$ 2 7

HCF of 84, 98 and 112 = $2 \times 7 = 14$

(j)
$195 = \boxed{3} \times \boxed{5} \times 13$
$270 = 2 \times \boxed{3} \times 3 \times 3 \times \boxed{5}$
$345 = \boxed{3} \times \boxed{5} \times 23$
$\downarrow$ 3 5

HCF of 195, 270 and 345 = $3 \times 5 = 15$

(k)
$147 = \boxed{3} \times \boxed{7} \times 7$
$231 = \boxed{3} \times \boxed{7} \times 11$
$273 = \boxed{3} \times \boxed{7} \times 13$
$\downarrow$ 3 7

HCF of 147, 231 and 273 = $3 \times 7 = 21$

(l)
$225 = \boxed{3} \times \boxed{3} \times \boxed{5} \times 5$
$495 = \boxed{3} \times \boxed{3} \times \boxed{5} \times 11$
$810 = 2 \times \boxed{3} \times \boxed{3} \times 3 \times \boxed{3} \times 5$
$\downarrow$ 3 3 5

HCF of 225, 495 and 810 = $3 \times 3 \times 5 = 45$

11. **(a)**
$48 = \boxed{2} \times \boxed{2} \times \boxed{2} \times 2 \times \boxed{3}$
$72 = \boxed{2} \times \boxed{2} \times \boxed{2} \times \boxed{3} \times 3$
$\downarrow$ 2 2 2 2 3 3

LCM of 48 and 72 = $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$

(b)
$75 = \boxed{3} \times \boxed{5} \times 5$
$105 = \boxed{3} \times \boxed{5} \times 7$
$\downarrow$ 3 5 5 7

LCM of 75 and 105 = $3 \times 5 \times 5 \times 7 = 525$

(c)
$243 = \boxed{3} \times \boxed{3} \times \boxed{3} \times \boxed{3} \times 3$
$405 = \boxed{3} \times \boxed{3} \times \boxed{3} \times \boxed{3} \times 5$
$\downarrow$ 3 3 3 3 3 5

LCM of 243 and 405 = $3 \times 3 \times 3 \times 3 \times 3 \times 5$
$= 1215$

(d) 261 = 3 × 3 × 29
435 = 3 × 5 × 29
3 3 5 29

LCM of 261 and 435 = 3 × 3 × 5 × 29 = 1305

(e) 144 = 2 × 2 × 2 × 2 × 3 × 3
306 = 2 × 3 × 3 × 17
2 2 2 2 3 3 17

LCM of 144 and 306 = 2 × 2 × 2 × 2 × 3 × 3 × 17
= 2448

(f) 264 = 2 × 2 × 2 × 3 × 11
504 = 2 × 2 × 2 × 3 × 3 × 7
2 2 2 3 3 7 11

LCM of 264 and 504 = 2 × 2 × 2 × 3 × 3 × 7 × 11
= 5544

(g) 1176 = 2 × 2 × 2 × 3 × 7 × 7
1960 = 2 × 2 × 2 × 5 × 7 × 7
2 2 2 3 5 7 7

LCM of 1176 and 1960 = 2 × 2 × 2 × 3 × 5 × 7 × 7
= 5880

(h) 56 = 2 × 2 × 2 × 7
72 = 2 × 2 × 2 × 3 × 3
104 = 2 × 2 × 2 × 13
2 2 2 3 3 7 13

LCM of 56, 72 and 104
= 2 × 2 × 2 × 3 × 3 × 7 × 13 = 6552

(i) 324 = 2 × 2 × 3 × 3 × 3 × 3
756 = 2 × 2 × 3 × 3 × 3 × 7
972 = 2 × 2 × 3 × 3 × 3 × 3 × 3
2 2 3 3 3 3 3 7

LCM of 324, 756 and 972
= 2 × 2 × 3 × 3 × 3 × 3 × 3 × 7
= 6804

(j) 450 = 2 × 3 × 3 × 5 × 5
720 = 2 × 2 × 2 × 2 × 3 × 3 × 5
1170 = 2 × 3 × 3 × 5 × 13
2 2 2 2 3 3 5 5 13

LCM of 450, 720 and 1170
= 2 × 2 × 2 × 2 × 3 × 3 × 5 × 5 × 13
= 46 800

12. (a) 84 = 2 × 2 × 3 × 7
189 = 3 × 3 × 3 × 7
3 7

HCF of 84 and 189 = 3 × 7 = 21

84 = 2 × 2 × 3 × 7
189 = 3 × 3 × 3 × 7
2 2 3 3 3 7

LCM of 84 and 189 = 2 × 2 × 3 × 3 × 3 × 7
= 756

(b) 315 = 3 × 3 × 5 × 7
720 = 2 × 2 × 2 × 2 × 3 × 3 × 5
3 3 5

HCF of 315 and 720 = 3 × 3 × 5 = 45

315 = 3 × 3 × 5 × 7
720 = 2 × 2 × 2 × 2 × 3 × 3 × 5
2 2 2 2 3 3 5 7

LCM of 315 and 720
= 2 × 2 × 2 × 2 × 3 × 3 × 5 × 7 = 5040

(c) 392 = 2 × 2 × 2 × 7 × 7
616 = 2 × 2 × 2 × 7 × 11
2 2 2 7

HCF of 392 and 616 = 2 × 2 × 2 × 7 = 56

392 = 2 × 2 × 2 × 7 × 7
616 = 2 × 2 × 2 × 7 × 11
2 2 2 7 7 11

LCM of 392 and 616 = 2 × 2 × 2 × 7 × 7 × 11
= 4312

(d) 1008 = 2 × 2 × 2 × 2 × 3 × 3 × 7
1764 = 2 × 2 × 3 × 3 × 7 × 7
2 2 3 3 7

HCF of 1008 and 1764 = 2 × 2 × 3 × 3 × 7
= 252

1008 = 2 × 2 × 2 × 2 × 3 × 3 × 7
1764 = 2 × 2 × 3 × 3 × 7 × 7
2 2 2 2 3 3 7 7

LCM of 1008 and 1764
= 2 × 2 × 2 × 2 × 3 × 3 × 7 × 7
= 7056

(e)

$140 = 2 \times 2 \times 5 \times 7$

$224 = 2 \times 2 \times 2 \times 2 \times 2 \times 7$

$560 = 2 \times 2 \times 2 \times 2 \times 5 \times 7$

$2 \quad 2 \quad 7$

HCF of 140, 224 and 560 = $2 \times 2 \times 7 = 28$

$140 = 2 \times 2 \times 5 \times 7$

$224 = 2 \times 2 \times 2 \times 2 \times 2 \times 7$

$560 = 2 \times 2 \times 2 \times 2 \times 5 \times 7$

$2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 5 \quad 7$

LCM of 140, 224 and 560

$= 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 7 = 1120$

(f)

$315 = 3 \times 3 \times 5 \times 7$

$525 = 3 \times 5 \times 5 \times 7$

$1400 = 2 \times 2 \times 2 \times 5 \times 5 \times 7$

$5 \quad 7$

HCF of 315, 525 and 1400 = $5 \times 7 = 35$

$315 = 3 \times 3 \times 5 \times 7$

$525 = 3 \times 5 \times 5 \times 7$

$1400 = 2 \times 2 \times 2 \times 5 \times 5 \times 7$

$2 \quad 2 \quad 2 \quad 3 \quad 3 \quad 5 \quad 5 \quad 7$

LCM of 315, 525 and 1400

$= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7$

$= 12\ 600$

(g)

$252 = 2 \times 2 \times 3 \times 3 \times 7$

$378 = 2 \times 3 \times 3 \times 3 \times 7$

$567 = 3 \times 3 \times 3 \times 3 \times 7$

$3 \quad 3 \quad 7$

HCF of 252, 378 and 567 = $3 \times 3 \times 7 = 63$

$252 = 2 \times 2 \times 3 \times 3 \times 7$

$378 = 2 \times 3 \times 3 \times 3 \times 7$

$567 = 3 \times 3 \times 3 \times 3 \times 7$

$2 \quad 2 \quad 3 \quad 3 \quad 3 \quad 3 \quad 7$

LCM of 252, 378 and 567

$= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 7 = 2268$

(h)

$330 = 2 \times 3 \times 5 \times 11$

$792 = 2 \times 2 \times 2 \times 3 \times 3 \times 11$

$1188 = 2 \times 2 \times 3 \times 3 \times 3 \times 11$

$2 \quad 3 \quad 11$

HCF of 330, 792 and 1188 = $2 \times 3 \times 11 = 66$

$330 = 2 \times 3 \times 5 \times 11$

$792 = 2 \times 2 \times 2 \times 3 \times 3 \times 11$

$1188 = 2 \times 2 \times 3 \times 3 \times 3 \times 11$

$2 \quad 2 \quad 2 \quad 3 \quad 3 \quad 3 \quad 5 \quad 11$

LCM of 330, 792 and 1188

$= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 11$

$= 11\ 880$

13. (a)

$2^2 \times 3^2 \times 11$

$2^4 \times 3 \times 7$

$2^2 \quad 3$

HCF = $2^2 \times 3$

$2^2 \times 3^2 \times 11$

$2^4 \times 3 \times 7$

$2^4 \quad 3^2 \quad 7 \quad 11$

LCM = $2^4 \times 3^2 \times 7 \times 11$

(b)

$3^4 \times 5^2 \times 7$

$3^3 \times 7^3 \times 11$

$3^3 \quad 7$

HCF = $3^3 \times 7$

$3^4 \times 5^2 \times 7$

$3^3 \times 7^3 \times 11$

$3^4 \quad 5^2 \quad 7^3 \quad 11$

LCM = $3^4 \times 5^2 \times 7^3 \times 11$

(c)

$72\ 900 = 2^2 \times 3^6 \times 5^2$

$2^3 \times 3^3 \times 5^6$

$2^2 \quad 3^3 \quad 5^2$

HCF = $2^2 \times 3^3 \times 5^2$

$72\ 900 = 2^2 \times 3^6 \times 5^2$

$2^3 \times 3^3 \times 5^6$

$2^3 \quad 3^6 \quad 5^6$

LCM = $2^3 \times 3^6 \times 5^6$

(d) $160\,083 = 3^3 \times 7^2 \times 11^2$

$3^5 \times 7^6 \times 11$

↓ 3^3 ↓ 7^2 ↓ 11

HCF = $3^3 \times 7^2 \times 11$

$160\,083 = 3^3 \times 7^2 \times 11^2$

$3^5 \times 7^6 \times 11$

↓ 3^5 ↓ 7^6 ↓ 11^2

LCM = $3^5 \times 7^6 \times 11^2$

Intermediate

14. (a) The first 7 odd numbers are 1, 3, 5, 7, 9, 11 and 13.

The sum of the first 7 odd numbers

$= 1 + 3 + 5 + 7 + 9 + 11 + 13 = 49$.

Difference between the square of 7 and the sum of the first 7 odd numbers = 0.

(b) The length of its edge = $\sqrt[3]{29\,791} = 31$.

The area of one side of box = $31^2 = 961$ cm^2.

(c) $585 = 3^2 \times 5 \times 13$

In order for $585p$ to be perfect square, $585p$ must be expressed as a product of the square of its prime factors.

Thus $3^2 \times 5^2 \times 13^2$ is a perfect square and $3^2 \times 5 \times 13 \times 5 \times 13 = 585 \times 5 \times 13$.

Thus the smallest $p = 5 \times 13 = 65$.

15. $720 = 2^4 \times 3^2 \times 5$

$1575 = 3^2 \times 5^2 \times 7$

(i) Largest prime factor of 720 and 1575 = 5

(ii) LCM of 720 and 1575 = $2^4 \times 3^2 \times 5^2 \times 7$

$= 25\,200$

16. $374 = 2 \times 11 \times 17$

$34 = 2 \quad \times 17$

So the smallest number that gives LCM of 374 is 11.

Thus $m = 11$.

17. (i) Divide 1764 by the smallest prime number until we get 1.

2	1764
2	882
3	441
3	147
7	49
7	7
	1

$1764 = 2^2 \times 3^2 \times 7^2 = (2 \times 3 \times 7)^2$

$\sqrt{1764} = 2 \times 3 \times 7 = 42$

(ii) Divide 3375 by the smallest prime number until we get 1.

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$3375 = 3^3 \times 5^3 = (3 \times 5)^3$

$\sqrt[3]{3375} = 3 \times 5 = 15$

(iii) Find the HCF and LCM of 15 and 42.

$15 = \quad 3 \times 5$

$42 = 2 \times 3 \quad \times 7$

↓ 3

HCF of 15 and 42 = 3

$15 = \quad 3 \times 5$

$42 = 2 \times 3 \quad \times 7$

↓ 2 ↓ 3 ↓ 5 ↓ 7

LCM = $2 \times 3 \times 5 \times 7 = 210$

18. **(a)** **(i)** Divide 216 000 by the smallest prime number until we get 1.

2	216 000
2	108 000
2	54 000
2	27 000
2	13 500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$216\ 000 = 2^6 \times 3^3 \times 5^3$

(ii) Divide 518 400 by the smallest prime number until we get 1.

2	518 400
2	259 200
2	129 600
2	64 800
2	32 400
2	16 200
2	8100
2	4050
3	2025
3	675
3	225
3	75
5	25
5	5
	1

$518\ 400 = 2^8 \times 3^4 \times 5^2$

(b) **(i)** $216\ 000 = 2^6 \times 3^3 \times 5^3 = (2^2 \times 3 \times 5)^3$

$\sqrt[3]{216\ 000} = 2^2 \times 3 \times 5 = 60$

(ii) $518\ 400 = 2^8 \times 3^4 \times 5^2 = (2^4 \times 3^2 \times 5)^2$

$\sqrt{518\ 400} = 2^4 \times 3^2 \times 5 = 720$

(iii) $216\ 000 = \boxed{2^6} \times \boxed{3^3} \times \boxed{5^3}$

$518\ 400 = \boxed{2^8} \times \boxed{3^4} \times \boxed{5^2}$

$\downarrow 2^6 \quad \downarrow 3^3 \quad \downarrow 5^2$

HCF of 216 000 and 518 400 $= 2^6 \times 3^3 \times 5^2$

$= 43\ 200$

(iv) $60 = \boxed{2^2} \times \boxed{3} \times \boxed{5}$

$720 = \boxed{2^4} \times \boxed{3^2} \times \boxed{5}$

$\downarrow 2^4 \quad \downarrow 3^2 \quad \downarrow 5$

LCM of 60 and 720 $= 2^4 \times 3^2 \times 5 = 720$

19. $84 = 2 \times 2 \times 3 \times 7$

$126 = 2 \times 3 \times 3 \times 7$

(i) To find the length of each square is to find the largest whole number which is a factor of both 84 and 126.

$84 = \boxed{2} \times 2 \times \boxed{3} \times \boxed{7}$

$126 = \boxed{2} \times \boxed{3} \times 3 \times \boxed{7}$

$\downarrow 2 \quad \downarrow 3 \quad \downarrow 7$

HCF of 84 and 126 $= 2 \times 3 \times 7 = 42$

Thus the length of each square is 42 cm.

(ii) Area of the rectangular sheet $= 84 \times 126$

$= 10\ 584\ cm^2$

Area of each square $= 42 \times 42 = 1764\ cm^2$

Number of squares that she can cut

$= 10\ 584 \div 1764 = 6$

20. $48 = \boxed{2} \times \boxed{2} \times \boxed{2} \times 2 \times \boxed{3}$

$72 = \boxed{2} \times \boxed{2} \times \boxed{2} \times \boxed{3} \times 3$

$96 = \boxed{2} \times \boxed{2} \times \boxed{2} \times 2 \times 2 \times \boxed{3}$

$\downarrow 2 \quad \downarrow 2 \quad \downarrow 2 \quad \downarrow 3$

(i) Greatest number of discussion topics

= HCF of 48, 72 and 96

$= 2 \times 2 \times 2 \times 3$

= 24

(ii) Number of participants from China in each discussion group

$= 96 \div 24$

= 4

21. $8 = 2 \times 2 \times 2$
$10 = 2 \times 5$
$12 = 2 \times 2 \times 3$

↓ 2, 2, 2, 3, 5

$\therefore$ LCM of 8, 10 and 12 $= 2 \times 2 \times 2 \times 3 \times 5 = 120$

$\therefore$ The three canteens will serve noodle soup again after 120 days.

22. (a) Volume of paper box $= 8 \times 12 \times 16 = 1536\ m^3$

Volume of each small cube $= 2 \times 2 \times 2 = 8\ m^3$

Number of small cubes that he is able to pack
$= 1536 \div 8$
$= 192$

(b) The length of each cube is the largest whole number which is a factor of 8, 12 and 16.

$8 = 2 \times 2 \times 2$
$12 = 2 \times 2 \times 3$
$16 = 2 \times 2 \times 2 \times 2$

↓ 2, 2

HCF of 8, 12 and 16 = 4

Thus the length of each cube is 4 m.

23. $160 = 2 \times 2 \times 2 \times 2 \times 2 \times 5$
$192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$
$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

↓ 2, 2, 2, 2

(i) Largest possible length of each piece of ribbon
$= 2 \times 2 \times 2 \times 2$
$= 16$ cm

(ii) Total number of ribbons
$= (160 \div 16) + (192 \div 16) + (240 \div 16)$
$= 37$

24. To find the time when they next meet again is to find the LCM of 126, 154 and 198 seconds.

$126 = 2 \times 3 \times 3 \times 7$
$154 = 2 \times 7 \times 11$
$198 = 2 \times 3 \times 3 \times 11$

↓ 2, 3, 3, 7, 11

LCM of 126, 154 and 198 $= 2 \times 3 \times 3 \times 7 \times 11$
$= 1386$

Time when they next meet again
= 4 pm + 23 min 6 s
= 4.23 pm

25. (i) To find the greatest number of hampers that can be packed is to find the HCF of the boxes of chocolates, the bottles of water and the tins of biscuits.

$420 = 2 \times 2 \times 3 \times 5 \times 7$
$630 = 2 \times 3 \times 3 \times 5 \times 7$
$1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7$

↓ 2, 3, 5, 7

HCF of 420, 630 and 1260 $= 2 \times 3 \times 5 \times 7 = 210$

(ii) Number of boxes of chocolate $= 1260 \div 210$
$= 6$

Number of bottles of water $= 420 \div 210 =$
2 Number of tins of biscuits $= 630 \div 210 =$

26. (i) Divide 13 824 by the smallest prime number until we get 1.

2	13 824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$13\ 824 = 2^9 \times 3^3$

5	42 875
5	8575
5	1715
7	343
7	49
7	7
	1

$42\ 875 = 5^3 \times 7^3$

$13\ 824 \times 42\ 875 = 2^9 \times 3^3 \times 5^3 \times 7^3$

(ii) $13\ 824 \times 42\ 875 = 2^9 \times 3^3 \times 5^3 \times 7^3$
$= (2^3 \times 3 \times 5 \times 7)^3$

$\sqrt[3]{13\ 824 \times 42\ 875} = 2^3 \times 3 \times 5 \times 7 = 840$

Advanced

27. **(a)** True

If a and b are two prime numbers, $a < b$ and $(a + b)$ is another prime number, the only possible set of a and b is 2 and another prime number. The only possible set of a and b are 2 and other prime numbers. When 2 is added to the number, the sum will turn out to be an odd number. As such, some of the numbers will turn out to be prime numbers. When an odd number (prime number) is added to another prime number, the sum is an even number, which will not be a prime number.

(b) False

Consider $1 \times 2 = 2$. 2 is a prime number.

(c) False

$a + b = 2$

(d) False

The digits of $c \times d = 56$ as $38^3 = 54\ 872$.

(e) False

When $x = 62$, the sum of the digits $= 6 + 2 = 8$. But 62 is not divisible by 8.

(f) True

One example to verify this statement is $12 \times 32 = 4 \times 96 = 384$.

(g) False

$2 \times 24 \neq 6 \times 8 \times 12$

28. **(i)** $15 = 3 \times 5$

$20 = 2 \times 2 \times 5$

$27 = 3 \times 3 \times 3$

LCM of 15, 20 and 27 $= 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= 540$

The next event will happen 540 seconds or 9 minutes later, i.e. at 12.09 am.

(ii) Since it happens after every 9 minutes and there are 60 minutes between midnight and 1 am, it will happen for another 6 times.

29. $24 = 2 \times 2 \times 2 \times 3$

$42 = 2 \times 3 \times 7$

$60 = 2 \times 2 \times 3 \times 5$

LCM of 24, 42 and 60 $= 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 840$

Shortest possible length = 840 cm

30. $36 = 2 \times 2 \times 3 \times 3$

$56 = 2 \times 2 \times 2 \times 7$

$1512 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7$

Smallest value of n

$= 3 \times 3 \times 3$

$= 27$

31. $A = 2^2 \times 3^4 \times 5^2 \times 7^4 \times 13^3$

$B = 2^4 \times 3^6 \times 5^2 \times 7^5 \times 11^{16}$

$C = 3^7 \times 5^2 \times 7 \times 17^2$

(i) **(a)** HCF of A, B and $C = 3^4 \times 5^2 \times 7$

(b) LCM of A, B and C

$= 2^4 \times 3^7 \times 5^2 \times 7^5 \times 11^{16} \times 13^3 \times 17^2$

(ii) For $B \times D$ to be a perfect square, the powers of $B \times D$ must be even. Hence $D = 7$ so that

$B \times D = 2^4 \times 3^6 \times 5^2 \times 7^5 \times 11^{16} \times 7$

$= (2^2 \times 3^3 \times 5 \times 7^3 \times 11^8)^2$.

(iii) $A \times C = 2^2 \times 3^4 \times 5^2 \times 7^4 \times 13^3 \times 3^7 \times 5^2 \times 7 \times 17^2$

$= 2^2 \times 3^{11} \times 5^4 \times 7^5 \times 13^3 \times 17^2$

In order for $A \times C \times E$ to be a perfect cube, the powers of $A \times C \times E$ must be multiples of 3.

Thus $E = 2 \times 3 \times 5^2 \times 7 \times 17$.

32. Consider multiples of 4 and they are 8, 12, 16 and 20. We can find the corresponding numbers which give HCF = 4 and LCM = 120.

Case 1

$4 = 2 \times 2$

LCM $= 2 \times 2 \times 30 = 120$. Thus the next number is $2 \times 2 \times 30 = 120$.

The first set of numbers is 4 and 120.

Case 2

$8 = 2 \times 2 \times 2$

LCM $= 2 \times 2 \times 2 \times 15 = 120$. Thus the next number is $2 \times 2 \times 15 = 60$.

The second set of numbers is 8 and 60.

Case 3

$12 = 2 \times 2 \times 3$

LCM $= 2 \times 2 \times 3 \times 10 = 120$. Thus the next number is $2 \times 2 \times 10 = 40$.

The third set of numbers is 12 and 40.

Case 4

$20 = 2 \times 2 \times 5$

LCM $= 2 \times 2 \times 5 \times 6 = 120$. Thus the next number is $2 \times 2 \times 6 = 24$.

The last set of numbers is 20 and 24.

33. By observation, $19 \times 11 = 209$ where $19 + 11 = 30$ but 209 does not contain all prime numbers.

So, we can try $19 \times 2 \times 3 \times 5$. But $19 + 2 + 3 + 5 \neq 30$ and $19 \times 2 \times 3 \times 5 = 570$ and 0 is not a prime number.

Therefore we can try $19 \times 2 \times 2 \times 7$.

$19 + 2 + 2 + 7 = 30$

and $19 \times 2 \times 2 \times 7 = 532$ and 5, 3 and 2 are prime numbers.

So, the 3-digit number that satisfies all the conditions is 532.

New Trend

34. (a) $504 = 2^3 \times 3^2 \times 7$

(b) HCF: 2×3

LCM: $2^3 \times 3^2 \times 7$

First number $= 2 \times 3 \times 7 = 42$

Second number $= 2^3 \times 3^2 = 72$

35. (a) Total surface area $= 2(10 \times 12 + 10 \times 8 + 12 \times 8)$

$= 592 \text{ cm}^2$

(b) $455 = 5 \times 7 \times 13$

Length of side of each cube = 2 cm

∴ Dimensions of the cuboid are 10 cm by 14 cm by 26 cm.

(c) Number of cubes required to form the largest cube

$= 7^3$

$= 343$

Number of cubes left $= 455 - 343$

$= 112$

36. (a)

2	3234
3	1617
7	537
7	77
11	11
	1

$3234 = 2 \times 3 \times 7 \times 7 \times 11 = 2 \times 3 \times 7^2 \times 11$

(b) $4 = 2 \times 2$

$30 = 2 \times 3 \times 5$

LCM of 4 and 30 $= 2 \times 2 \times 3 \times 5$

$= 60$

Factors of 60 = 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

37. (a) Divide 1200 by the smallest prime number until we get 1.

2	1200
2	600
2	300
2	150
3	75
5	25
5	5
	1

$1200 = 2^4 \times 3 \times 5^2$

$= 2^a \times 3^b \times 5^c$

∴ $a = 4, b = 1, c = 2$

(b) (i) Divide 3240 by the smallest prime number until we get 1.

2	3240
2	1620
2	810
3	405
3	135
3	45
3	15
5	5
	1

$3240 = 2^3 \times 3^4 \times 5$

Divide 4212 by the smallest prime number until we get 1.

2	4212
2	2106
3	1053
3	351
3	117
3	39
13	13
	1

$4212 = 2^2 \times 3^4 \times 13$

(ii) The largest whole number which is a factor of both 3240 and 4212 is $2^2 \times 3^4 = 324$.

(c) $3240 = 2^3 \times 3^4 \times 5$

In order for $\frac{3240}{k}$ to be a square number, the powers of $\frac{3240}{k}$ must be even.

Thus, $k = 2 \times 5 = 10$.

38. $A = 2^2 \times 3^4 \times 5^2 \times 7^2$

$B = 2^4 \times 3^6 \times 5^2 \times 11^{16}$

$C = 3^7 \times 5^2 \times 7$

(i) LCM of A, B and $C = 2^4 \times 3^7 \times 5^2 \times 7^2 \times 11^{16}$

(ii) HCF of A, B and $C = 3^4 \times 5^2 = 2025$

∴ The greatest number that will divide A, B and C exactly is 2025

(iii) $A \times B = 2^6 \times 3^{10} \times 5^4 \times 7^2 \times 11^{16}$

$= (2^3 \times 3^5 \times 5^2 \times 7 \times 11^8)^2$

$\sqrt{A \times B} = 2^3 \times 3^5 \times 5^2 \times 7 \times 11^8$

(iv) In order for Ck to be a perfect cube, the powers of Ck have to be multiples of 3.

Thus Ck has to be $3^9 \times 5^3 \times 7^3$ which means $3^7 \times 5^2 \times 7 \times 3^2 \times 5 \times 7^2 = C \times 3^2 \times 5 \times 7^2$.

Thus $k = 3^2 \times 5 \times 7^2 = 2205$.

Chapter 2 Integers, Rational Numbers and Real Numbers

Basic

1. **(a)** If –15 represents 15 m below sea level, then +20 represents 20 m above sea level.

(b) If –10 represents the distance of 10 km of a car travelling south, then +10 represents the distance of 10 km of a car travelling north.

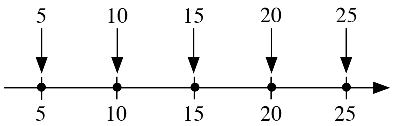
(c) If +100 represents a profit of \$100 on the sale of a mobile phone, then –91 represents a loss of \$91 on the sale.

(d) If +90° represents a clockwise rotation of 90°, then –90° represents rotating 90° anticlockwise.

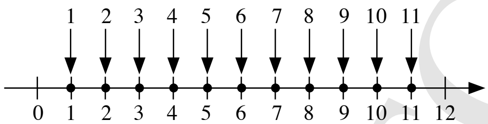
(e) If –5 represents 5 flights down the stairs, then 14 flights up the stairs is represented by +14.

(f) If +600 represents a deposit of \$600 in the bank, then a withdrawal of \$60 is represented by –60.

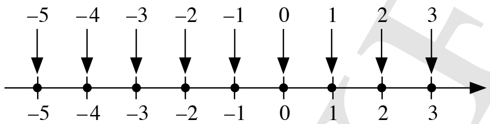
2. **(a)**



(b)



(c)



3. **(a)** $6 \boxed{>} -6$

(b) $0 \boxed{>} -\dfrac{4}{5}$

(c) $(-1)^3 = -1$

Since $-1 < 3$,

Therefore, $(-1)^3 \boxed{<} 3$

(d) $-12 \boxed{>} -16$

(e) $-\dfrac{12}{2} = -6$

Since $-6 < -5$,

Therefore, $-\dfrac{12}{2} \boxed{<} -5$

(f) $-5.6 \boxed{<} -3.4$

(g) $\sqrt[3]{-64} = -4$

$-\sqrt{16} = -4$

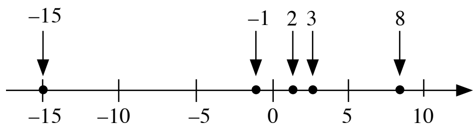
Therefore, $\sqrt[3]{-64} \boxed{=} -\sqrt{16}$

(h) $-\sqrt{25} = -5$

$-\sqrt{15} = -3.873$

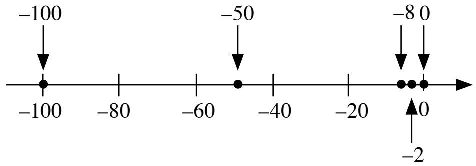
Therefore, $-\sqrt{25} \boxed{<} -\sqrt{15}$

4. **(a)**



$\therefore -15, -1, 2, 3, 8$

(b)



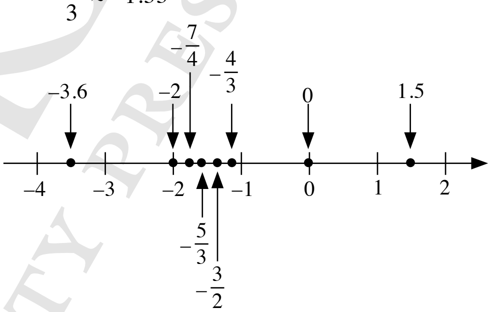
$\therefore -100, -50, -8, -2, 0$

(c) $-\dfrac{7}{4} = -1.75$

$-\dfrac{5}{3} \approx -1.67$

$-\dfrac{3}{2} = -1.5$

$-\dfrac{4}{3} \approx -1.33$



$\therefore -3.6, -2, -\dfrac{7}{4}, -\dfrac{5}{3}, -\dfrac{3}{2}, -\dfrac{4}{3}, 0, 1.5$

5. **(a)** $-5 + 13 = 13 - 5$
$= 8$

(b) $-25 + 12 = 12 - 25$
$= -13$

(c) $5 + (-4) = 5 - 4$
$= 1$

(d) $19 + (-26) = 19 - 26$
$= -7$

(e) $-2 + (-2) = -2 - 2$
$= -4$

(f) $-5 + (-3) = -5 - 3$
$= -8$

(g) $-11 + (-10) = -11 - 10$
$= -21$

(h) $-25 + (-65) = -25 - 65$
$= -90$

6. **(a)** $14 - 18 = -4$

(b) $-5 - 3 = -8$

(c) $-12 - 13 = -25$

(d) $-(-13) = 13$

(e) $6 - (-11) = 6 + 11$
$= 17$

(f) $-8 - (-11) = -8 + 11$
$= 11 - 8$
$= 3$

(g) $(-17) - (-35) = -17 + 35$
$= 35 - 17$
$= 18$

(h) $(-25) - (-10) = -25 + 10$
$= 10 - 25$
$= -15$

7. (a) $5 \times (-4) = 5 \times (-1 \times 4)$
$= 5 \times (-1) \times 4$
$= (-1) \times 20$
$= -20$

(b) $-3 \times 8 = (-1 \times 3) \times 8$
$= (-1) \times 3 \times 8$
$= (-1) \times 24$
$= -24$

(c) $(-4) \times (-12) = (-1 \times 4) \times (-12)$
$= (-1 \times 4) \times (-1 \times 12)$
$= (-1) \times 4 \times (-1) \times 12$
$= (-1) \times (-1) \times 4 \times 12$
$= 1 \times 48$
$= 48$

(d) $-5(-16) = (-1 \times 5) \times (-16)$
$= (-1 \times 5) \times (-1 \times 16)$
$= (-1) \times 5 \times (-1) \times 16$
$= (-1) \times (-1) \times 5 \times 16$
$= 1 \times 80$
$= 80$

(e) $-10(-20) = (-1 \times 10) \times (-20)$
$= (-1 \times 10) \times (-1 \times 20)$
$= (-1) \times 10 \times (-1) \times 20$
$= (-1) \times (-1) \times 10 \times 20$
$= 1 \times 200$
$= 200$

(f) $0 \times (-18) = 0 \times (-1) \times 18$
$= (-1) \times 0 \times 18$
$= (-1) \times 0$
$= 0$

(g) $56 \div (-7) = \frac{56}{-7}$
$= 56 \times \frac{1}{-7}$
$= 56 \times \left(-\frac{1}{7}\right)$
$= -8$

(h) $0 \div (-12) = \frac{0}{-12}$
$= 0 \times \frac{1}{-12}$
$= 0$

(i) $-100 \div (-4) = \frac{-100}{-4}$
$= -100 \times \frac{1}{-4}$
$= -100 \times \left(-\frac{1}{4}\right)$
$= 25$

(j) $(-75) \div (-25) = \frac{-75}{-25}$
$= -75 \times \frac{1}{-25}$
$= -75 \times \left(-\frac{1}{25}\right)$
$= 3$

(k) $\frac{70}{-14} = 70 \times \frac{1}{-14}$
$= 70 \times \left(-\frac{1}{14}\right)$
$= -5$

(l) $\frac{-90}{-15} = -90 \times \frac{1}{-15}$
$= -90 \times \left(-\frac{1}{15}\right)$
$= 6$

8. (a) $(-2) \times (-3) \times (-4) \times (-5) = 6 \times (-4) \times (-5)$
$= -(6 \times 4) \times (-5)$
$= (-24) \times (-5)$
$= 120$

(b) $(-8) \times (-3) \times 5 \times (-6) = 24 \times 5 \times (-6)$
$= 120 \times (-6)$
$= -(120 \times 6)$
$= -720$

(c) $(-2) \times 5 \times (-9) \times (-7) = -(2 \times 5) \times (-9) \times (-7)$
$= -10 \times (-9) \times (-7)$
$= 90 \times (-7)$
$= -630$

(d) $4 \times (-4) \times (-5) \times (-16)$
$= -(4 \times 4) \times (-5) \times (-16)$
$= -16 \times (-5) \times (-16)$
$= 80 \times (-16)$
$= -1280$

(e) $5 \times 6 \times (-1) \times (-12) = 30 \times (-1) \times (-12)$
$= (-30 \times 1) \times (-12)$
$= -30 \times (-12)$
$= 360$

(f) $(-1) \times (-8) \times 3 \times 5 = 8 \times 3 \times 5$
$= 24 \times 5$
$= 120$

(g) $140 \div (-7) \div 4 = \left(\frac{140}{-7}\right) \div 4$
$= \left(140 \times \frac{-1}{7}\right) \div 4$
$= (-20) \div 4$
$= \frac{-20}{4}$
$= -20 \times \left(\frac{1}{4}\right)$
$= -5$

(h) $(-264) \div 11 \div 8 = \left(\frac{-264}{11}\right) \div 8$
$= \left(-264 \times \frac{1}{11}\right) \div 8$
$= (-24) \div 8$
$= -3$

(i) $(-390) \div (-13) \div (-5) = \left(\frac{-390}{-13}\right) \div (-5)$
$= \left(-390 \times \frac{1}{-13}\right) \div (-5)$
$= (30) \div (-5)$
$= -(30 \div 5)$
$= -6$

(j) $(-9) \times (-4) \div (-12) = (36) \div (-12)$
$= \left(\frac{36}{-12}\right)$
$= (36) \times \left(-\frac{1}{12}\right)$
$= -3$

(k) $(-56) \div (-8) \times 15 = \left(\frac{-56}{-8}\right) \times 15$
$= \left(-56 \times \frac{1}{-8}\right) \times 15$
$= 7 \times 15$
$= 105$

(l) $\sqrt{-288 \div (-2)} \times (-3)^2$
$= \left(\sqrt{\frac{-288}{-2}}\right) \times (-3)^2$
$= \left(\sqrt{-288 \times \left(\frac{1}{-2}\right)}\right) \times (-3)^2$
$= \left(\sqrt{144}\right) \times (-3)^2$
$= 12 \times (-3)^2$
$= 12 \times 9$
$= 108$

9. **(a)** $(-2) \times (-3) \times (-4) \times (-5) = 120$
(b) $(-8) \times (-3) \times 5 \times (-6) = -720$
(c) $(-2) \times 5 \times (-9) \times (-7) = -630$
(d) $4 \times (-4) \times (-5) \times (-16) = -1280$
(e) $5 \times 6 \times (-1) \times (-12) = 360$
(f) $(-1) \times (-8) \times 3 \times 5 = 120$
(g) $140 \div (-7) \div 4 = -5$
(h) $(-264) \div 11 \div 8 = -3$
(i) $(-390) \div (-13) \div (-5) = -6$
(j) $(-9) \times (-4) \div (-12) = -3$
(k) $(-56) \div (-8) \times 15 = 105$
(l) $\sqrt{-288 \div (-2)} \times (-3)^2 = 108$

10. **(a)** $[(-3) + (-4)] \div 7 = [(-3) - 4] \div 7$
$= (-7) \div 7$
$= \frac{-7}{7}$
$= -7 \times \frac{1}{7}$
$= -1$

(b) $(-56) \div [7 + (-14)] = (-56) \div [7 - 14]$
$= (-56) \div (-7)$
$= \frac{-56}{-7}$
$= -56 \times \left(-\frac{1}{7}\right)$
$= 8$

(c) $(-72) \div [-14 - (-23)] = (-72) \div (-14 + 23)$
$= (-72) \div (9)$
$= \frac{-72}{9}$
$= -72 \times \frac{1}{9}$
$= -8$

(d) $32 + (-16) \div (-2)^2 = 32 + (-16) \div 4$
$= 32 + \left(\frac{-16}{4}\right)$
$= 32 + \left(-16 \times \frac{1}{4}\right)$
$= 32 + (-4)$
$= 32 - 4$
$= 28$

(e) $5 \times (-4)^2 - (-3)^3 = 5 \times (16) - (-27)$
$= 80 - (-27)$
$= 80 + 27$
$= 107$

(f) $(47 + 19 - 36) \div (-5) = (66 - 36) \div (-5)$
$= 30 \div (-5)$
$= \frac{30}{-5}$
$= 30 \times -\frac{1}{5}$
$= -6$

(g) $6 - (-3)^2 + 6 \div (-3) = 6 - 9 + 6 \div (-3)$
$= 6 - 9 + \left(\frac{6}{-3}\right)$
$= 6 - 9 + \left(6 \times -\frac{1}{3}\right)$
$= 6 - 9 + (-2)$
$= -3 - 2$
$= -5$

(h) $(-2)^3 \times (-2)^2 - 8 \div (-2)^3 = (-8) \times 4 - 8 \div (-8)$
$= -32 - 8 \div (-8)$
$= -32 + \left(\frac{-8}{-8}\right)$
$= -32 + \left(-8 \times -\frac{1}{8}\right)$
$= -32 + 1$
$= -31$

11. (a) $[(-3) + (-4)] \div 7 = -1$
(b) $(-56) \div [7 + (-14)] = 8$
(c) $(-72) \div [-14 - (-23)] = -8$
(d) $32 + (-16) \div (-2)^2 = 28$
(e) $5 \times (-4)^2 - (-3)^3 = 107$
(f) $(47 + 19 - 36) \div (-5) = -6$
(g) $6 - (-3)^2 + 6 \div (-3) = -5$
(h) $(-2)^3 \times (-2)^2 - 8 \div (-2)^3 = -31$

12. (a) $2\frac{5}{9} - 3\frac{1}{4} = \frac{23}{9} - \frac{13}{4}$
$= \frac{23 \times 4}{9 \times 4} - \frac{13 \times 9}{4 \times 9}$
$= \frac{92}{36} - \frac{117}{36}$
$= \frac{92 - 117}{36}$
$= -\frac{25}{36}$

(b) $2\frac{1}{4} + \left(-1\frac{3}{5}\right) = 2\frac{1}{4} - 1\frac{3}{5}$
$= \frac{9}{4} - \frac{8}{5}$
$= \frac{9 \times 5}{4 \times 5} - \frac{8 \times 4}{5 \times 4}$
$= \frac{45 - 32}{20}$
$= \frac{13}{20}$

(c) $9\frac{1}{4} + \left(-7\frac{3}{5}\right) = 9\frac{1}{4} - 7\frac{3}{5}$
$= \frac{37}{4} - \frac{38}{5}$
$= \frac{37 \times 5}{4 \times 5} - \frac{38 \times 4}{5 \times 4}$
$= \frac{185 - 152}{20}$
$= \frac{33}{20}$
$= 1\frac{13}{20}$

(d) $\frac{2}{5} - \left(-\frac{1}{6}\right) = \frac{2}{5} + \frac{1}{6}$
$= \frac{2 \times 6}{5 \times 6} + \frac{1 \times 5}{6 \times 5}$
$= \frac{12 + 5}{30}$
$= \frac{17}{30}$

(e) $-1\frac{1}{5} + \left(-1\frac{1}{3}\right) = -\frac{6}{5} - 1\frac{1}{3}$
$= -\frac{6}{5} - \frac{4}{3}$
$= \frac{-6 \times 3}{5 \times 3} - \frac{4 \times 5}{3 \times 5}$
$= \frac{-18 - 20}{15}$
$= \frac{-38}{15}$
$= -2\frac{8}{15}$

(f) $-2\frac{1}{3} - \left(-1\frac{1}{2}\right) = -2\frac{1}{3} + 1\frac{1}{2}$
$= -\frac{7}{3} + \frac{3}{2}$
$= \frac{-7 \times 2}{3 \times 2} + \frac{3 \times 3}{2 \times 3}$
$= \frac{-14 + 9}{6}$
$= -\frac{5}{6}$

(g) $-4\frac{2}{9} - \left(-1\frac{1}{6}\right) = -4\frac{2}{9} + 1\frac{1}{6}$
$= -\frac{38}{9} + \frac{7}{6}$
$= \frac{-38 \times 6}{9 \times 6} + \frac{7 \times 9}{6 \times 9}$
$= \frac{-228 + 63}{54}$
$= -\frac{165}{54}$
$= -\frac{55}{18}$

(h) $-\left(-\frac{7}{8}\right) - 1\frac{3}{4} = \frac{7}{8} - 1\frac{3}{4}$

$= \frac{7}{8} - \frac{7}{4}$

$= \frac{7 \times 4}{8 \times 4} - \frac{7 \times 8}{4 \times 8}$

$= \frac{28 - 56}{32}$

$= -\frac{28}{32}$

$= -\frac{7}{8}$

13. (a) $2\frac{5}{9} - 3\frac{1}{4} = -\frac{25}{36}$

(b) $2\frac{1}{4} + \left(-1\frac{3}{5}\right) = \frac{13}{20}$

(c) $9\frac{1}{4} + \left(-7\frac{3}{5}\right) = 1\frac{13}{20}$

(d) $\frac{2}{5} - \left(-\frac{1}{6}\right) = \frac{17}{30}$

(e) $-1\frac{1}{5} + \left(-1\frac{1}{3}\right) = -2\frac{8}{15}$

(f) $-2\frac{1}{3} - \left(-1\frac{1}{2}\right) = -\frac{5}{6}$

(g) $-4\frac{2}{9} - \left(-1\frac{1}{6}\right) = -\frac{55}{18}$

(h) $-\left(-\frac{7}{8}\right) - 1\frac{3}{4} = -\frac{7}{8}$

14. (a) $5 \times \left(-2\frac{2}{5}\right) = {}^{1}\cancel{5} \times \left(\frac{-12}{\cancel{5}_{1}}\right)$

$= -12$

(b) $\left(-\frac{4}{5}\right) \div (-16) = \left(-\frac{{}^{1}\cancel{4}}{5}\right) \times \left(\frac{-1}{\cancel{16}_{4}}\right)$

$= \frac{1}{20}$

(c) $16\frac{3}{10} \times \left(-\frac{5}{8}\right) = \left(\frac{163}{{}_{2}\cancel{10}}\right) \times \left(-\frac{\cancel{5}^{1}}{8}\right)$

$= -\frac{163}{16}$

$= -10\frac{3}{16}$

(d) $-\frac{4}{9} \times \frac{3}{14} = -\frac{{}^{2}\cancel{4}}{{}_{3}\cancel{9}} \times \frac{\cancel{3}^{1}}{\cancel{14}_{7}}$

$= -\frac{2}{21}$

(e) $\left(-3\frac{1}{2}\right) \times 2\frac{3}{5} = -\frac{7}{2} \times \frac{13}{5}$

$= -\frac{91}{10}$

$= -9\frac{1}{10}$

(f) $\left(-7\frac{1}{3}\right) \div 1\frac{5}{6} = -\frac{22}{3} \div \frac{11}{6}$

$= -\frac{{}^{2}\cancel{22}}{{}_{1}\cancel{3}} \times \frac{\cancel{6}^{2}}{\cancel{11}_{1}}$

$= -4$

(g) $-\frac{{}^{1}\cancel{7}}{{}_{2}\cancel{18}} \times \left(-\frac{\cancel{9}^{1}}{\cancel{14}_{2}}\right) = -\frac{1}{2} \times \left(-\frac{1}{2}\right)$

$= -\left(-\frac{1}{4}\right)$

$= \frac{1}{4}$

(h) $\left(-\frac{5}{6}\right) \div \left(-1\frac{3}{4}\right) = -\frac{5}{6} \div \left(-\frac{7}{4}\right)$

$= -\frac{5}{{}_{3}\cancel{6}} \times \left(-\frac{\cancel{4}^{2}}{7}\right)$

$= \frac{10}{21}$

15. (a) $5 \times \left(-2\frac{2}{5}\right) = -12$

(b) $\left(-\frac{4}{5}\right) \div (-16) = \frac{1}{20}$

(c) $16\frac{3}{10} \times \left(-\frac{5}{8}\right) = -10\frac{3}{16}$

(d) $-\frac{4}{9} \times \frac{3}{14} = -\frac{2}{21}$

(e) $\left(-3\frac{1}{2}\right) \times 2\frac{3}{5} = -9\frac{1}{10}$

(f) $\left(-7\frac{1}{3}\right) \div 1\frac{5}{6} = -4$

(g) $-\frac{{}^{1}\cancel{7}}{{}_{2}\cancel{18}} \times \left(-\frac{\cancel{9}^{1}}{\cancel{14}_{2}}\right) = \frac{1}{4}$

(h) $\left(-\frac{5}{6}\right) \div \left(-1\frac{3}{4}\right) = \frac{10}{21}$

16. (a)

```
    1 4.8
×     6.2
---------
    2 9 6
+ 8 8 8
---------
  9 1.7 6
```

$\therefore 14.8 \times 6.2 = 91.76$

(b)

```
    1 4 4.7 3 5
×         0.1 5
---------------
    7 2 3 6 7 5
+ 1 4 4 7 3 5
---------------
  2 1.7 1 0 2 5
```

$\therefore 144.735 \times 0.15 = 21.710\ 25$

(c)
$$\begin{array}{r} 0.35 \\ \times\ 0.096 \\ \hline 210 \\ +\ 315 \\ \hline 0.03360 \\ \hline \end{array}$$

$\therefore 0.35 \times 0.096 = 0.0336$

(d)
$$\begin{array}{r} 1.84 \\ \times\ 0.092 \\ \hline 368 \\ +1656 \\ \hline 0.16928 \end{array}$$

$\therefore 1.84 \times 0.092 = 0.169\ 28$

(e) $1.45 \div 0.16$

$= \dfrac{1.45}{0.16}$

$= \dfrac{145}{16}$

$$\begin{array}{r} 9.0625 \\ 16\overline{)\ 145.0000} \\ -\ 144 \\ \hline 1\ 00 \\ -\ \ 96 \\ \hline 40 \\ -\ 32 \\ \hline 80 \\ -\ 80 \\ \hline 0 \\ \hline \end{array}$$

$\therefore 1.45 \div 0.16 = 9.0625$

(f) $4.86 \div 1.20$

$= \dfrac{4.86}{1.20}$

$= \dfrac{486}{120}$

$$\begin{array}{r} 4.05 \\ 120\overline{)\ 486} \\ -480 \\ \hline 60 \\ -\ \ 0 \\ \hline 600 \\ -\ 600 \\ \hline 0 \\ \hline \end{array}$$

$\therefore 486 \div 120 = 4.05$

(g) $1.921\ 68 \div 62.8$

$= \dfrac{1.92168}{62.8}$

$= \dfrac{19.2168}{628}$

$$\begin{array}{r} 0.0306 \\ 628\overline{)\ 19.2168} \\ -0 \\ \hline 192 \\ -\ \ 0 \\ \hline 1921 \\ -1884 \\ \hline 376 \\ -\ \ 0 \\ \hline 3768 \\ -\ 3768 \\ \hline 0 \\ \hline \end{array}$$

$\therefore 1.92168 \div 62.8 = 0.0306$

(h) $0.003\ 48 \div 0.048$

$= \dfrac{0.00348}{0.048}$

$= \dfrac{3.48}{48}$

$$\begin{array}{r} 0.0725 \\ 48\overline{)\ 3.48} \\ -0 \\ \hline 34 \\ -\ \ 0 \\ \hline 348 \\ -336 \\ \hline 120 \\ -\ \ 96 \\ \hline 240 \\ -\ 240 \\ \hline 0 \\ \hline \end{array}$$

$\therefore 0.003\ 48 \div 0.048 = 0.0725$

17. (a) $5.3 - (-4.9)$
$= 5.3 + 4.9$
$= 10.2$

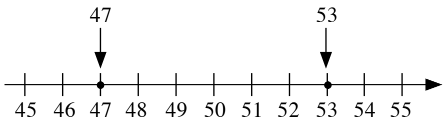
(b) $3.3 + (-2.7)$
$= 3.3 - 2.7$
$= 0.6$

(c) $-15.4 + 8.9$
$= -(15.4 - 8.9)$
$= -6.5$

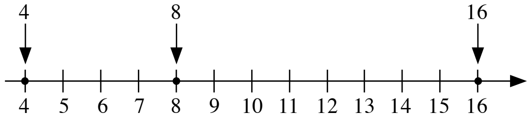
(d) $-17.3 - 6.25$
$= -(17.3 + 6.25)$
$= -23.55$

Intermediate

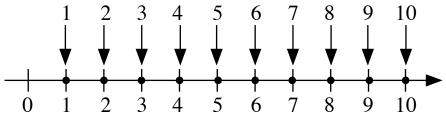
18. (a)



(b) Factors of 64 and 80 are 1, 2, 4, 5, 8 and 16.
Composite numbers that are factors of both 64 and 80 are 4, 8, 16.



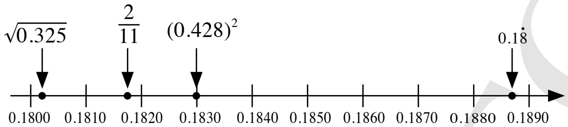
(c) Natural numbers are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10……



19. (a) $\frac{2}{11} = 0.1818$

$\sqrt{0.325} = 0.1803$

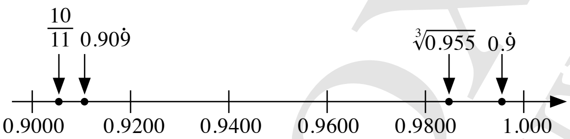
$(0.428)^2 = 0.1830$



$\therefore 0.1\dot{8}, (0.428)^2, \frac{2}{11}, \sqrt{0.325}$

(b) $\frac{10}{11} = 0.9090$

$\sqrt[3]{0.955} = 0.984$



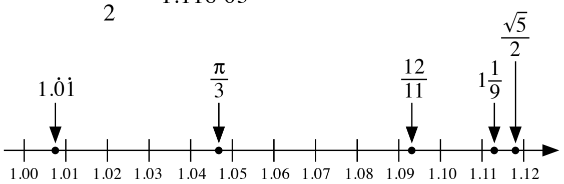
$\therefore 0.\dot{9}, \sqrt[3]{0.955}, 0.90\dot{9}, \frac{10}{11}$

(c) $\frac{\pi}{3} = 1.047\ 20$

$1\frac{1}{9} = 1.1111$

$\frac{12}{11} = 1.0909$

$\frac{\sqrt{5}}{2} = 1.118\ 03$



$\therefore \frac{\sqrt{5}}{2}, 1\frac{1}{9}, \frac{12}{11}, \frac{\pi}{3}, 1.\dot{0}\dot{1}$

20. (a) $4 + (-15) - 21$
$= 4 - 15 - 21$
$= -11 - 21$
$= -(11 + 21)$
$= -32$

(b) $-4 + (-12) + 10$
$= -4 - 12 + 10$
$= -16 + 10$
$= -(16 - 10)$
$= -6$

(c) $-5 + (-7) - (-13)$
$= -5 - 7 + 13$
$= -12 + 13$
$= -(12 - 13)$
$= -(-1)$
$= 1$

(d) $20 + (-9) - (-16)$
$= 20 - 9 - (-16)$
$= 20 - 9 + 16$
$= 11 + 16$
$= 27$

(e) $3 - (-7) - 4 + (-4)$
$= 3 + 7 - 4 - 4$
$= 10 - 4 - 4$
$= 6 - 4$
$= 2$

(f) $-27 - (-35) - 5 + (-9)$
$= -27 + 35 - 5 - 9$
$= -(27 - 35) - 5 - 9$
$= -(-8) - 5 - 9$
$= 8 - 5 - 9$
$= 3 - 9$
$= -6$

(g) $35 - (-5) + (-12) - (-8)$
$= 35 + 5 - 12 + 8$
$= 40 - 12 + 8$
$= 28 + 8$
$= 36$

(h) $23 + (-3) - (-7) + (-22)$
$= 23 - 3 + 7 - 22$
$= 20 + 7 - 22$
$= 27 - 22$
$= 5$

(i) $-14 - [-6 + (-15)]$
$= -14 - (-6 - 15)$
$= -14 - (-21)$
$= -14 + 21$
$= 7$

(j) $[-4 + (-14)] + [-8 - (-26)]$
$= (-4 - 14) + (-8 + 26)$
$= (-18) + (26 - 8)$
$= -18 + 18$
$= 0$

21. $[-2 + (-14) - 10] - [(-6)^2 + (-17) - (-9)]$
$= (-2 -14 - 10) - [36 + (-17) - (-9)]$
$= (-26) - (36 - 17 + 9)$
$= -26 - 28$
$= -(26 + 28)$
$= -54$

22. (a) $\dfrac{(-2) \times (-5) + (-20)}{(-10)}$
$= \dfrac{10 + (-20)}{-10}$
$= \dfrac{10 - 20}{-10}$
$= \dfrac{-10}{-10}$
$= 1$

(b) $\dfrac{(-123) \times [19 + (-19)]}{38}$
$= \dfrac{(-123) \times (19 - 19)}{38}$
$= \dfrac{-123 \times 0}{38}$
$= \dfrac{0}{38}$
$= 0$

(c) $(-11) \times [-52 + (-17) - (-39)]$
$= (-11) \times (-52 - 17 + 39)$
$= (-11) \times (-69 + 39)$
$= (-11) \times (-30)$
$= -(-330)$
$= 330$

(d) $[109 - (-19)] \div (-2)^3 \times (-5)$
$= (109 + 19) \div (-2)^3 \times (-5)$
$= 128 \div (-8) \times (-5)$
$= \left(\dfrac{128}{-8}\right) \times (-5)$
$= -16 \times (-5)$
$= -(-80)$
$= 80$

(e) $(13 - 9)^2 - 5^2 - (28 - 31)^3$
$= 4^2 - 5^2 - (-3)^3$
$= 16 - 25 - (-27)$
$= 16 - 25 + 27$
$= 18$

(f) $16 + (-21) \div 7 \times \{9 + [56 \div (-8)]\}$
$= 16 + (-21) \div 7 \times [9 + (-7)]$
$= 16 + (-21) \div 7 \times (9 - 7)$
$= 16 + (-21) \div 7 \times 2$
$= 16 + \left(\dfrac{-21}{7} \times 2\right)$
$= 16 + (-3 \times 2)$
$= 16 + (-6)$
$= 16 - 6$
$= 10$

(g) $8 \div [3 + (-15)] \div [(-2) \times 4 \times (-3)]$
$= 8 \div (3 - 15) \div [(-2) \times (-12)]$
$= 8 \div (-12) \div (24)$
$= \left(\dfrac{8}{-12}\right) \div 24$
$= \left(-\dfrac{2}{3}\right) \times \dfrac{1}{24}$
$= -\dfrac{1}{36}$

(h) $[(-5) \times (-8)^2 - (-2)^3 \times 7] \div (-11)$
$= [(-5) \times 64 - (-8) \times 7] \div (-11)$
$= [-(5 \times 64) - [-(8 \times 7)]] \div (-11)$
$= [-320 - (-56)] \div (-11)$
$= (-320 + 56) \div (-11)$
$= (-264) \div (-11)$
$= 24$

(i) $\{[(-23) - (-11)] \div 6 - 7 \div (-7)\} \times 1997$
$= [(-23 + 11) \div 6 - 7 \div (-7) \times 1997$
$= [(-12) \div 6 - 7 \div (-7)] \times 1997$
$= \left[\left(\dfrac{-12}{6}\right) - \left(\dfrac{7}{-7}\right)\right] \times 1997$
$= [(-2) - (-1)] \times 1997$
$= (-2 + 1) \times 1997$
$= (-1) \times 1997$
$= -1997$

(j) $(-7)^3 + (-2)^3 - [(-21) + 35 - \sqrt[3]{125} \times (-8)]$
$= -343 + (-8) - [(-21) + 35 - 5 \times (-8)]$
$= -343 + (-8) - [(-21) + 35 - (-40)]$
$= -343 + (-8) - [(-21) + 35 + 40]$
$= -343 + (-8) - (14 + 40)$
$= -343 - 8 - 54$
$= -(343 + 8 + 54)$
$= -405$

23. **(a)** $\dfrac{(-2)\times(-5)+(-20)}{(-10)} = 1$

(b) $\dfrac{(-123)\times[19+(-19)]}{38} = 0$

(c) $(-11)\times[-52+(-17)-(-39)] = 330$

(d) $[109-(-19)] \div (-2)^3 \times (-5) = 80$

(e) $(13-9)^2 - 5^2 - (28-31)^3 = 18$

(f) $16 + (-21) \div 7 \times \{9 + [56 \div (-8)]\} = 10$

(g) $8 \div [3 + (-15)] \div [(-2)\times 4 \times (-3)] = -\dfrac{1}{36}$

(h) $[(-5)\times(-8)^2 - (-2)^3 \times 7] \div (-11) = 24$

(i) $\{[(-23)-(-11)] \div 6 - 7 \div (-7)\} \times 1997 = -1997$

(j) $(-7)^3 + (-2)^3 - [(-21) + 35 - \sqrt[3]{125} \times (-8)] = -405$

24. **(a)** $-5\dfrac{2}{9} - 3\dfrac{1}{4} - 3\dfrac{5}{9}$

$= -5\dfrac{8}{36} - 3\dfrac{9}{36} - 3\dfrac{20}{36}$

$= (-5-3-3) - \dfrac{8}{36} - \dfrac{9}{36} - \dfrac{20}{36}$

$= -11 - \dfrac{(8+9+20)}{36}$

$= -11 - \dfrac{37}{36}$

$= -12\dfrac{1}{36}$

(b) $-3\dfrac{4}{5} - 1\dfrac{3}{10} - \left(-2\dfrac{3}{4}\right)$

$= -3\dfrac{16}{20} - 1\dfrac{6}{20} - \left(-2\dfrac{15}{20}\right)$

$= -3\dfrac{16}{20} - 1\dfrac{6}{20} + \left(2\dfrac{15}{20}\right)$

$= (-3-1+2) - \dfrac{16}{20} - \dfrac{6}{20} + \dfrac{15}{20}$

$= -2 + \dfrac{(-16-6+15)}{20}$

$= -2 + \left(\dfrac{-7}{20}\right)$

$= -2 - \dfrac{7}{20}$

$= -2\dfrac{7}{20}$

(c) $-2\dfrac{3}{4} + \left(-1\dfrac{1}{2}\right) - \left(-1\dfrac{2}{3}\right)$

$= -2\dfrac{3}{4} - 1\dfrac{1}{2} + 1\dfrac{2}{3}$

$= -2\dfrac{9}{12} - 1\dfrac{6}{12} + 1\dfrac{8}{12}$

$= (-2-1+1) - \dfrac{9}{12} - \dfrac{6}{12} + \dfrac{8}{12}$

$= -2 - \dfrac{7}{12}$

$= -2\dfrac{7}{12}$

(d) $-\left(-3\dfrac{5}{7}\right) + 1\dfrac{3}{5} - \left(-\dfrac{3}{7}\right)$

$= 3\dfrac{5}{7} + 1\dfrac{3}{5} + \dfrac{3}{7}$

$= 3\dfrac{25}{35} + 1\dfrac{21}{35} + \dfrac{15}{35}$

$= (3+1) + \dfrac{25}{35} + \dfrac{21}{35} + \dfrac{15}{35}$

$= 4 + \dfrac{61}{35}$

$= 4 + 1\dfrac{26}{35}$

$= 5\dfrac{26}{35}$

(e) $\left(-\dfrac{1}{5}+\dfrac{1}{3}\right) + \left[\dfrac{1}{10}+\left(-\dfrac{1}{5}\right)\right] + \left(-\dfrac{1}{25}\right)$

$= \left(-\dfrac{3}{15}+\dfrac{5}{15}\right) + \left[\dfrac{1}{10}+\left(-\dfrac{2}{10}\right)\right] + \left(-\dfrac{1}{25}\right)$

$= \dfrac{2}{15} + \left(-\dfrac{1}{10}\right) - \dfrac{1}{25}$

$= \dfrac{2}{15} - \dfrac{1}{10} - \dfrac{1}{25}$

$= \dfrac{20}{150} - \dfrac{15}{150} - \dfrac{6}{150}$

$= -\dfrac{1}{150}$

25. **(a)** $-5\dfrac{2}{9} - 3\dfrac{1}{4} - 3\dfrac{5}{9} = -12\dfrac{1}{36}$

(b) $-3\dfrac{4}{5} - 1\dfrac{3}{10} - \left(-2\dfrac{3}{4}\right) = -2\dfrac{7}{20}$

(c) $-2\dfrac{3}{4} + \left(-1\dfrac{1}{2}\right) - \left(-1\dfrac{2}{3}\right) = -2\dfrac{7}{12}$

(d) $-\left(-3\dfrac{5}{7}\right) + 1\dfrac{3}{5} - \left(-\dfrac{3}{7}\right) = 5\dfrac{26}{35}$

(e) $\left(-\dfrac{1}{5}+\dfrac{1}{3}\right) + \left[\dfrac{1}{10}+\left(-\dfrac{1}{5}\right)\right] + \left(-\dfrac{1}{25}\right) = -\dfrac{1}{150}$

26. **(a)** $(-4) \div \left(-\dfrac{1}{4}\right) \times (-4)$

$= (-4)\times(-4)\times(-4)$

$= 16 \times (-4)$

$= -64$

(b) $\left(-2\dfrac{2}{5}\right) \times \left(\dfrac{5}{6}\right) \div (-13)$

$= \left(-\dfrac{{}^{2}\cancel{12}}{{}_{1}\cancel{5}}\right) \times \left(\dfrac{\cancel{5}^{1}}{\cancel{6}_{1}}\right) \div (-13)$

$= -2 \div (-13)$

$= \dfrac{-2}{-13}$

$= \dfrac{2}{13}$

(c) $\left(1\frac{7}{15}\right) \div \left(-17\frac{2}{7}\right) \times \left(3\frac{3}{14}\right)$

$= \left(\frac{22}{15}\right) \div \left(\frac{-121}{7}\right) \times \left(\frac{45}{14}\right)$

$= \left(\frac{22}{{}_1\cancel{15}}\right) \times \left(-\frac{\cancel{7}^1}{121}\right) \times \left(\frac{\cancel{45}^3}{\cancel{14}_2}\right)$

$= -\frac{66}{242}$

$= -\frac{3}{11}$

(d) $\left(-2\frac{5}{7}\right) \div \left(1\frac{1}{3} \times \frac{3}{4}\right)$

$= \left(-2\frac{5}{7}\right) \div \left(\frac{4}{3} \times \frac{3}{4}\right)$

$= \left(-2\frac{5}{7}\right) \div 1$

$= -2\frac{5}{7}$

(e) $\left(3\frac{3}{5}\right) \times (-6) \div \left(-4\frac{4}{5}\right)$

$= \left(\frac{18}{5}\right) \times (-6) \div \left(-\frac{24}{5}\right)$

$= \left(\frac{18}{{}_1\cancel{5}}\right) \times ({}^1\cancel{-6}) \times \left(-\frac{\cancel{5}^1}{\cancel{24}_4}\right)$

$= \frac{18}{4}$

$= 4\frac{2}{4}$

$= 4\frac{1}{2}$

(f) $\frac{1}{4} + \left(-\frac{3}{4}\right) \times \left(-1\frac{1}{4}\right)$

$= \frac{1}{4} + \left(-\frac{3}{4}\right) \times \left(-\frac{5}{4}\right)$

$= \frac{1}{4} + \left(\frac{15}{16}\right)$

$= \frac{4}{16} + \left(\frac{15}{16}\right)$

$= \frac{19}{16}$

$= 1\frac{3}{16}$

(g) $\left[\left(-9\frac{1}{4} - \left(-7\frac{3}{5}\right)\right)\right] \div 2\frac{3}{4}$

$= \left[\left(-9\frac{1}{4} + 7\frac{3}{5}\right)\right] \div 2\frac{3}{4}$

$= \left[\left(-9\frac{5}{20} + 7\frac{12}{20}\right)\right] \div 2\frac{3}{4}$

$= \left[(-9 + 7) - \frac{5}{20} + \frac{12}{20}\right] \div 2\frac{3}{4}$

$= \left[(-2) + \frac{7}{20}\right] \div 2\frac{3}{4}$

$= \left(-\frac{33}{20}\right) \div 2\frac{3}{4}$

$= \left(-\frac{33}{20}\right) \div \frac{11}{4}$

$= \left(-\frac{33}{20}\right) \times \frac{4}{11}$

$= -\frac{3}{5}$

(h) $\left[\left(-1\frac{1}{4}\right) + 1\frac{2}{5}\right] \div \left[(-6) - \frac{4}{7} \times \left(-2\frac{3}{4}\right)\right]$

$= \left[\left(-1\frac{5}{20}\right) + 1\frac{8}{20}\right] \div \left[(-6) - \frac{4}{7} \times \left(-\frac{11}{4}\right)\right]$

$= \left[(-1 + 1) - \frac{5}{20} + \frac{8}{20}\right] \div \left[(-6) - \frac{4}{7} \times \left(-\frac{11}{4}\right)\right]$

$= \left(\frac{3}{20}\right) \div \left[(-6) - \left(-\frac{11}{7}\right)\right]$

$= \left(\frac{3}{20}\right) \div \left[\left(-\frac{42}{7}\right) + \left(\frac{11}{7}\right)\right]$

$= \frac{3}{20} \div \left(-\frac{31}{7}\right)$

$= \frac{3}{20} \times \left(-\frac{7}{31}\right)$

$= -\frac{21}{620}$

(i) $\left(-\frac{3}{4}\right) \times 1\frac{1}{2} + \left(-\frac{3}{4}\right) \times \left(-2\frac{1}{2}\right)$

$= \left(-\frac{3}{4}\right) \times \frac{3}{2} + \left(-\frac{3}{4}\right) \times \left(-\frac{5}{2}\right)$

$= \left(-\frac{9}{8}\right) + \frac{15}{8}$

$= \frac{6}{8}$

$= \frac{3}{4}$

27. **(a)** $(-4) \div \left(-\frac{1}{4}\right) \times (-4) = -64$

(b) $\left(-2\frac{2}{5}\right) \times \left(\frac{5}{6}\right) \div (-13) = \frac{2}{13}$

(c) $\left(1\frac{7}{15}\right) \div \left(-17\frac{2}{7}\right) \times \left(3\frac{3}{14}\right) = -\frac{3}{11}$

(d) $\left(-2\frac{5}{7}\right) \div \left(1\frac{1}{3} \times \frac{3}{4}\right) = -2\frac{5}{7}$

(e) $\left(3\frac{3}{5}\right) \times (-6) \div \left(-4\frac{4}{5}\right) = 4\frac{1}{2}$

(f) $\frac{1}{4} + \left(-\frac{3}{4}\right) \times \left(-1\frac{1}{4}\right) = 1\frac{3}{16}$

(g) $\left[\left(-9\frac{1}{4} - \left(-7\frac{3}{5}\right)\right)\right] \div 2\frac{3}{4} = -\frac{3}{5}$

(h) $\left[\left(-1\frac{1}{4}\right) + 1\frac{2}{5}\right] \div \left[(-6) - \frac{4}{7} \times \left(-2\frac{3}{4}\right)\right] = -\frac{21}{620}$

(i) $\left(-\frac{3}{4}\right) \times 1\frac{1}{2} + \left(-\frac{3}{4}\right) \times \left(-2\frac{1}{2}\right) = \frac{3}{4}$

28. **(a)** $\frac{0.25}{0.05} \times \left(-\frac{0.18}{1.3}\right)$

$= \frac{25}{5} \times \left(\frac{-1.8}{13}\right)$

$= 5 \times \left(\frac{-1.8}{13}\right)$

$= -\frac{9}{13}$

(b) $\frac{0.0064}{0.04} \times \left(-\frac{1.8}{0.16}\right)$

$= \frac{0.64}{4} \times \left(-\frac{180}{16}\right)$

$= 0.16 \times \left(-\frac{45}{4}\right)$

$= -1.8$

(c) $(-0.3)^2 \times \left(\frac{-1.4}{0.07}\right) - 0.78$

$= \left(-\frac{3}{10}\right)^2 \times \left(\frac{-140}{7}\right) - 0.78$

$= \left(\frac{9}{100}\right) \times (-20) - 0.78$

$= -1.8 - 0.78$

$= -2.58$

(d) $(-0.4)^3 \times \left(\frac{-3.3}{0.11}\right) + 0.123$

$= \left(-\frac{4}{10}\right)^3 \times \left(\frac{-3.3}{0.11}\right) + 0.123$

$= \left(-\frac{64}{1000}\right) \times \left(\frac{-330}{11}\right) + 0.123$

$= \left(-\frac{64}{1000}\right) \times (-30) + 0.123$

$= 1.92 + 0.123$

$= 2.043$

29. **(a)** $\dfrac{1\frac{8}{13} \times \frac{13}{42} + 5\frac{1}{5} \div \frac{7}{45}}{\left(\frac{7}{9} + \frac{7}{18}\right) \div \frac{1}{18} \times \frac{1}{7}} = 11\frac{13}{42}$

(b) $\dfrac{\sqrt[3]{13} - \sqrt{7}}{\sqrt{48} - \sqrt[3]{101}} = -0.130$ (to 3 d.p.)

(c) $\dfrac{\sqrt[3]{42.7863} \times (41.567)^2}{94\ 536.721} = 0.064$ (to 3 d.p.)

(d) $\sqrt[3]{\dfrac{9206 \times (29.5)^3}{(11.86)^2}} = 118.884$ (to 3 d.p.)

(e) $\sqrt{\dfrac{46.3^2 + 85.9^2 - 70.7^2}{2 \times 46.3 \times 85.9}} = 0.754$ (to 3 d.p.)

(f) $\sqrt{\dfrac{18 \times (4.359)^2 + 10 \times (3.465)^2}{(4.359)^3 + 3 \times (3.465)^3}} = 1.492$ (to 3 d.p.)

30. Altitude at which the plane is flying now
= 650 – 150 + 830
= 500 + 830
= 1330 m

31. Temperature of Singapore after rain stops
= 24°C + 8°C – 12°C + 6°C
= 32°C – 12°C + 6°C
= 20°C + 6°C
= 26°C

32. Let x be the number of boys.
Number of sweets each boy will have = 6 – 1
= 5
Since Raj took the last sweet,
total number of sweets = 41

$5x + 1 = 41$

$5x = 40$

$x = \frac{40}{5}$

$x = 8$

∴ 8 boys were seated around the table.

33. **(a)**

Packet	1	2	3	4	5
Mass above or below the standard mass (g)	–28	–13	+10	–19	+5
Actual mass (g)	1000 – 28 = 972 g	1000 – 13 = 987 g	1000 + 10 = 1010 g	1000 – 19 = 981 g	1000 + 5 = 1005 g

Packet 5

(b) **(i)** Difference = 1005 – 972
= 33 g

(ii) Difference = 1005 – 981
= 24 g

(iii) Difference = 1005 – 987
= 18 g

Packet 5 and packet 1 have the largest difference.

(c) Mass of rice in packet 6

$= \frac{972 + 1010}{2}$

$= \frac{1982}{2}$

= 991 g

34.

Reservoir	A	B	C	D
Water level	–2 + 6 + 8 = 12	+1 + 3 – 7 = –3	–3 – 1 – 2 = –6	–5 + 9 – 1 = 3

(a) Reservoir **A** caught the most rain.

(b) Reservoir **C** caught the least rain.

(c) Reservoir **D** because 3 > –3.

35. (i) Cost of ride = \$5 – \$3.36
= \$1.64

(ii) Total value of card = \$20 + \$3.36
= \$23.36

Total cost in a day = $\$1.83 \times 2$
= \$3.66

Therefore, number of days before he needs to top up his card

$= \frac{23.36}{3.66}$

= 6.38 (to 3 s.f.)

Hence, he will need to top up his card next Sunday.

36. Let the length of shorter piece of rope be x m.

Therefore, length of the longer piece of rope = $\frac{5}{4}x$ m.

$$x + \frac{5}{4}x = 6.3$$

$$\frac{4}{4}x + \frac{5}{4}x = 6.3$$

$$\frac{9}{4}x = 6.3$$

$$x = 6.3 \times \frac{4}{9}$$

$$x = 2.8$$

∴ Length of the shorter piece of rope is 2.8 m

37. Number of cups of flour Priya used

$= \left(2\frac{1}{2} \times 9\right) + \left(2\frac{3}{4} \times 3\right)$

$= \left(\frac{5}{2} \times 9\right) + \left(\frac{11}{4} \times 3\right)$

$= \left(\frac{45}{2}\right) + \left(\frac{33}{4}\right)$

$= \frac{90}{4} + \frac{33}{4}$

$= \frac{123}{4}$

$= 30\frac{3}{4}$

38. Number of students in class A

$= \frac{4}{19} \times 247$

= 52

Number of students in class that travel to school by bus

$= \left(\frac{8}{13} \times 52\right) + 7$

= 32 + 7

= 39

Therefore, number of students in class A who do not travel by bus

= 52 – 39

= 13

39. (i) Fraction of cost price of refrigerator that Huixian pays

$= 1 - \frac{3}{10} - \frac{9}{20}$

$= \frac{20}{20} - \frac{6}{20} - \frac{9}{20}$

$= \frac{5}{20}$

$= \frac{1}{4}$

(ii) Cost of refrigerator

$= \$525 \div \frac{1}{4}$

$= \$525 \times 4$

= \$2100

40. Fraction of students who failed the test

$= 1 - \frac{1}{7} - \frac{1}{3} - \frac{1}{2}$

$= \frac{42}{42} - \frac{6}{42} - \frac{14}{42} - \frac{21}{42}$

$= \frac{1}{42}$

Fraction of students who scored A and B

$= \frac{1}{7} + \frac{1}{3}$

$= \frac{3+7}{21}$

$= \frac{10}{21}$

$= \frac{20}{42}$

Therefore, number of students who failed the test

$= \frac{100}{20} \times 1$

$= 5$

41. Let the money that Junwei has be $\$x$.

His wife will receive $\$\frac{3}{7}x$.

Rui Feng will receive $\left(x - \frac{3}{7}x\right) \times \frac{1}{2}$

$= \frac{4}{7}x \times \frac{1}{2}$

$= \$\frac{2}{7}x$

Fraction of money distributed to each child

$= \left[x - \left(\frac{3}{7}x + \frac{2}{7}x\right)\right] \div 3$

$= \left(x - \frac{5}{7}x\right) \div 3$

$= \frac{2}{7}x \div 3$

$= \frac{2}{7}x \times \frac{1}{3}$

$= \frac{2x}{21}$

Therefore,

$\frac{2x}{21} = 400$

$2x = 8400$

$x = 4200$

Hence, his wife will receive

$= \frac{3}{7} \times 4200$

$= \$1800$

Advanced

42. $\sqrt[4]{-4 \times (-5.5) - [-2 \times (-3) + 8(-2) - 8 \times 2] + 12^2 - (-4)^3}$

$= \sqrt[4]{-4 \times (-5.5) - [6 + (-16) - 16] + 12^2 - (-4)^3}$

$= \sqrt[4]{-4 \times (-5.5) - (6 - 16 - 16) + 144 - (-64)}$

$= \sqrt[4]{-4 \times (-5.5) - (-26) + 144 + 64}$

$= \sqrt[4]{22 + 26 + 144 + 64}$

$= \sqrt[4]{256}$

$= 4$

43. Fraction of land used for phase 1

$= \frac{11}{18} + \left(\frac{3}{7}\right)\left(1 - \frac{11}{18}\right)$

$= \frac{11}{18} + \left(\frac{3}{7}\right)\left(\frac{7}{18}\right)$

$= \frac{11}{18} + \left(\frac{1}{6}\right)$

$= \frac{11}{18} + \frac{3}{18}$

$= \frac{14}{18}$

$= \frac{7}{9}$

Fraction of land used for phase 2

$= \frac{1}{4} \times \left(1 - \frac{7}{9}\right)$

$= \frac{1}{4} \times \frac{2}{9}$

$= \frac{1}{18}$

Fraction of land used for shopping malls and medical facilities

$= 1 - \frac{7}{9} - \frac{1}{18}$

$= \frac{18}{18} - \frac{14}{18} - \frac{1}{18}$

$= \frac{3}{18}$

$= \frac{1}{6}$

New Trend

44. Arranging in ascending order,

$0.85^{\frac{3}{2}}, \frac{\pi}{4}, \sqrt{0.64}, 0.801$

Chapter 3 Approximation and Estimation

Basic

1. **(a)** 789 500 (to the nearest 100)
   (b) 790 000 (to the nearest 1000)
   (c) 790 000 (to the nearest 10 000)
2. **(a)** 2.5 (to 1 d.p.)
   (b) 18.5 (to 1 d.p.)
   (c) 36.1 (to 1 d.p.)
   (d) 138.1 (to 1 d.p.)
3. **(a)** 4.70 (to 2 d.p.)
   (b) 14.94 (to 2 d.p.)
   (c) 26.80 (to 2 d.p.)
   (d) 0.05 (to 2 d.p.)
4. **(a)** 4.826 (to 3 d.p.)
   (b) 6.828 (to 3 d.p.)
   (c) 7.450 (to 3 d.p.)
   (d) 8.445 (to 3 d.p.)
   (e) 11.639 (to 3 d.p.)
   (f) 13.451 (to 3 d.p.)
   (g) 32.929 (to 3 d.p.)
   (h) 0.038 (to 3 d.p.)
5. **(a)** 36.3 (to 1 d.p.)
   (b) 36 (to the nearest whole number)
   (c) 36.260 (to 3 d.p.)
6. **(a)** All zeros between non-zero digits are significant.
   5 significant figures
   (b) In a decimal, all zeros before a non-zero digit are not significant.
   4 significant figures
   (c) 5 significant figures
   (d) 9 or 10 significant figures.
7. **(a)** 3.9 (to 2 s.f.)
   (b) 20 (to 2 s.f.)
   (c) 38 (to 2 s.f.)
   (d) 4.07 (to 3 s.f.)
   (e) 18.1 (to 3 s.f.)
   (f) 0.0326 (to 3 s.f.)
   (g) 0.0770 (to 3 s.f.)
   (h) 0.008 17 (to 3 s.f.)
   (i) 18.14 (to 4 s.f.)
   (j) 240.0 (to 4 s.f.)
   (k) 5004 (to 4 s.f.)
   (l) 0.054 45 (to 4 s.f.)
8. **(a)** 20 (to 1 s.f.)
   (b) 19.1 (to 1 d.p.)
   (c) 19 (to 2 s.f.)
9. **(a)** 0.007 (to 1 s.f.)
   (b) 0.007 (to 3 d.p.)
   (c) 0.007 20 (to 3 s.f.)
10. **(a)** 984.61 (to 2 d.p.)
    (b) 984.6 (to 4 s.f.)
    (c) 984.608 (to 3 d.p.)
    (d) 984.61 (to the nearest hundredth)
11. **(a)** 0.000 143 (to 3 s.f.)
    (b) 5.1 (to 1 d.p.)
    (c) 1000 (to 2 s.f.)
12. **(a)** 0.3403 (to 4 s.f.)
    (b) 10.255 (to 5 s.f.)
    (c) 64 704 800 (to 6 s.f.)
13. **(a)** 428.2 (to 4 s.f.)
    The number of decimal places in the answer is 1.
    (b) 0.000 90 (to 5 d.p.)
    The number of significant figures is 1 or 2, depending on whether the last zero is included or otherwise.
14. **(a)** 4 cm (to the nearest cm)
    (b) 24 cm (to the nearest cm)
    (c) 107 cm (to the nearest cm)
    (d) 655 cm (to the nearest cm)
15. **(a)** 14.0 kg (to the nearest 0.1 kg)
    (b) 57.5 kg (to the nearest 0.1 kg)
    (c) 108.4 kg (to the nearest 0.1 kg)
    (d) 763.2 kg (to the nearest 0.1 kg)
16. **(a)** 7.0 cm^2 (to the nearest $\frac{1}{10}$ cm^2)
    (b) 40.1 cm^2 (to the nearest $\frac{1}{10}$ cm^2)
    (c) 148.3 cm^2 (to the nearest $\frac{1}{10}$ cm^2)
    (d) 168.4 cm^2 (to the nearest $\frac{1}{10}$ cm^2)
17. **(a)** 5620 km (to the nearest 10 km)
    (b) 900 cm (to the nearest 100 cm)
    (c) 2.45 g (to the nearest $\frac{1}{100}$ g)
    (d) $50 000 (to the nearest $10 000)
18. **(a)** 61.994 06 – 29.980 78
    = 32.013 28
    = 30 (to 1 s.f.)
    (b) 64.967 02 – 36.230 87
    = 28.736 15
    = 30 (to 1 s.f.)
    (c) 4987×91.2
    = 454 814.4
    = 500 000 (to 1 s.f.)

(d) 30.9×98.6
$= 3046.74$
$= 3000$ (to 1 s.f.)

(e) 0.0079×21.7
$= 0.171\ 43$
$= 0.2$ (to 1 s.f.)

(f) $1793 \times 0.000\ 97$
$= 1.739\ 21$
$= 2$ (to 1 s.f.)

(g) 9801×0.0613
$= 600.8013$
$= 600$ (to 1 s.f.)

(h) $(8.907)^2$
$= 79.334\ 649$
$= 80$ (to 1 s.f.)

(i) $(398)^2 \times 0.062$
$= 9821.048$
$= 10\ 000$ (to 1 s.f.)

(j) $81.09 \div 1.592$
$= 50.935\ldots$
$= 50$ (to 1 s.f.)

(k) $\dfrac{49.82}{9.784}$
$= 5.091\ 98\ldots$
$= 5$ (to 1 s.f.)

(l) $\dfrac{163.4}{0.0818}$
$= 1997.555\ 012\ldots$
$= 2000$ (to 1 s.f.)

(m) $15.002 \div 0.019\ 99 - 68.12$
$= 682.355\ 237\ 6\ldots$
$= 700$ (to 1 s.f.)

(n) $\dfrac{59.26 \times 5.109}{3.817}$
$= \dfrac{302.759\ 34}{3.817}$
$= 79.318\ 663\ 87\ldots$
$= 80$ (to 1 s.f.)

(o) $\dfrac{4.18 \times 0.0309}{0.0212}$
$= \dfrac{0.129\ 162}{0.0212}$
$= 6.092\ 547\ 17$
$= 6$ (to 1 s.f.)

(p) $\dfrac{16.02 \times 0.0341}{0.079\ 21}$
$= \dfrac{0.546\ 282}{0.079\ 21}$
$= 6.896\ 629\ 213\ldots$
$= 7$ (to 1 s.f.)

(q) $\sqrt{\dfrac{35.807}{101.09}}$
$= \sqrt{0.354\ 209\ 12}$
$= 0.595\ 154\ 703\ldots$
$= 0.6$ (to 1 s.f.)

(r) $\sqrt{\dfrac{18.01 \times 36.01}{1.989}}$
$= \sqrt{\dfrac{648.5401}{1.989}}$
$= \sqrt{326.063\ 398\ 7}$
$= 18.057\ 225\ 66\ldots$
$= 20$ (to 1 s.f.)

19. $340 \div 21$
$\approx 340 \div 20$
$= 34 \div 2$
$= 17$
$\therefore$ Rui Feng's answer is wrong.
Using a calculator, the actual answer is 16.190 476 19.
Hence, his estimated value 15 is close to actual value 16.190 476 19.
He has underestimated the value by using the estimation $300 \div 20$.

20. **(i)** **(a)** $45.3125 = 45$ (to 2 s.f.)
(b) $3.9568 = 4.0$ (to 2 s.f.)

(ii) $45.3125 \div 3.9568$
$\approx 45 \div 4.0$
$= 11.25$

(iii) Using a calculator, the actual value is 11.451 804 49. The estimated value is close to the actual value. The estimated value is approximately 0.20 less than the actual value.

21. **(a)** $0.052\ 639\ 81 = 0.052\ 640$ (to 5 s.f.)

(b) $1793 \times 0.000\ 979$
$= 1.755\ 347$
$= 1.8$ (to 1 d.p.)

(c) $\dfrac{31.205 \times 4.97}{1.925}$
$= \dfrac{155.088\ 85}{1.925}$
$= 80.565\ 636\ 36\ldots$
$= 80$ (to 1 s.f.)

22. The calculation is $297 \div 19.91$.
$297 \div 19.91$
$\approx 300 \div 20$
$= 15$ (to 2 s.f.)
15 litres of petrol is used to travel 1 km.

23. Total cost of set meals = \$6.90 × 9
= \$7 × 9
= \$63

Ethan should pay less than \$63 for the set meals.

Therefore, he has paid the wrong amount.

Intermediate

24. **(a)** $(16.245 - 5.001)^3 \times \sqrt{122.05}$
= 15 704.76…
= 20 000 (to 1 s.f.)

(b) $\frac{6.01 \times 0.0312}{0.0622}$
= 3.014 66…
= 3 (to 1 s.f.)

(c) $\frac{29.12 \times 5.167}{1.895}$
= 79.400…
= 80 (to 1 s.f.)

(d) $\frac{41.41}{10.02 \times 0.018\,65}$
= 221.594 344 8
= 200 (to 1 s.f.)

(e) $\frac{\pi(8.5^2 - 7.5^2) \times 26}{169.8}$
= 7.6967…
= 8 (to 1 s.f.)

(f) $\frac{\sqrt{24.997} \times 28.0349}{19.897}$
= 7.044 58…
= 7 (to 1 s.f.)

(g) $\frac{2905 \times (0.512)^3}{0.004\,987}$
= 78 183.77…
= 80 000 (to 1 s.f.)

(h) $\frac{59.701 + 41.098}{\sqrt[3]{998.07}}$
= 10.086 393 09…
= 10 (to 1 s.f.)

(i) $\frac{4.311 - 2.9016}{\sqrt[3]{981} \times 0.0231}$
= 6.140 437 069…
= 6 (1 s.f.)

(j) $\frac{(20.315)^3 - \sqrt{82.0548}}{\sqrt[3]{85.002 - 21.997}}$
= 2104.695 751…
= 2000 (to 1 s.f.)

25. **(i)** $\frac{12.01 \times 4.8}{2.99}$
$\approx \frac{12 \times 4.8}{3.0}$
= 19.2
= 20 (to 1 s.f.)

(ii) $\frac{12.01 \times 0.048}{0.299}$
$\approx \frac{12 \times 4.8 \div 100}{3.0 \div 10}$
= 20 ÷ 10
= 2

26. **(a)** **(i)** 24.988 = 25 (to 2 s.f.)
(ii) 39.6817 = 40 (to 2 s.f.)
(iii) 198.97 = 200 (to 2 s.f.)

(b) $\frac{\sqrt{24.988} \times 39.6817}{198.97}$
$\approx \frac{\sqrt{25} \times 40}{200}$
$= \frac{5 \times 40}{200}$
= 1 (to 1 s.f.)

27. **(a)** $\frac{17.47 \times 6.87}{5.61 - 3.52}$
= 57.425 311
= 57.425 (to 5 s.f.)

(b) $\frac{1.743 \times 5.3 \times 2.9454}{(11.71)^2}$
= 0.198 428 362…
= 0.198 43 (to 5 s.f.)

(c) $7.593 - 6.219 \times \frac{1.47}{(1.4987)^3}$
= 4.877 225 103…
= 4.8772 (to 5 s.f.)

(d) $\frac{119.73 - 13.27 \times 4.711}{88.77 \div 66.158}$
= 42.640 891 68…
= 42.641 (to 5 s.f.)

(e) $\left(\frac{32.41 - 10.479}{7.218}\right) \times \left(\frac{4.7103 \times 21.483}{8.4691}\right)$
= 36.303 441 14…
= 36.303 (to 5 s.f.)

(f) $\frac{(0.629)^2 - \sqrt{7.318}}{2.873}$
= −0.803 877 207…
= −0.803 88 (to 5 s.f.)

(g) $\sqrt[3]{\frac{11.84 \times 0.871}{0.9542}}$
= 2.210 939 278…
= 2.2109 (to 5 s.f.)

(h) $\dfrac{7.295 - \sqrt{7.295}}{(7.295)^2} + \dfrac{(6.98)^3 - 6.98}{\sqrt[3]{6.98}}$

= 0.086 327 152 + 174.290 757 4

= 174.377 084 6…

= 174.38 (to 5 s.f.)

28. **(a)** **(i)** 271.569 = 270 (to 2 s.f.)

(ii) 9.9068 = 10 (to the nearest whole number)

(iii) 3.0198 = 3.0 (to 1 d.p.)

(b) $\dfrac{271.569 \times (9.9068)^2}{(3.0198)^3}$

$\approx \dfrac{270 \times (10)^2}{(3.0)^3}$

$= \dfrac{270 \times 100}{27}$

= 1000 (to 1 s.f.)

(c) $\dfrac{271.569 \times (9.9068)^2}{(3.0198)^3}$

= 967.859 777 4…

= 970 (to 2 s.f.)

(d) No, the answers are close but not the same.

The estimated value is 30 more than the actual value.

29. **(a)** Perimeter of the rectangular sheet of metal

= 2(9.96 + 5.08)

= 2(15.04)

= 30.08

= 30 m (to 1 s.f.)

(b) Area of rectangular sheet of metal

= 9.96 × 5.08

= 50.5968

= 50.6 m^2

30. **(a)** Smallest possible number of customers = 250

(b) Largest possible number of customers = 349

31. Total number of students that the school can accommodate

= 33 × 37

= 1221

= 1200 (to 2 s.f.)

The school can accommodate approximately 1200 students.

32. Number of pens bought

= 815 ÷ 85

= 9.588…

= 9 (to 1 s.f.)

The greatest number of pens that he can buy is 9.

33. **(i)** Thickness of each piece of paper

$= \dfrac{60 \div 10}{500}$

$= \dfrac{6}{500}$

= 0.012

= 0.01 cm (to 1 d.p.)

(ii) Thickness of a piece of paper

= 0.012 cm

= 0.000 12 m

= 0.0001 m (to 1 s.f.)

34. **(i)** Length of the carpet

$= \dfrac{11.9089}{4.04}$

= 2.947 747 525…

= 2.95 m (to 3 s.f.)

(ii) Perimeter of the carpet

≈ 2(2.9477 + 4.04)

= 2(6.9877)

= 13.9754

= 13.98 m (to 4 s.f.)

35. **(i)** 18 905 = 19 000 (to 2 s.f.)

(ii) Cost of each ticket

$= \dfrac{7\,000\,000}{19\,000}$

$= \dfrac{7\,000}{19}$

≈ 368.421 052 6

= $368 (to the nearest dollar)

36. **(a)** **(i)** Radius

= 497

= 500 mm (to 2 s.f)

Circumference of circle

= 2π(500)

= 1000π

= 3141.59…

= 3000 mm (to 1 s.f.)

(ii) Radius

= 5.12

= 5.1 m (to 2 s.f.)

Circumference of circle

= 2π(5.1)

= 10.2π

= 32.044…

= 30 m (to 1 s.f.)

(b) **(i)** Radius

= 10.09

= 10 m (to 2 s.f.)

Area of circle

= $\pi(10)^2$

= 100π

= 314.159…

= 300 m^2 (to 1 s.f.)

(ii) Radius
= 98.4
= 98 mm (to 2 s.f.)
Area of circle
= $\pi(98)^2$
= 9604π
= 30 171.855 …
= 30 000 mm^2 (to 1 s.f.)

37. Total value of 20-cent coins
= 31×0.2
= \$6.20
Total value of 5-cent coins
= \$7.35 – \$6.20
= \$1.15
Number of 5-cent coins
$= \frac{1.15}{0.05}$
$= \frac{1.2}{0.05}$ (to 2 s.f.)
$= \frac{120}{5}$
= 24
There are about 24 5-cent coins in the box.

38. Total amount that Lixin has to pay
$= 18 \times (0.99 \div 3) + 1.2 \times 1.5 + 2 \times 0.81 + 2.2 \times 3.4$
$= 18 \times 0.33 + 1.2 \times 1.5 + 2 \times 0.8 + 2.2 \times 3.4$
$= 5.94 + 1.8 + 1.6 + 7.48$
= \$16.84
The total amount she has to pay, to the nearest dollar, is \$17.

39. KRW 900 ≈ S\$1
Price of a shirt in KRW = KRW 27 800
≈ KRW 27 900
Price of shirt in S\$ $= \frac{27\,900}{900}$
$= \frac{279}{9}$
= S\$31

40. For Airline *A*,
cost = $0.8 \times$ \$88.020
= $0.8 \times$ \$90
= \$72
For Airline *B*,
cost = \$93 – \$35
= \$58
For Airline *C*,
cost = $0.9 \times$ \$75
= \$67.50
∴ Airline *B*'s offer is the best.

41. For option *A*,
700 m*l* costs about \$4.00.
For option *B*,
1400 m*l* costs \$8.90.
Thus 700 m*l* will cost about $(8.90 \div 2)$ = \$4.45
For option *C*,
950 m*l* costs \$9.90.
Thus 700 m*l* will cost about $(9.90 \div 950) \times 700$
≈ \$7.00
∴ Option *A* is better value for money.

Advanced

42. **(a)** 406 *A*45 when correct to 3 significant figures is 406 000, so $A < 5$.
∴ The maximum prime value of *A* is 3.
(b) 398 200 is the estimated value for
398 150 to 398 199, if corrected to 4 significant figures;
398 195 to 398 204, if corrected to 5 significant figures;
398 200.1 to 398 200.4, if corrected to 6 significant figures.
∴ m = 4, 5 or 6

43. 2000 is the estimated value for 1999 to 2004, if corrected to 1, 2 and 3 significant figures.
∴ The smallest number is 1999 and the largest number is 2004.

44. Rp 7872.5300 = S\$1
Rp 8000 ≈ S\$1
Price of cup noodle in Rp
= Rp 27 800
≈ Rp 28 000
Price of cup noodle in S\$ = S\$ $\frac{28\,000}{8000}$
= S\$3.50
The cup noodle costs S\$3.50.

45. $$\frac{\sqrt{16\,500.07 \times 39.59 - \left(119\,999.999 + \frac{485\,200.023}{(2.6)^2}\right)}}{\sqrt[3]{1.02 \times (13.5874 + 19.0007)^2 - 99.998}}$$

$$\approx \frac{\sqrt{17\,000 \times 40 - \left(120\,000 + \frac{490\,000}{(2.6)^2}\right)}}{\sqrt[3]{1.0 \times (14 + 20)^2 - 100}}$$

$$= \frac{\sqrt{17\,000 \times 40 - \left(120\,000 + \frac{490\,000}{6.76}\right)}}{\sqrt[3]{989}}$$

$$\approx \frac{\sqrt{680\,000 - \left(120\,000 + \frac{490\,000}{7}\right)}}{\sqrt[3]{1000}}$$

(Note: 6.76 and 989 are estimated so that the division and cube root can be carried out, without the use of calculator)

$$= \frac{\sqrt{680\,000 - 190\,000}}{10}$$

$$= \frac{\sqrt{490\,000}}{10}$$

$$= \frac{700}{10}$$

= 70 (to 1 s.f.)

New Trend

46. (a) $\frac{16.8^5}{3(7.1) - 1.55} \approx 67\,760$

(b) 67 760 = 67 800 (to 3 s.f.)

47. (a) $\frac{(0.984\,52)^3 \times \sqrt{2525}}{102.016}$

$\approx \frac{(1.0)^3 \times \sqrt{2500}}{100}$

= 0.5 (to 1 s.f.)

(b) $\frac{(0.984\,52)^3 \times \sqrt{2525}}{102.016}$

= 0.470 041 311

= 0.47 (to 2 s.f.)

48. $\sqrt[3]{\frac{(1.92)^2}{(4.3)^3 - \sqrt{4.788}}}$

= 0.362 609 371

= 0.362 61 (to 5 s.f.)

49. (a) 8.5 kg

(b) Greatest possible mass of 1 m^3 of wood

$= \frac{9.5}{2.5}$

= 3.8 kg

Chapter 4 Basic Algebra and Algebraic Manipulation

Basic

1. (a) $(2x + 5y) - 4 = 2x + 5y - 4$
   (b) $(3x)(7y) + 9z = 21xy + 9z$
   (c) $(7x)(11y) \times 2z = 77xy \times 2z$
   $= 154xyz$
   (d) $(3z + 7s) \div 5a = \dfrac{3z + 7s}{5a}$
   (e) $r^3 - (p \div 3q) = r^3 - \dfrac{p}{3q}$
   (f) $3w \div (3x + 7y) = \dfrac{3w}{3x + 7y}$
   (g) $(k \div 2y) - 9(x)(3h) = \dfrac{k}{2y} - 27xh$

2. (a) $7b - 3c + 4a$
   $= 7(2) - (3)(-1) + 4(3)$
   $= 14 + 3 + 12$
   $= 29$
   (b) $3a^3$
   $= 3(3)^3$
   $= 3(27)$
   $= 81$
   (c) $(5b)^2$
   $= (5 \times 2)^2$
   $= (10)^2$
   $= 100$
   (d) $(2a + b + c)(5b - 3a)$
   $= (2 \times 3 + 2 + (-1))(5 \times 2 - 3 \times 3)$
   $= (7)(1)$
   $= 7$
   (e) $(a - b)^2 - (b - c)^2$
   $= (3 - 2)^2 - (2 - (-1))^2$
   $= 1^2 - (3)^2$
   $= -8$
   (f) $2a^2 - 3b^2 + 3abc$
   $= 2(3)^2 - 3(2)^2 + 3(3)(2)(-1)$
   $= 18 - 12 - 18$
   $= -12$
   (g) $(a + 3b)^3$
   $= (3 + 3(2))^3$
   $= 9^3$
   $= 729$
   (h) $a^b - c^a + b^c$
   $= (3)^2 - (-1)^3 + (2)^{(-1)}$
   $= 9 + 1 + \dfrac{1}{2}$
   $= 10\dfrac{1}{2}$
   (i) $\dfrac{a}{b} - \dfrac{b}{c}$
   $= \dfrac{3}{2} - \dfrac{2}{-1}$
   $= 1\dfrac{1}{2} + 2$
   $= 3\dfrac{1}{2}$
   (j) $\dfrac{8b - (3a)^2}{c}$
   $= \dfrac{8(2) - (3 \times 3)^2}{(-1)}$
   $= \dfrac{16 - 9^2}{-1}$
   $= \dfrac{16 - 81}{-1}$
   $= 65$
   (k) $\dfrac{b + c}{a} + \dfrac{a + bc}{b}$
   $= \dfrac{2 + (-1)}{3} + \dfrac{3 + (2)(-1)}{2}$
   $= \dfrac{1}{3} + \dfrac{1}{2}$
   $= \dfrac{5}{6}$
   (l) $\dfrac{a^2 - b^2}{c^2} - \dfrac{a^3 - c}{c - 3b}$
   $= \dfrac{3^2 - 2^2}{(-1)^2} - \dfrac{3^3 - (-1)}{(-1) - 3(2)}$
   $= \dfrac{5}{1} - \dfrac{28}{-7}$
   $= 5 + 4$
   $= 9$

3. (a) $3x + 9y + (-11y)$
   $= 3x + 9y - 11y$
   $= 3x - 2y$
   (b) $-a - 3b + 7a - 10b$
   $= 7a - a - 3b - 10b$
   $= 6a - 13b$
   (c) $13d + 5c + (-13c + 5d)$
   $= 13d + 5c - 13c + 5d$
   $= 13d + 5d + 5c - 13c$
   $= 18d - 8c$
   $= -8c + 18d$
   (d) $7pq - 11hk + (-3pq - 21kh)$
   $= 7pq - 11hk - 3pq - 21kh$
   $= 4pq - 32hk$

4. (a) $5x + 7y - 2x - 4y$
   $= 5x - 2x + 7y - 4y$
   $= 3x + 3y$

(b) $-3a - 7b + 11a + 11b$
$= -3a + 11a - 7b + 11b$
$= 11a - 3a + 11b - 7b$
$= 8a + 4b$

(c) $5u - 7v - 7u - 9v$
$= 5u - 7u - 7v - 9v$
$= -2u - 16v$

(d) $5p + 4q - 7r - 5q + 4p$
$= 5p + 4p + 4q - 5q - 7r$
$= 9p - q - 7r$

(e) $5pq - 7qp + 21 - 7$
$= -2pq + 14$

(f) $15x + 9y + 5x - 3y - 13$
$= 15x + 5x + 9y - 3y - 13$
$= 20x + 6y - 13$

(g) $8ab - 5bc + 21ba - 7cb$
$= 8ab + 21ab - 5bc - 7cb$
$= 29ab - 12bc$

(h) $-7 + mn + 9mn - 3mn - 25$
$= mn + 9mn - 3mn - 25 - 7$
$= 7mn - 32$

(i) $3h - 4gh + \frac{2}{3}h - \frac{1}{3}gh$
$= 3h + \frac{2}{3}h - 4gh - \frac{1}{3}gh$
$= 3\frac{2}{3}h - 4\frac{1}{3}gh$

(j) $\frac{3}{5}x - \frac{2}{3}xy + \frac{1}{4}x - \frac{1}{5}xy$
$= \frac{3}{5}x + \frac{1}{4}x - \frac{2}{3}xy - \frac{1}{5}xy$
$= \frac{17}{20}x - \frac{13}{15}xy$

(k) $2.5p - 3.6q + 1.1p - 6.3q$
$= 2.5p + 1.1p - 3.6q - 6.3q$
$= 3.6p - 9.9q$

(l) $-0.5a - 0.65b + 0.375a - 0.258b$
$= -0.5a + 0.375a - 0.65b - 0.258b$
$= -0.125a - 0.908b$

5. (a) $3(3x - 5)$
$= 9x - 15$

(b) $7(5 - 7x)$
$= 35 - 49x$

(c) $11(4x + 5y)$
$= 44x + 55y$

(d) $-3(9k - 2)$
$= -27k + 6$

(e) $-7(-3h - 5)$
$= 21h + 35$

(f) $4(3a - 2b + c)$
$= 12a - 8b + 4c$

(g) $-5\left(\frac{1}{4}p - \frac{2}{5}q + \frac{1}{2}r\right)$
$= -\frac{5}{4}p + 2q - \frac{5}{2}r$

(h) $-\frac{1}{4}(8a - 5b + 3c)$
$= -2a + \frac{5}{4}b - \frac{3}{4}c$

6. (a) $5a - 3(2p + 3)$
$= 5a - 6p - 9$

(b) $3x - 5(x - y)$
$= 3x - 5x + 5y$
$= -2x + 5y$

(c) $5(a + 4) + 7(b - 2)$
$= 5a + 20 + 7b - 14$
$= 5a + 7b + 20 - 14$
$= 5a + 7b + 6$

(d) $3(2p - 3q) - 5(3p - 5q)$
$= 6p - 9q - 15p + 25q$
$= 6p - 15p + 25q - 9q$
$= -9p + 16q$

(e) $r(3x - y) - 3r(x - 7y)$
$= 3xr - ry - 3xr + 21ry$
$= 3xr - 3xr + 21ry - ry$
$= 20ry$

(f) $3(x + y + z) + 5y - 4z$
$= 3x + 3y + 3z + 5y - 4z$
$= 3x + 3y + 5y + 3z - 4z$
$= 3x + 8y - z$

7. In 4 years' time,
Rui Feng will be $(x + 4)$ years old.
$\therefore$ His brother will be $3(x + 4)$ years old.

8. Let the largest odd integer be x.
Then the previous odd integer will be $(x - 2)$.
The smallest odd integer is $(x - 2) - 2 = x - 4$.
Sum of three consecutive odd integers
$= x + (x - 2) + (x - 4)$
$= x + x - 2 + x - 4$
$= x + x + x - 2 - 4$
$= 3x - 6$

9. (a) $\frac{1}{3}x + \frac{1}{5}y - \frac{1}{9}x - \frac{1}{15}y$
$= \frac{1}{3}x - \frac{1}{9}x + \frac{1}{5}y - \frac{1}{15}y$
$= \frac{3}{9}x - \frac{1}{9}x + \frac{3}{15}y - \frac{1}{15}y$
$= \frac{2}{9}y + \frac{2}{15}y$

(b) $\frac{3}{4}a - \frac{1}{5}b + 3a - \frac{4}{7}b$

$= \frac{3}{4}a + 3a - \frac{4}{7}b - \frac{1}{5}b$

$= 3\frac{3}{4}a - \frac{20}{35}b - \frac{7}{35}b$

$= 3\frac{3}{4}a - \frac{27}{35}b$

(c) $\frac{5}{6}c + \frac{8}{7}d - \frac{2}{9}c - \frac{5}{3}d$

$= \frac{5}{6}c - \frac{2}{9}c + \frac{8}{7}d - \frac{5}{3}d$

$= \frac{15}{18}c - \frac{4}{18}c + \frac{24}{21}d - \frac{35}{21}d$

$= \frac{11}{18}c - \frac{11}{21}d$

(d) $5f - \frac{5}{7}h + \frac{7}{8}k - \frac{4}{3}f - \frac{4}{5}h + \frac{12}{11}k$

$= 5f - \frac{4}{3}f - \frac{5}{7}h - \frac{4}{5}h + \frac{12}{11}k + \frac{7}{8}k$

$= 3\frac{2}{3}f - \frac{25}{35}h - \frac{28}{35}h + \frac{96}{88}k + \frac{77}{88}k$

$= 3\frac{2}{3}f - 1\frac{18}{35}h + 1\frac{85}{88}k$

10. Amount of money spent on buying apples

$= 10 \times \$\frac{x}{4}$

$= \$\frac{5}{2}x$

Amount of money spent on buying bananas

$= \$1.25 \times m$

$= \$1.25m$

Amount of money spent on buying oranges

$= \$\frac{3}{4} \times (3n + 1)$

$= \$\frac{3(3n+1)}{4}$

Total money spent $= \$\frac{5}{2}x + \$1.25m + \$\frac{3(3n+1)}{4}$

$= \$\left(\frac{5x}{2} + 1.25m + \frac{3(3n+1)}{4}\right)$

11. (a) $5a + 3b - 2c + \left(3\frac{1}{2}a + 2\frac{1}{2}b - 3\frac{1}{2}c\right)$

$= 5a + 3b - 2c + 3\frac{1}{2}a + 2\frac{1}{2}b - 3\frac{1}{2}c$

$= 5a + 3\frac{1}{2}a + 3b + 2\frac{1}{2}b - 2c - 3\frac{1}{2}c$

$= 8\frac{1}{2}a + 5\frac{1}{2}b - 5\frac{1}{2}c$

(b) $\frac{1}{2}[5y - 2(x - 3y)]$

$= \frac{5}{2}y - (x - 3y)$

$= \frac{5}{2}y - x + 3y$

$= \frac{5}{2}y + 3y - x$

$= \frac{11}{2}y - x$

(c) $\frac{3}{4}[8q - 7p - 3(p - 2q)]$

$= \frac{3}{4}[8q - 7p - 3p + 6q)]$

$= \frac{3}{4}[-7p - 3p + 6q + 8q]$

$= \frac{3}{4}[-10p + 14q]$

$= -\frac{30}{4}p + \frac{42}{4}q$

$= \frac{21q - 15p}{2}$

(d) $\frac{3}{10}[3(5a - b) - 7(2a - 5b)]$

$= \frac{3}{10}[15a - 3b - 14a + 35b]$

$= \frac{3}{10}[15a - 14a + 35b - 3b]$

$= \frac{3}{10}[a + 32b]$

$= \frac{3}{10}a + \frac{96}{10}b$

$= \frac{3}{10}(a + 32b)$

12. (a) $\frac{2(5x-1)}{3} - \frac{x-3}{5}$

$= \frac{2(5x-1) \times 5}{3 \times 5} - \frac{(x-3) \times 3}{5 \times 3}$

$= \frac{50x - 10}{15} - \frac{(3x - 9)}{15}$

$= \frac{50x - 10 - 3x + 9}{15}$

$= \frac{50x - 3x + 9 - 10}{15}$

$= \frac{47x - 1}{15}$

(b) $\frac{x}{2}+\frac{x-3}{5}-\frac{x-4}{4}$

$=\frac{5x}{10}+\frac{2(x-3)}{10}-\frac{x-4}{4}$

$=\frac{5x+2x-6}{10}-\frac{(x-4)}{4}$

$=\frac{7x-6}{10}-\frac{(x-4)}{4}$

$=\frac{(7x-6)\times 2}{10\times 2}-\frac{(x-4)\times 5}{4\times 5}$

$=\frac{2(7x-6)}{20}-\frac{5(x-4)}{20}$

$=\frac{14x-12-5x+20}{20}$

$=\frac{9x+8}{20}$

(c) $\frac{x+5}{3}-\frac{2x-7}{6}+\frac{x}{2}$

$=\frac{2(x+5)}{6}-\frac{(2x-7)}{6}+\frac{x}{2}$

$=\frac{2x+10-2x+7}{6}+\frac{x}{2}$

$=\frac{17}{6}+\frac{3x}{6}$

$=\frac{3x+17}{6}$

(d) $\frac{3x-7}{2}-\frac{x+4}{5}-\frac{3}{4}$

$=\frac{5(3x-7)}{10}-\frac{2(x+4)}{10}-\frac{3}{4}$

$=\frac{15x-35-2x-8}{10}-\frac{3}{4}$

$=\frac{15x-2x-8-35}{10}-\frac{3}{4}$

$=\frac{13x-43}{10}-\frac{3}{4}$

$=\frac{2(13x-43)}{20}-\frac{15}{20}$

$=\frac{2(13x-43)-15}{20}$

$=\frac{26x-86-15}{20}$

$=\frac{26x-101}{20}$

(e) $\frac{x+y}{3}-\frac{2}{5}-\frac{3x-2y}{6}$

$=\frac{5(x+y)}{15}-\frac{6}{15}-\frac{3x-2y}{6}$

$=\frac{5x+5y-6}{15}-\frac{3x-2y}{6}$

$=\frac{2(5x+5y-6)}{30}-\frac{5(3x-2y)}{30}$

$=\frac{10x+10y-12-15x+10y}{30}$

$=\frac{10x-15x+10y+10y-12}{30}$

$=\frac{-5x+20y-12}{30}$

13. (a) $15x-3=3(5x-1)$

(b) $-21y-48=-3(7y+16)$

(c) $64b-27bc=b(64-27c)$

(d) $18ax+6a-36az=6a(3x+1-6z)$

(e) $14p-56pq-42pr$

$=7p(2-8q-6r)$

$=14p(1-4q-3r)$

Intermediate

14. (a)

$k+8$	Add 8 to a number k
$5(k+8)$	Multiply the sum by 5
$5(k+8)-(2k-1)$	Subtract $(2k-1)$ from the result
$=5k+40-2k+1$	
$=5k-2k+40+1$	
$=3k+41$	

(b) Cost of 7 pencils

$=7\times p$

$=7p$ cents

Change after buying the pencils

$=\$4.20$

$=420$ cents

Amount Kate had before buying the pencils

$=(420+7p)$ cents

(c) Cost price of the apples

$=x(y+3)$ cents

Selling price of the apples

$=x(2y-5)$ cents

Profit

= selling price – cost price

$=x(2y-5)-x(y+3)$

$=2xy-5x-xy-3x$

$=2xy-xy-5x-3x$

$=(xy-8x)$ cents

(d) Cost price of the microchips

$= (n)(2x)$

$= \$2nx$

Selling price of the microchips

$= (n)(n - x)$

$= \$n(n - x)$

Loss

= cost price – selling price

$= 2nx - n(n - x)$

$= 2nx - n^2 + nx$

$= \$(3nx - n^2)$

15. (a) When $a = 4$, $m = -2$ and $n = -1$,

$4(-2)^2 - 3(4) - 5(-1)$

$= 16 - 12 + 5$

$= 4 + 5$

$= 9$

(b) When $a = 4$, $m = -2$ and $n = -1$,

$7(-1) + 3\frac{3}{4}(4) - (-2 - 4)$

$= -7 + 15 - (-6)$

$= 8 + 6$

$= 14$

16. (a) When $a = 2$, $c = -1$, $d = 5$ and $e = -4$,

$(2) - (-1)(5 - (-4))$

$= 2 + (5 + 4)$

$= 2 + 5 + 4$

$= 11$

(b) When $a = 2$, $c = -1$, $d = 5$ and $e = -4$,

$\frac{2(-4) - 2}{(-1)^2 - 5(-4)}$

$= \frac{-8 - 2}{1 + 20}$

$= \frac{10}{21}$

17. When $a = 2$, $b = -1$, $c = 0$ and $d = \frac{1}{2}$

(a) $(2a - b)^2$

$= (2 \times 2 - (-1))^2$

$= (4 + 1)^2$

$= 5^2$

$= 25$

(b) $(3a - b)(2c + d)$

$= [3(2) - (-1)]\left[2(0) + \frac{1}{2}\right]$

$= (6 + 1)\left(\frac{1}{2}\right)$

$= \frac{7}{2}$

$= 3\frac{1}{2}$

(c) $(5a - b)(2c + d) - b(ab + bc - 4cd)$

$[5(2) - (-1)]\left[2(0) + \frac{1}{2}\right]$

$- (-1)\left[(2)(-1) + (-1)(0) - 4(0)\left(\frac{1}{2}\right)\right]$

$= (10 + 1)\left(\frac{1}{2}\right) + (-2)$

$= 3\frac{1}{2}$

18. When $x = -3$,

$(2x - 1)(2x + 1)(2x + 3)$

$= (2(-3) - 1)(2(-3) + 1)(2(-3) + 3)$

$= (-6 - 1)(-6 + 1)(-6 + 3)$

$= (-7)(-5)(-3)$

$= -105$

19. When $x = -2$,

$\frac{(-2) + 1}{(-2) - 1} + \frac{2(-2) - 1}{2(-2) + 1}$

$= \frac{-1}{-3} + \left(\frac{-5}{-3}\right)$

$= \frac{1}{3} + \frac{5}{3}$

$= 2$

20. When $x = -2$,

$\frac{(-2) - 5}{(-2) + 7} - 3(-2)^2$

$= \frac{-7}{5} - 12$

$= -13\frac{2}{5}$

21. When $x = 2$ and $y = -1$,

$(2)^3 + 2(2)(-1)^2 + (-1)^3$

$= 8 + 4 - 1$

$= 11$

22. When $a = -2$, $b = 3$ and $c = -5$,

$\frac{3(-2)^2(3)(-5)}{2(3) - 3(-5)} - \frac{(3)(-5)}{(-2)}$

$= \frac{3(4)(3)(-5)}{6 - (-15)} - \left(\frac{-15}{-2}\right)$

$= \frac{-180}{21} - \frac{15}{2}$

$= -16\frac{1}{14}$

23. When $y = -3$ and $z = -1\frac{1}{2}$,

$5x = (-3)^2 - \dfrac{(-3)^3}{\left(-1\frac{1}{2}\right)}$

$5x = 9 - \left[-27 \div \left(-\frac{3}{2}\right)\right]$

$5x = 9 - \left[-27 \times \left(-\frac{2}{3}\right)\right]$

$5x = 9 - 18$

$5x = -9$

$\therefore x = \frac{-9}{5} = -1\frac{4}{5}$

24. When $y = -3$,

$\dfrac{x + 5(-3)}{5x - 7(-3)} = \dfrac{1}{4}$

$\dfrac{x - 15}{5x + 21} = \dfrac{1}{4}$

$4(x - 15) = 5x + 21$

$4x - 60 = 5x + 21$

$5x - 4x = -60 - 21$

$x = -81$

25. **(a)** $a + b + c + (2b - c) + (3c + a)$

$= a + b + c + 2b - c + 3c + a$

$= a + a + b + 2b + c - c + 3c$

$= 2a + 3b + 3c$

(b) $2ab + 3bc + (5ac - 5ba) + (2cb + 5ab)$

$= 2ab + 3bc + 5ac - 5ba + 2cb + 5ab$

$= 2ab + 5ab - 5ba + 3bc + 2cb + 5ac$

$= 2ab + 5ab - 5ab + 3bc + 2bc + 5ac$

$= 2ab + 5bc + 5ac$

(c) $\frac{1}{2}xy + \left(\frac{1}{3}xz - \frac{1}{4}yx\right) + \left(\frac{1}{6}xz + xy\right)$

$= \frac{1}{2}xy + \frac{1}{3}xz - \frac{1}{4}yx + \frac{1}{6}xz + xy$

$= 1\frac{1}{4}xy + \frac{1}{2}xz$

(d) $a + b - c + (2c - b + a) + (5a + 7c)$

$= a + b - c + 2c - b + a + 5a + 7c$

$= a + a + 5a + b - b - c + 2c + 7c$

$= 7a + 8c$

(e) $5abc - 7cb + 4ac + (4cba - 4bc + 3ca)$

$= 5abc - 7cb + 4ac + 4cba - 4bc + 3ca$

$= 5abc + 4cba + 4ac + 3ca - 7cb - 4bc$

$= 5abc + 4abc + 4ac + 3ac - 7bc - 4bc$

$= 9abc + 7ac - 11bc$

26. **(a)** $5(2x - 7y) - 4(y - 3x)$

$= 10x - 35y - 4y + 12x$

$= 10x + 12x - 35y - 4y$

$= 22x - 39y$

(b) $3a + 5ac - 2c - 4c - 6a - 8ca$

$= 3a - 6a + 5ac - 8ca - 2c - 4c$

$= -3a - 3ac - 6c$

(c) $5p + 3q - 4r - (6q - 3p + r)$

$= 5p + 3q - 4r - 6q + 3p - r$

$= 5p + 3p + 3q - 6q - 4r - r$

$= 8p - 3q - 5r$

(d) $3b + 5a - 2(a - 2b)$

$= 3b + 5a - 2a + 4b$

$= 3a + 7b$

(e) $2(z - 5x) - 7(y + z - 1)$

$= 2z - 10x - 7y - 7z + 7$

$= -10x - 7y + 2z - 7z + 7$

$= -10x - 7y - 5z + 7$

(f) $7m - 2[6m - (3m - 4p)]$

$= 7m - 2[6m - 3m + 4p]$

$= 7m - 12m + 6m - 8p$

$= m - 8p$

(g) $7x - \{3x - [4x - 2(x + 3y)]\}$

$= 7x - \{3x - [4x - 2x - 6y]\}$

$= 7x - \{3x - [2x - 6y]\}$

$= 7x - \{3x - 2x + 6y\}$

$= 7x - \{x + 6y\}$

$= 7x - x - 6y$

$= 6x - 6y$

(h) $8a - \{2a - [3c - 6(a - 2c)]\}$

$= 8a - \{2a - [3c - 6a + 12c]\}$

$= 8a - \{2a - [3c + 12c - 6a]\}$

$= 8a - \{2a - [15c - 6a]\}$

$= 8a - \{2a - 15c + 6a\}$

$= 8a - \{2a + 6a - 15c\}$

$= 8a - \{8a - 15c\}$

$= 8a - 8a + 15c$

$= 15c$

(i) $12a - 3\{a - 4[c - 5(a - c)]\}$

$= 12a - 3\{a - 4[c - 5a + 5c]\}$

$= 12a - 3\{a - 4[c + 5c - 5a]\}$

$= 12a - 3\{a - 4[6c - 5a]\}$

$= 12a - 3\{a - 24c + 20a\}$

$= 12a - 3\{a + 20a - 24c\}$

$= 12a - 3\{21a - 24c\}$

$= 12a - 63a + 72c$

$= 72c - 51a$

(j) $7m - 4n - 5(m - 3n) + 4(n - 5)$
$= 7m - 4n - 5m + 15n + 4n - 20$
$= 7m - 5m - 4n + 15n + 4n - 20$
$= 2m + 15n - 20$

(k) $2a - 5(3ab - 4b) - 2(a - 2ba)$
$= 2a - 15ab + 20b - 2a + 4ab$
$= 2a - 2a - 15ab + 4ab + 20b$
$= -11ab + 20b$
$= 20b - 11ab$

(l) $4(x - 5y) - 5(2y - 3x) - (2x - 5y)$
$= 4x - 20y - 10y + 15x - 2x + 5y$
$= 4x + 15x - 2x - 20y - 10y + 5y$
$= 17x - 25y$

(m) $2(3x + y) - 5[3(x - 3y) - 4(2x - y)]$
$= 2(3x + y) - 5[3x - 9y - 8x + 4y]$
$= 2(3x + y) - 5[3x - 8x - 9y + 4y]$
$= 2(3x + y) - 5[- 5x - 5y]$
$= 6x + 2y + 25x + 25y$
$= 6x + 25x + 2y + 25y$
$= 31x + 27y$

(n) $\frac{1}{2}\left[14x - \frac{2}{3}(9x - 21y) - 2(x + y)\right]$
$= \frac{1}{2}[14x - 6x + 14y - 2x - 2y]$
$= \frac{1}{2}[14x - 6x - 2x + 14y - 2y]$
$= \frac{1}{2}[6x + 12y]$
$= 3x + 6y$

27. (a) $3a - 2b - 11 - (10a + 5b - 7)$
$= 3a - 2b - 11 - 10a - 5b + 7$
$= 3a - 10a - 2b - 5b - 11 + 7$
$= -7a - 7b - 4$

(b) $4x - 2z + 7 - (x - 3y - 5z + 5)$
$= 4x - 2z + 7 - x + 3y + 5z - 5$
$= 4x - x + 3y - 2z + 5z + 7 - 5$
$= 3x + 3y + 3z + 2$

(c) $4p + 2q - 5r - 1 - (7p - q + 3r + 3)$
$= 4p + 2q - 5r - 1 - 7p + q - 3r - 3$
$= 4p - 7p + 2q + q - 5r - 3r - 1 - 3$
$= -3p + 3q - 8r - 4$

(d) $6(2 + 3n + 5m) - 4m(n + 5) - [2(3m - 5n) + 5mn]$
$= 12 + 18n + 30m - 4mn - 20m - (6m - 10n + 5mn)$
$= 12 + 18n + 30m - 20m - 4mn - 6m + 10n - 5mn$
$= 12 + 18n + 10n + 30m - 20m - 6m - 4mn - 5mn$
$= 12 + 28n + 4m - 9mn$

28. (i) Let the second number be n.
Then the first number is $n - 2$.
Then the third number is $n + 2$.
Lastly, the fourth number is $(n + 2) + 2 = n + 4$.

(ii) Sum of the four numbers
$= n - 2 + n + n + 2 + n + 4$
$= n + n + n + n - 2 + 2 + 4$
$= 4n + 4$

29. Perimeter of Figure 1
$= 7y + 3x + 7y + 3x$
$= 7y + 7y + 3x + 3x$
$= 14y + 6x$
$= (6x + 14y)$ cm
Perimeter of Figure 2
$= 5x + (x + 5y) + 7y$
$= 5x + x + 5y + 7y$
$= (6x + 12y)$ cm
$\therefore$ Since $(6x + 14y) > (6x + 12y)$, Figure 1 has a larger perimeter.

30. (i) Amount of money spent on the fruits
$= (120h + 180k)$ cents

(ii) Number of bags in which each bag contains 2 apples and 3 oranges
$= 120 \div 2$
$= 60$
Total amount of money for which he sold all bags of fruits
$= [60(3h + 4k)]$
$= (180h + 240k)$ cents

(iii) Amount earned from selling the fruits
$= [60(3h + 4k)] - (120h + 180k)$
$= 180h + 240k - 120h - 180k$
$= 180h - 120h + 240k - 180k$
$= (60h + 60k)$ cents or $\$(0.6h + 0.6k)$

31. (i) Number of 50-cent coins Shirley has
$= n - x - 3x$
$= n - 4x$

(ii) Since the number of 10-cent coins is x, then the number of 50-cent coins is $\frac{1}{4}x$.
Total value of all the coins
$= 10x + 20(3x) + 50\left(\frac{1}{4}x\right)$
$= 10x + 60x + \frac{50}{4}x$
$= 82\frac{1}{2}x$ cents

(iii) Ratio of number of 20-cent coins to 50-cent coins

= 5 : 3

5 parts is $3x$.

1 part is $\frac{3x}{5}$.

3 parts is $\frac{3x}{5} \times 3 = \frac{9x}{5}$.

Total value of all the coins

$= 10x + 20(3x) + 50\left(\frac{9x}{5}\right)$

$= 10x + 60x + 90x$

$= 160x$ cents

32. (a) $\frac{3(x-2)}{3} + \frac{2(x+3)}{4}$

$= \frac{12(x-2)}{12} + \frac{6(x+3)}{12}$

$= \frac{12(x-2)+6(x+3)}{12}$

$= \frac{12x-24+6x+18}{12}$

$= \frac{18x-6}{12}$

$= \frac{3x-1}{2}$

(b) $\frac{5(3x+1)}{4} - \frac{7(5x-3)}{12}$

$= \frac{15(3x+1)}{12} - \frac{7(5x-3)}{12}$

$= \frac{45x+15-35x+21}{12}$

$= \frac{45x-35x+15+21}{12}$

$= \frac{10x+36}{12}$

$= \frac{5x+18}{6}$

(c) $1 + \frac{2x+1}{3} + \frac{4(x-3)}{6}$

$= \frac{3}{3} + \frac{2x+1}{3} + \frac{2(x-3)}{3}$

$= \frac{3+2x+1+2(x-3)}{3}$

$= \frac{3+2x+1+2x-6}{3}$

$= \frac{2x+2x+1+3-6}{3}$

$= \frac{4x-2}{3}$

(d) $\frac{3x-4y}{6} + \frac{x-2y}{4} - \frac{x+y}{5}$

$= \frac{2(3x-4y)}{12} + \frac{3(x-2y)}{12} - \frac{x+y}{5}$

$= \frac{6x-8y+3x-6y}{12} - \frac{x+y}{5}$

$= \frac{5(9x-14y)}{60} - \frac{12(x+y)}{60}$

$= \frac{5(9x-14y)-12(x+y)}{60}$

$= \frac{45x-70y-12x-12y}{60}$

$= \frac{33x-82y}{60}$

(e) $\frac{2x-5}{3} - \frac{x+4}{6} + \frac{3(5-x)}{9}$

$= \frac{2(2x-5)}{6} - \frac{x+4}{6} + \frac{3(5-x)}{9}$

$= \frac{2(2x-5)-(x+4)}{6} + \frac{3(5-x)}{9}$

$= \frac{4x-10-x-4}{6} + \frac{3(5-x)}{9}$

$= \frac{4x-x-10-4}{6} + \frac{3(5-x)}{9}$

$= \frac{3x-14}{6} + \frac{3(5-x)}{9}$

$= \frac{3(3x-14)}{18} + \frac{6(5-x)}{18}$

$= \frac{3(3x-14)+6(5-x)}{18}$

$= \frac{9x-42+30-6x}{18}$

$= \frac{9x-6x-42+30}{18}$

$= \frac{3x-12}{18}$

$= \frac{x-4}{6}$

(f) $\frac{4(3x+4)}{10} - \frac{x+7}{15} - \frac{2x-1}{5}$

$= \frac{12(3x+4)}{30} - \frac{2(x+7)}{30} - \frac{2x-1}{5}$

$= \frac{12(3x+4)-2(x+7)}{30} - \frac{2x-1}{5}$

$= \frac{36x+48-2x-14}{30} - \frac{2x-1}{5}$

$= \frac{36x-2x+48-14}{30} - \frac{6(2x-1)}{30}$

$= \frac{34x+34}{30} - \frac{6(2x-1)}{30}$

$= \frac{34x+34-12x+6}{30}$

$= \frac{34x-12x+34+6}{30}$

$= \frac{22x+40}{30}$

$= \frac{11x+20}{15}$

(g) $-1 - \frac{3(x+7)}{7} - \frac{4(2x-1)}{5}$

$= \frac{-7}{7} - \frac{3(x+7)}{7} - \frac{4(2x-1)}{5}$

$= \frac{-7-3(x+7)}{7} - \frac{4(2x-1)}{5}$

$= \frac{-7-3x-21}{7} - \frac{4(2x-1)}{5}$

$= \frac{-3x-28}{7} - \frac{4(2x-1)}{5}$

$= \frac{5(-3x-28)}{35} - \frac{28(2x-1)}{35}$

$= \frac{5(-3x-28)-28(2x-1)}{35}$

$= \frac{-15x-140-56x+28}{35}$

$= \frac{-15x-56x-140+28}{35}$

$= \frac{-71x-112}{35}$

(h) $\frac{3x-7}{4} - (x-5) - \frac{x-1}{3}$

$= \frac{3x-7}{4} - \frac{x-1}{3} - (x-5)$

$= \frac{3(3x-7)}{12} - \frac{4(x-1)}{12} - (x-5)$

$= \frac{3(3x-7)-4(x-1)}{12} - (x-5)$

$= \frac{9x-21-4x+4}{12} - (x-5)$

$= \frac{9x-4x-21+4}{12} - (x-5)$

$= \frac{5x-17}{12} - \frac{12(x-5)}{12}$

$= \frac{5x-17-12x+60}{12}$

$= \frac{5x-12x-17+60}{12}$

$= \frac{-7x+43}{12}$

(i) $\frac{2(3x-1)}{5} - (x-3) - \frac{2x+1}{3}$

$= \frac{2(3x-1)}{5} - \frac{2x+1}{3} - (x-3)$

$= \frac{6(3x-1)-5(2x+1)}{15} - (x-3)$

$= \frac{18x-6-10x-5}{15} - (x-3)$

$= \frac{18x-10x-6-5}{15} - (x-3)$

$= \frac{8x-11}{15} - \frac{15(x-3)}{15}$

$= \frac{8x-11-15(x-3)}{15}$

$= \frac{8x-11-15x+45}{15}$

$= \frac{8x-15x-11+45}{15}$

$= \frac{-7x+34}{15}$

Advanced

33. (a) $a(5b - 3) - b(4a - 1) + a(1 - 2b)$
$= 5ab - 3a - 4ab + b + a - 2ab$
$= 5ab - 4ab - 2ab - 3a + a + b$
$= -ab - 2a + b$

(b) $3x - \{2x - 4(x - 3y) - [(3x - 4y) - (y - 2x)]\}$
$= 3x - \{2x - 4(x - 3y) - [3x - 4y - y + 2x]\}$
$= 3x - \{2x - 4(x - 3y) - [3x + 2x - 4y - y]\}$
$= 3x - \{2x - 4x + 12y - [5x - 5y]\}$
$= 3x - \{- 2x + 12y - 5x + 5y]\}$
$= 3x - \{- 2x - 5x + 12y + 5y]\}$
$= 3x - \{- 7x + 17y\}$
$= 3x + 7x - 17y$
$= 10x - 17y$

34. Let Raj's present age be p years.
Then Ethan's present age is $5p$ years.
In 5 years' time,
Raj is $(p + 5)$ years old and Ethan is $(5p + 5)$ years old.
$p + 5 + (5p + 5) = x$
$p + 5 + 5p + 5 = x$
$p + 5p + 5 + 5 = x$
$6p = x - 5 - 5$
$6p = x - 10$
$p = \frac{x - 10}{6}$

Raj's present age is $\frac{x - 10}{6}$ years old.

35. Total age of the girls
$= (n + 5)q$ years
Total age of the group of boys and girls
$= (m + 2 + n + 5)p$
$= (m + n + 7)p$ years
Total age of the boys
$= p(m + n + 7) - q(n + 5)$ years
Average age of the boys
$= \frac{p(m + n + 7) - q(n + 5)}{(m + 2)}$ years old

36. (a) $3ac - ad + 2ba - 15a$
$= a(3c - d + 2b - 15)$

(b) $2x + 4xy - 7xyz + 2xz$
$= x(2 + 4y - 7yz + 2z)$

(c) $4ba + 5bca + 9dab$
$= ab(4 + 5c + 9d)$

(d) $5m + 20pmn - 10mn + 35pm$
$= 5m(1 + 4pn - 2n + 7p)$

(e) $6pqr - 3(p + q - 2r)$
$= 6pqr - 3p - 3q + 6r$
$= 3(2pqr - p - q + 2r)$

37. (a) $\frac{2(x - 3y)}{4} - \frac{4y - x}{12} - \frac{4(x - 5y)}{3}$

$= \frac{6(x - 3y)}{12} - \frac{4y - x}{12} - \frac{4(x - 5y)}{3}$

$= \frac{6(x - 3y) - (4y - x)}{12} - \frac{4(x - 5y)}{3}$

$= \frac{6x - 18y - 4y + x}{12} - \frac{4(x - 5y)}{3}$

$= \frac{7x - 22y}{12} - \frac{16(x - 5y)}{12}$

$= \frac{7x - 22y - 16(x - 5y)}{12}$

$= \frac{7x - 22y - 16x + 80y}{12}$

$= \frac{7x - 16x - 22y + 80y}{12}$

$= \frac{-9x + 58}{12}$

(b) $\frac{-4(p - 3q)}{5} - \left[\frac{2(q - p)}{20} - \frac{3(p - 5q)}{4}\right]$

$= \frac{-4(p - 3q)}{5} - \left[\frac{2(q - p)}{20} - \frac{15(p - 5q)}{20}\right]$

$= \frac{-4(p - 3q)}{5} - \left[\frac{2(q - p) - 15(p - 5q)}{20}\right]$

$= \frac{-4(p - 3q)}{5} - \left[\frac{2q - 2p - 15p + 75q}{20}\right]$

$= \frac{-16(p - 3q)}{20} - \left[\frac{-17p + 77q}{20}\right]$

$= \frac{-16(p - 3q) - (-17p + 77q)}{20}$

$= \frac{-16p + 48q + 17p - 77q}{20}$

$= \frac{-16p + 17p + 48q - 77q}{20}$

$= \frac{p - 29q}{20}$

(c) $-3 + \frac{2(f-3h)}{21} - \frac{5(h-f)}{7} + \frac{2(-2f-3h)}{3}$

$= -3 + \frac{2(f-3h)}{21} - \frac{15(h-f)}{21} + \frac{2(-2f-3h)}{3}$

$= -3 + \frac{2(f-3h)-15(h-f)}{21} + \frac{2(-2f-3h)}{3}$

$= -3 + \frac{2f-6h-15h+15f}{21} + \frac{2(-2f-3h)}{3}$

$= -3 + \frac{17f-21h}{21} + \frac{2(-2f-3h)}{3}$

$= -3 + \frac{17f-21h}{21} + \frac{14(-2f-3h)}{21}$

$= -3 + \frac{17f-21h+14(-2f-3h)}{21}$

$= -3 + \frac{17f-21h-28f-42h}{21}$

$= -3 + \frac{17f-28f-42h-21h}{21}$

$= \frac{-63}{21} + \frac{-11f-63h}{21}$

$= \frac{-11f-63h-63}{21}$

(d) $\frac{x}{5} - \frac{4}{3x}$

$= \frac{3x^2}{15x} - \frac{20}{15x}$

$= \frac{3x^2-20}{15x}$

(e) $\frac{1}{x} + \frac{1}{2x} + \frac{1}{3x}$

$= \frac{6}{6x} + \frac{3}{6x} + \frac{2}{6x}$

$= \frac{11}{6x}$

(f) $\frac{5}{2x} - \frac{3}{3x} + \frac{7}{x}$

$= \frac{15}{6x} - \frac{6}{6x} + \frac{42}{6x}$

$= \frac{15-6+42}{6x}$

$= \frac{51}{6x}$

$= \frac{17}{2x}$

(g) $\frac{2x-3}{5y} - \frac{5-2x}{10y} + \frac{x}{y}$

$= \frac{2(2x-3)-(5-2x)+10x}{10y}$

$= \frac{4x-6-5+2x+10x}{10y}$

$= \frac{4x+2x+10x-6-5}{10y}$

$= \frac{16x-11}{10y}$

New Trend

38. $\frac{a}{5} - \frac{2(3a-5c)}{6}$

$= \frac{a\times 6}{5\times 6} - \frac{2(3a-5c)\times 5}{6\times 5}$

$= \frac{6a}{30} - \frac{10(3a-5c)}{30}$

$= \frac{6a-10(3a-5c)}{30}$

$= \frac{6a-30a+50c}{30}$

$= \frac{-24a+50c}{30}$

$= \frac{2(-12a+25c)}{30}$

$= \frac{25c-12a}{15}$

39. (a) $BC = 23x - 2 - (3x - 2) - (5x + 1) - (6x - 7)$
$= 23x - 3x - 5x - 6x - 2 + 2 - 1 + 7$
$= (9x + 6)$ cm

(b) Since $BC = 2AD$,
$9x + 6 = 2(5x + 1)$
$9x + 6 = 10x + 2$
$x = 4$
Perimeter of trapezium $= 23x - 2$
$= 23(4) - 2$
$= 90$ cm

40. (a) $2(3x - 5) - 3(7 - 4x)$
$= 6x - 10 - 21 + 12x$
$= 6x + 12x - 10 - 21$
$= 18x - 31$

(b) $4(2x + 3y) - 7(x - 2y) = 8x + 12y - 7x + 14y$
$= x + 26y$

41. $\frac{3x+4}{10} - \frac{x+7}{15} - \frac{2x-1}{5}$

$= \frac{3(3x+4)}{30} - \frac{2(x+7)}{30} - \frac{2x-1}{5}$

$= \frac{9x+12-2x-14}{30} - \frac{2x-1}{5}$

$= \frac{9x-2x+12-14}{30} - \frac{2x-1}{5}$

$= \frac{7x-2}{30} - \frac{6(2x-1)}{30}$

$= \frac{7x-2-6(2x-1)}{30}$

$= \frac{7x-2-12x+6}{30}$

$= \frac{7x-12x-2+6}{30}$

$= \frac{-5x+4}{30}$

42. x ¢ $\rightarrow$ 1 gram

$\$y = (100 \times y)$ ¢

$= 100y$ ¢

$100y$ ¢ $\rightarrow \frac{1}{x} \times 100y$

$= \frac{100y}{x}$ grams

Revision Test A1

1. (a) $36 = 2^2 \times \boxed{3^2}$

$54 = 2 \times \boxed{3^3}$

$63 = \boxed{3^2} \times 7$

$\downarrow$

3^2

HCF of 36, 54 and 63 $= 3^2 = 9$

(b) $63 = \boxed{3^2} \times 7$

$105 = 3 \times \boxed{5} \times 7$

$420 = 2^2 \times 3 \times 5 \times 7$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$2^2 \quad 3^2 \quad 5 \quad 7$

LCM of 63, 105 and 420 $= 2^2 \times 3^2 \times 5 \times 7 = 1260$

2. (i) (a)

2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

$576 = 2^6 \times 3^2$

(b)

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$5832 = 2^3 \times 3^6$

(ii) $\sqrt{576} = \sqrt{2^6 \times 3^2}$

$= \sqrt{(2^3 \times 3)^2}$

$= 2^3 \times 3$

$= 24$

$\sqrt[3]{5832} = \sqrt[3]{2^3 \times 3^6}$

$= \sqrt[3]{(2 \times 3^2)^3}$

$= 2 \times 3^2$

$= 18$

3. (a) $\dfrac{(-24) + (-30)}{-6}$

$= \dfrac{-24 - 30}{-6}$

$= \dfrac{-54}{-6}$

$= 9$

(b) $\dfrac{\left(\frac{1}{5} + \frac{1}{4}\right) \div \left(-\frac{1}{20}\right)}{\left(-1\frac{2}{5}\right) \times \left(-1\frac{1}{4}\right)}$

$= \dfrac{\left(\frac{9}{20}\right) \div \left(-\frac{1}{20}\right)}{\left(-1\frac{2}{5}\right) \times \left(-1\frac{1}{4}\right)}$

$= \dfrac{\left(\frac{9}{\not{20}_1}\right) \times \left(-\frac{\not{20}^1}{1}\right)}{\left(-\frac{7}{\not{5}_1}\right) \times \left(-\frac{\not{5}^1}{4}\right)}$

$= \dfrac{-9}{\left(\frac{7}{4}\right)}$

$= -9 \times \dfrac{4}{7}$

$= -5\dfrac{1}{7}$

4. (a) $79.12^2 + 56.19^2 - 2 \times 79.12 \times 56.19 \times 0.8716$

$= 79.12^2 + 56.19^2 - 7749.836\ 281$

$= 1667.45$ (to 2 d.p.)

(b) $\dfrac{245 \times \sqrt[3]{269.78} + 966 \times \sqrt{294.81}}{4 \times (54.783)^3}$

$= \dfrac{1583.0793 + 16\ 586.250\ 94}{657\ 653.9379}$

$= 0.03$ (to 2 d.p.)

5. (a) Each book costs \$4 approximately.

Number of books that can be bought with \$10

$= \dfrac{10}{4}$

$= 2.5$

The number of books that can be purchased is 2.

(b) Number of litres of petrol

$= \dfrac{600}{12.1}$

$= \dfrac{600}{12}$

$= 50\ l$

50 l of petrol are consumed for a 600-km journey.

(c) $\sqrt[3]{7.95} \times 25.04$
$= \sqrt[3]{8} \times 25$
$= 2 \times 25$
$= 50$

6. (a) $3a - (2 - 5a) - 7$
$= 3a - 2 + 5a - 7$
$= 3a + 5a - 2 - 7$
$= 8a - 9$

(b) $8x - 3(x - y)$
$= 8x - 3x + 3y$
$= 5x + 3y$

(c) $4(3x - 7) - 2(6x - 7)$
$= 12x - 28 - 12x + 14$
$= 12x - 12x - 28 + 14$
$= -14$

(d) $\frac{2x}{5} - \frac{3(2x-5)}{3}$
$= \frac{2x}{5} - \frac{5(2x-5)}{5}$
$= \frac{2x - 5(2x-5)}{5}$
$= \frac{2x - 10x + 25}{5}$
$= \frac{-8x + 25}{5}$

7. (a) $5x + 15y = 5(x + 3y)$

(b) $4cx - 8dx + 2cdx - 2x$
$= 2x(2c - 4d + cd - 1)$

8. (a) The breadth of the rectangle is x cm.
Then the length of the rectangle is $(x + 7)$ cm.
Perimeter of rectangle
$= 2[(x + 7) + x]$
$= 2[2x + 7]$
$= (4x + 14)$ cm
Area of rectangle
$= (x)(x + 7)$
$= (x^2 + 7x)$ cm^2

(b) Let the smaller number be y.
Then the larger number is $4y$.
$y + 4y = p$
$5y = p$
$y = \frac{p}{5}$

The smaller number is $\frac{p}{5}$ and the larger number is $\frac{4p}{5}$.

Revision Test A2

1. **(a)**

$$\begin{array}{c} \boxed{\begin{matrix}2^4\\2^2\end{matrix}} \times \boxed{\begin{matrix}3^3\\3^4\end{matrix}} \times \boxed{\begin{matrix}5\\5^3\end{matrix}} \\ \downarrow \quad \downarrow \quad \downarrow \\ 2^2 \quad 3^3 \quad 5 \end{array}$$

HCF of the two numbers $= 2^2 \times 3^3 \times 5$
$= 540$

(b)

$$\begin{array}{c} 2^2 \times \boxed{\begin{matrix}3^2\\3\end{matrix}} \times \boxed{\begin{matrix}5\\5^2\end{matrix}} \\ \downarrow \quad \downarrow \quad \downarrow \\ 2^2 \quad 3^2 \quad 5^2 \end{array}$$

LCM of the two numbers $= 2^2 \times 3^2 \times 5^2$
$= 900$

(c)

3	11 025
3	3675
5	1225
5	245
7	49
7	7
	1

$11\,025 = 3^2 \times 5^2 \times 7^2 = (3 \times 5 \times 7)^2$

$\therefore \sqrt{11\,025} = \sqrt{(3 \times 5 \times 7)^2}$
$= 3 \times 5 \times 7$
$= 105$

2. **(a)** $(-2)^3 \times (-5) - 4 \times (-5)^2 - (-7)^2$
$-8 \times (-5) - 4 \times 25 - 49$
$= 40 - 100 - 49$
$= -109$

(b) $-2\frac{1}{3} \div \left[\left(-\frac{1}{5} - \frac{2}{3}\right) \div \frac{1}{15}\right] - \left(-\frac{1}{4}\right)$

$= -2\frac{1}{3} \div \left[\left(-\frac{13}{15}\right) \div \frac{1}{15}\right] - \left(-\frac{1}{4}\right)$

$= -2\frac{1}{3} \div \left[-\frac{13}{\cancel{15}_1} \times \frac{\cancel{15}^1}{1}\right] - \left(-\frac{1}{4}\right)$

$= -2\frac{1}{3} \div (-13) - \left(-\frac{1}{4}\right)$

$= -\frac{7}{3} \times \left(-\frac{1}{13}\right) + \frac{1}{4}$

$= \frac{7}{39} + \frac{1}{4}$

$= \frac{67}{156}$

(c) $\dfrac{75 \times \left(-\frac{1}{2}\right) \times (-13.4)}{(0.5) \times 7.5}$

$= \frac{502.5}{3.75}$

$= 134$

3. **(a)** $\sqrt{29.76^3 + (8.567 - 0.914)^2}$

$= \sqrt{26\,415.738\,59}$
$= 29.78$ (to 2 d.p.)

(b) $\sqrt{\dfrac{121.56^2 + 78.94^2 - 99.18^2}{2 \times 121.56 \times 78.94}}$

$= \sqrt{\dfrac{11\,171.6848}{19\,191.8926}}$

$= 0.76$ (to 2 d.p.)

4. **(a)** $8.4454 = 8.45$ (to 2 d.p.)
(b) $0.070\,49 = 0.070$ (to 2 s.f.)
(c) $25\,958 = 26\,000$ (to the nearest 100)
(d) $15\,997 = 16\,000$ (to the nearest 10)

5. **(a)** $2a + 5b - 3c - (4b - 3a + 6c)$
$= 2a + 5b - 3c - 4b + 3a - 6c$
$= 2a + 3a + 5b - 4b - 3c - 6c$
$= 5a + b - 9c$

(b) $[2a - b(a + 3)] + b(3 + 2a)$
$= [2a - ab - 3b] + 3b + 2ab$
$= 2a - ab + 2ab - 3b + 3b$
$= 2a + ab$

(c) $(2x + 1)(x - 3) - (3 - x)(1 - 5x)$
$= (2x + 1)(x - 3) + (x - 3)(1 - 5x)$
$= (x - 3)(2x + 1 + 1 - 5x)$
$= (x - 3)(-3x + 2)$

(d) $\frac{2(x + 3)}{3} - (x - 2) - 4 - \frac{3(x - 4)}{6}$

$= \frac{4(x + 3)}{6} - \frac{3(x - 4)}{6} - (x - 2) - 4$

$= \frac{4(x + 3) - 3(x - 4)}{6} - (x - 2) - 4$

$= \frac{4x + 12 - 3x + 12}{6} - (x - 2) - 4$

$= \frac{4x - 3x + 12 + 12}{6} - (x - 2) - 4$

$= \frac{x + 24}{6} - \frac{6(x - 2)}{6} - \frac{24}{6}$

$= \frac{x + 24 - 6(x - 2) - 24}{6}$

$= \frac{x + 24 - 6x + 12 - 24}{6}$

$= \frac{x - 6x + 24 + 12 - 24}{6}$

$= \frac{-5x + 12}{6}$

6. **(a)** When $x = -2$, $y = -1$, $z = 0$,

$(x - y)^{z - x}$

$= (-2 - (-1))^{0 - (-2)}$

$= (-2 + 1)^2$

$= (-1)^2$

$= 1$

(b) When $a = 3$, $b = -2$ and $c = 5$,

(i) $a + b + c$

$= 3 + (-2) + 5$

$= 3 - 2 + 5$

$= 1 + 5$

$= 6$

(ii) abc

$= (3)(-2)(5)$

$= -30$

(iii) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

$= \frac{1}{3} + \frac{1}{-2} + \frac{1}{5}$

$= \frac{10}{30} - \frac{15}{30} + \frac{6}{30}$

$= \frac{1}{30}$

7. **(a)** $7q + 5p - 4r - 5 - (2p + 5q - 4r + 3)$

$= 7q + 5p - 4r - 5 - 2p - 5q + 4r - 3$

$= 5p - 2p + 7q - 5q - 4r + 4r - 5 - 3$

$= 3p + 2q - 8$

(b) **(i)** $xy + 2x - 5zxy - 10xz$

$= x(y + 2 - 5zy - 10z)$

(ii) $6ap - 6bp - 3pc + 24pab$

$= 3p(2a - 2b - c + 8ab)$

(c) Total height of the boys = (mp) cm

Total height of the girls = (nq) cm

Total height of the students

= $(mp + nq)$ cm

Average height of the students

$= \left(\frac{mp + nq}{m + n}\right)$ cm

Chapter 5 Linear Equations and Simple Inequalities

Basic

1. (a)
$$\begin{aligned} 5x + 2 &= 7 \\ 5x + 2 - 2 &= 7 - 2 \\ 5x &= 5 \\ \frac{5x}{5} &= \frac{5}{5} \\ x &= 1 \end{aligned}$$

(b)
$$\begin{aligned} 2x - 7 &= 3 \\ 2x - 7 + 7 &= 3 + 7 \\ 2x &= 10 \\ \frac{2x}{2} &= \frac{10}{2} \\ x &= 5 \end{aligned}$$

(c)
$$\begin{aligned} 15 - 2x &= 9 \\ 15 - 2x + 2x &= 9 + 2x \\ 15 &= 9 + 2x \\ 9 + 2x - 9 &= 15 - 9 \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} \\ x &= 3 \end{aligned}$$

(d)
$$\begin{aligned} 17 + 3x &= -3 \\ 17 + 3x - 17 &= -3 - 17 \\ 3x &= -20 \\ \frac{3x}{3} &= \frac{-20}{3} \\ x &= -6\frac{2}{3} \end{aligned}$$

(e)
$$\begin{aligned} -4x + 7 &= -15 \\ -4x + 7 - 7 &= -15 - 7 \\ -4x &= -22 \\ \frac{-4x}{-4} &= \frac{-22}{-4} \\ x &= 5\frac{1}{2} \end{aligned}$$

(f)
$$\begin{aligned} 2x - 3 &= x + 5 \\ 2x - 3 + 3 &= x + 5 + 3 \\ 2x &= x + 8 \\ 2x - x &= x + 8 - x \\ x &= 8 \end{aligned}$$

(g)
$$\begin{aligned} 9x + 4 &= 3x - 9 \\ 9x + 4 - 4 &= 3x - 9 - 4 \\ 9x &= 3x - 13 \\ 9x - 3x &= 3x - 13 - 3x \\ 6x &= -13 \\ \frac{6x}{6} &= \frac{-13}{6} \\ x &= -2\frac{1}{6} \end{aligned}$$

(h)
$$\begin{aligned} 7x - 14 &= 18 - 4x \\ 7x - 14 + 14 &= 18 - 4x + 14 \\ 7x &= 32 - 4x \\ 7x + 4x &= 32 - 4x + 4x \\ 11x &= 32 \\ \frac{11x}{11} &= \frac{32}{11} \\ x &= 2\frac{10}{11} \end{aligned}$$

2. (a)
$$\begin{aligned} 3(x - 4) &= 7 \\ 3x - 12 &= 7 \\ 3x - 12 + 12 &= 7 + 12 \\ 3x &= 19 \\ \frac{3x}{3} &= \frac{19}{3} \\ x &= 6\frac{1}{3} \end{aligned}$$

(b)
$$\begin{aligned} 5(2x + 3) &= 35 \\ 10x + 15 &= 35 \\ 10x + 15 - 15 &= 35 - 15 \\ 10x &= 20 \\ \frac{10x}{10} &= \frac{20}{10} \\ x &= 2 \end{aligned}$$

(c)
$$\begin{aligned} 4(3 - x) &= -15 \\ 12 - 4x &= -15 \\ 12 - 4x - 12 &= -15 - 12 \\ -4x &= -27 \\ \frac{-4x}{-4} &= \frac{-27}{-4} \\ x &= 6\frac{3}{4} \end{aligned}$$

(d)
$$\begin{aligned} 2(7 - 2x) &= 11 \\ 14 - 4x &= 11 \\ 14 - 4x - 14 &= 11 - 14 \\ -4x &= -3 \\ \frac{-4x}{-4} &= \frac{-3}{-4} \\ x &= \frac{3}{4} \end{aligned}$$

(e) $2(x-5) = 5x+7$

$2x-10 = 5x+7$

$2x-10-7 = 5x+7-7$

$2x-17 = 5x$

$2x-17-2x = 5x-2x$

$-17 = 3x$

$3x = -17$

$\frac{3x}{3} = \frac{-17}{3}$

$x = -5\frac{2}{3}$

(f) $6-4x = 5(x-6)$

$6-4x = 5x-30$

$6-4x+4x = 5x-30+4x$

$6 = 9x-30$

$6+30 = 9x-30+30$

$36 = 9x$

$9x = 36$

$\frac{9x}{9} = \frac{36}{9}$

$x = 4$

(g) $2x-3(5-x) = 35$

$2x-15+3x = 35$

$2x+3x-15 = 35$

$5x-15 = 35$

$5x-15+15 = 35+15$

$5x = 50$

$\frac{5x}{5} = \frac{50}{5}$

$x = 10$

(h) $7(x+4) = 2(x-4)$

$7x+28 = 2x-8$

$7x+28-2x = 2x-8-2x$

$5x+28 = -8$

$5x+28-28 = -8-28$

$5x = -36$

$\frac{5x}{5} = \frac{-36}{5}$

$x = -7\frac{1}{5}$

(i) $2(5-2x) = 4(2-3x)$

$10-4x = 8-12x$

$10-4x+12x = 8-12x+12x$

$8x+10 = 8$

$8x+10-10 = 8-10$

$8x = -2$

$\frac{8x}{8} = \frac{-2}{8}$

$x = -\frac{1}{4}$

(j) $(5x+3)-(4x-9) = 0$

$5x+3-4x+9 = 0$

$5x-4x+3+9 = 0$

$x+12 = 0$

$x+12-12 = 0-12$

$x = -12$

(k) $7(3-4x)-5(2x+8) = 0$

$21-28x-10x-40 = 0$

$21-40-28x-10x = 0$

$-19-38x = 0$

$-19-38x+19 = 0+19$

$-38x = 19$

$\frac{-38x}{-38} = \frac{19}{-38}$

$x = -\frac{1}{2}$

(l) $5(2x-3)-3(x-2) = 0$

$10x-15-3x+6 = 0$

$10x-3x-15+6 = 0$

$7x-9 = 0$

$7x-9+9 = 0+9$

$7x = 9$

$\frac{7x}{7} = \frac{9}{7}$

$x = 1\frac{2}{7}$

3. (a) $\frac{3}{4}x = 15$

$\frac{3}{4}x \times 4 = 15 \times 4$

$3x = 60$

$\frac{3x}{3} = \frac{60}{3}$

$x = 20$

(b) $\frac{2}{5}x-1 = 4$

$\frac{2}{5}x-1+1 = 4+1$

$\frac{2}{5}x = 5$

$\frac{2}{5}x \times 5 = 5 \times 5$

$2x = 25$

$\frac{2x}{2} = \frac{25}{2}$

$x = 12\frac{1}{2}$

(c) $5 - \frac{3}{4}x = -1$

$5 - \frac{3}{4}x + \frac{3}{4}x = -1 + \frac{3}{4}x$

$5 = -1 + \frac{3}{4}x$

$5 + 1 = -1 + \frac{3}{4}x + 1$

$\frac{3}{4}x = 6$

$\frac{3}{4}x \times 4 = 6 \times 4$

$3x = 24$

$\frac{3x}{3} = \frac{24}{3}$

$x = 8$

(d) $3 + \frac{4}{7}x = 1\frac{1}{3}$

$3 + \frac{4}{7}x - 3 = 1\frac{1}{3} - 3$

$\frac{4}{7}x = -1\frac{2}{3}$

$\frac{4}{7}x \times 7 = -1\frac{2}{3} \times 7$

$4x = -11\frac{2}{3}$

$\frac{4x}{4} = \frac{-11\frac{2}{3}}{4}$

$x = -2\frac{11}{12}$

(e) $2x = 0.4x + 12.8$

$2x - 0.4x = 0.4x + 12.8 - 0.4x$

$1.6x = 12.8$

$\frac{1.6x}{1.6} = \frac{12.8}{1.6}$

$x = 8$

(f) $0.3x + 1.2 = 0.25 - 0.2x$

$0.3x + 1.2 + 0.2x = 0.25 - 0.2x + 0.2x$

$0.5x + 1.2 = 0.25$

$0.5x + 1.2 - 1.2 = 0.25 - 1.2$

$0.5x = -0.95$

$\frac{0.5x}{0.5} = \frac{-0.95}{0.5}$

$x = -1.9$

(g) $\frac{2}{3}x + 15 = 4x$

$\frac{2}{3}x + 15 - \frac{2}{3}x = 4x - \frac{2}{3}x$

$15 = 3\frac{1}{3}x$

$3\frac{1}{3}x = 15$

$\frac{3\frac{1}{3}x}{3\frac{1}{3}} = \frac{15}{3\frac{1}{3}}$

$x = 4\frac{1}{2}$

(h) $1.3x - 3.6 = \frac{4}{5}x + 2$

$1.3x - 3.6 - \frac{4}{5}x = \frac{4}{5}x + 2 - \frac{4}{5}x$

$0.5x - 3.6 = 2$

$0.5x - 3.6 + 3.6 = 2 + 3.6$

$0.5x = 5.6$

$\frac{0.5x}{0.5} = \frac{5.6}{0.5}$

$x = 11.2$

(i) $1.5 - \frac{7}{8}x = 2.6x + \frac{1}{5}$

$1.5 - \frac{7}{8}x + \frac{7}{8}x = 2.6x + \frac{1}{5} + \frac{7}{8}x$

$1.5 = 3.475x + \frac{1}{5}$

$1.5 - \frac{1}{5} = 3.475x + \frac{1}{5} - \frac{1}{5}$

$3.475x = 1.3$

$\frac{3.475x}{3.475} = \frac{1.3}{3.475}$

$x = \frac{52}{139}$

4. (a) $\frac{2x - 3}{5} = 7$

$\frac{2x - 3}{5} \times 5 = 7 \times 5$

$2x - 3 = 35$

$2x - 3 + 3 = 35 + 3$

$2x = 38$

$\frac{2x}{2} = \frac{38}{2}$

$x = 19$

(b) $\frac{3x-4}{5} - 7 = 0$

$\frac{3x-4}{5} = 7$

$\frac{3x-4}{5} \times 5 = 7 \times 5$

$3x - 4 = 35$

$3x - 4 + 4 = 35 + 4$

$3x = 39$

$\frac{3x}{3} = \frac{39}{3}$

$x = 13$

(c) $\frac{x+1}{3} = \frac{3x}{5}$

$15 \times \frac{x+1}{3} = 15 \times \frac{3x}{5}$

$5(x+1) = 3(3x)$

$5x + 5 = 9x$

$5x + 5 - 5x = 9x - 5x$

$5 = 4x$

$4x = 5$

$\frac{4x}{4} = \frac{5}{4}$

$x = 1\frac{1}{4}$

(d) $\frac{2x-1}{3} = 1 - x$

$\frac{2x-1}{3} \times 3 = (1-x) \times 3$

$2x - 1 = 3(1-x)$

$2x - 1 = 3 - 3x$

$2x - 1 + 3x = 3 - 3x + 3x$

$5x - 1 = 3$

$5x - 1 + 1 = 3 + 1$

$5x = 4$

$\frac{5x}{5} = \frac{4}{5}$

$x = 0.8$

(e) $\frac{2}{3}(5x-7) = \frac{4}{5}$

$15 \times \frac{2}{3}(5x-7) = 15 \times \frac{4}{5}$

$10(5x-7) = 12$

$50x - 70 = 12$

$50x - 70 + 70 = 12 + 70$

$50x = 82$

$\frac{50x}{50} = \frac{82}{50}$

$x = 1\frac{16}{25}$

(f) $\frac{2}{3}(6x+5) = 7(x-4.5)$

$4x + 3\frac{1}{3} = 7x - 31.5$

$4x + 3\frac{1}{3} - 4x = 7x - 31.5 - 4x$

$3\frac{1}{3} = 3x - 31.5$

$3\frac{1}{3} + 31.5 = 3x - 31.5 + 31.5$

$\frac{209}{6} = 3x$

$3x = \frac{209}{6}$

$\frac{3x}{3} = \frac{\left(\frac{209}{6}\right)}{3}$

$x = 11\frac{11}{18}$

(g) $\frac{1}{4}(3x+5) = \frac{1}{3}(5x-4)$

$12 \times \frac{1}{4}(3x+5) = 12 \times \frac{1}{3}(5x-4)$

$3(3x+5) = 4(5x-4)$

$9x + 15 = 20x - 16$

$9x + 15 - 15 = 20x - 16 - 15$

$9x = 20x - 31$

$9x - 20x = 20x - 31 - 20x$

$-11x = -31$

$11x = 31$

$\frac{11x}{11} = \frac{31}{11}$

$x = 2\frac{9}{11}$

(h) $\frac{1}{5}(4-3x) = \frac{1}{7}(3x-4)$

$35 \times \frac{1}{5}(4-3x) = 35 \times \frac{1}{7}(3x-4)$

$7(4-3x) = 5(3x-4)$

$28 - 21x = 15x - 20$

$28 - 21x + 21x = 15x - 20 + 21x$

$28 = 36x - 20$

$28 + 20 = 36x - 20 + 20$

$48 = 36x$

$36x = 48$

$\frac{36x}{36} = \frac{48}{36}$

$x = 1\frac{1}{3}$

(i) $\frac{4x-3}{5} = \frac{2x-7}{8}$

$8(4x-3) = 5(2x-7)$

$32x - 24 = 10x - 35$

$32x - 10x = -35 + 24$

$22x = -11$

$\frac{22x}{22} = \frac{-11}{22}$

$x = -\frac{1}{2}$

5. **(a)** $y = a(4a - 5)$

When $a = 3$, $y = 3(4 \times 3 - 5)$

$= 3(7) = 21$

(b) $y = (x + p)(3x - p - 4)$

When $x = 3$, $p = 4$,

$y = (3 + 4)(3 \times 3 - 4 - 4)$

$= (7)(1) = 7$

(c) $y = \frac{2x-1}{3}$

When $x = 5$, $y = \frac{2(5)-1}{3} = \frac{9}{3} = 3$

(d) $y = \frac{2r+5}{7r-9}$

When $r = 6$,

$y = \frac{2(6)+5}{7(6)-9}$

$= \frac{17}{33}$

6. $xy - 3y^2 = 15$

When $y = 2$,

$x(2) - 3(2)^2 = 15$

$2x - 12 = 15$

$2x = 15 + 12$

$2x = 27$

$x = 13\frac{1}{2}$

7. $y = \frac{2}{3}(24 - x) + 5xy$

When $x = -3\frac{1}{3}$,

$y = \frac{2}{3}\left[24 - \left(-3\frac{1}{3}\right)\right] + 5\left(-3\frac{1}{3}\right)y$

$y = \frac{2}{3}\left(27\frac{1}{3}\right) - 16\frac{2}{3}y$

$y = 18\frac{2}{9} - 16\frac{2}{3}y$

$y + 16\frac{2}{3}y = 18\frac{2}{9}$

$17\frac{2}{3}y = 18\frac{2}{9}$

$y = 1\frac{5}{159}$

8. $p - 5q = 4qr$

When $q = 4$, $r = -1$,

$p - 5(4) = 4(4)(-1)$

$p - 20 = -16$

$p = -16 + 20 = 4$

9. **(a)** $D = a^2 - b^2$

(b) The three consecutive numbers are d, $d + 2$ and $d + 4$.

$S = d + (d + 2) + (d + 4) = 3d + 6 = 3(d + 2)$

(c) Perimeter of square $= m + m + m + m = 4m$

Perimeter of rectangle $= 2(n + s)$

Perimeter of figure,

$P = 4m + 4(n + s)$

10. **(a)** Let the smallest odd number be n.

The next odd number is $n + 2$.

The largest odd number is $(n + 2) + 2 = n + 4$.

$\therefore S = n + n + 2 + n + 4 = 3n + 6$

$3n + 6 = 243$

$3n = 243 - 6 = 237$

$n = 79$

$\therefore$ The largest odd number is $79 + 4 = 83$.

(b) Let the smallest even number be n.

The next even number is $n + 2$.

The next even number is $(n + 2) + 2 = n + 4$.

The next even number is $(n + 4) + 2 = n + 6$.

The largest even number is $(n + 6) + 2 = n + 8$.

$\therefore S = n + n + 2 + n + 4 + n + 6 + n + 8$

$= 5n + 20$

$5n + 20 = 220$

$5n = 220 - 20 = 200$

$n = 40$

$\therefore$ The smallest of the five numbers is 40.

(c) Let the smaller odd number be n.

The next odd number is $n + 2$.

$3(n + 2) - n = 56$

$3n + 6 - n = 56$

$2n = 56 - 6$

$2n = 50$

$n = 25$

$\therefore$ The two numbers are 25 and 27.

(d) Let the smaller even number be n.

The next even number is $n + 2$.

$n + 2 + 3n = 42$

$4n = 40$

$n = 10$

$\therefore$ The two numbers are 10 and 12.

11. (a) Let the age of Raj be x years old.
Then Rui Feng is $2x$ years old.
Khairul is $(2x - 7)$ years old.

$$x + 2x + (2x - 7) = 38$$
$$5x = 38 + 7$$
$$5x = 45$$
$$x = 9$$

Raj is 9 years old.
Rui Feng is $2 \times 9 = 18$ years old.
Khairul is $(2 \times 9 - 7) = 11$ years old.

(b) Let the number of years ago in which Kate's father is three times as old as her be n.

$$50 - n = 3(24 - n)$$
$$50 - n = 72 - 3n$$
$$2n = 72 - 50$$
$$2n = 22$$
$$n = 11$$

$\therefore$ Kate's father was three times as old as Kate 11 years ago.

(c) Let the age of Farhan be x years old.
Then Farhan's brother's age is $3x$ years old.
In 12 years' time,
Farhan will be $(x + 12)$ years old and his brother will be $(3x + 12)$ years old.

$$(x + 12) + (3x + 12) = 10x$$
$$4x + 24 = 10x$$
$$6x = 24$$
$$x = 4$$

$\therefore$ Farhan's present age is 4 years old and his brother is 12 years old.

12. (a) Let the first number be x.
Then the second number is $120 - x$.

$$120 - x = 4x$$
$$5x = 120$$
$$x = 24$$

$\therefore$ The smaller number is 24.

(b) Let the number be x.

$$12 - \frac{x}{4} = \frac{1}{6}x$$
$$12 = \frac{1}{6}x + \frac{x}{4}$$
$$12 = \frac{5}{12}x$$
$$144 = 5x$$
$$x = 28\frac{4}{5}$$

$\therefore$ The number is $28\frac{4}{5}$.

13. (a) The cost of 12 pears is equal to the cost of 36 apples.
A pear costs 3 times an apple.
Let the cost of an apple be $\$x$.
Then the cost of a pear is $\$3x$.
The amount of money Michael has is $\$36x$.
Cost of 1 apple and 1 pear
$= \$3x + \$x$
$= \$4x$
No. of each fruit Michael can buy
$= \dfrac{36x}{4x}$
$= 9$

(b) Amount of money spent on pencils
$= 15 \times \dfrac{2x}{100} = \$\dfrac{3x}{10}$
Amount of money spent on pens
$= 24 \times \dfrac{4y}{100} = \$\dfrac{24y}{25}$
Total amount spent on pencils and pens
$= \dfrac{3x}{10} + \dfrac{24y}{25}$
$= \$\dfrac{15x + 48y}{50}$

14. (a)

$$3x > 33$$
$$x > \frac{33}{3}$$
$$x > 11$$

(b)

$$11x \leqslant 25$$
$$x \leqslant \frac{25}{11}$$
$$x \leqslant 2\frac{3}{11}$$

(c)

$$\frac{1}{2}x > 3$$
$$2 \times \frac{1}{2}x > 3 \times 2$$
$$x > 6$$

(d)

$$\frac{3x}{4} \leqslant \frac{3}{8}$$
$$4 \times \frac{3x}{4} \leqslant 4 \times \frac{3}{8}$$
$$3x \leqslant \frac{3}{2}$$
$$3x \div 3 \leqslant \frac{3}{2} \div 3$$
$$x \leqslant \frac{1}{2}$$

(e) $\frac{4}{5}x \leqslant 1\frac{1}{2}$

$5 \times \frac{4}{5}x \leqslant 5 \times 1\frac{1}{2}$

$4x \leqslant 7\frac{1}{2}$

$4x \div 4 \leqslant 7\frac{1}{2} \div 4$

$x \leqslant 1\frac{7}{8}$

(f) $0.4x < 3.2$

$0.4x \div 4 < 3.2 \div 0.4$

$x < 8$

Intermediate

15. (a) $5(3x - 2) - 7(x - 1) = 12$

$15x - 10 - 7x + 7 = 12$

$15x - 7x - 10 + 7 = 12$

$8x - 3 = 12$

$8x = 12 + 3$

$8x = 15$

$x = \frac{15}{8}$

$= 1\frac{7}{8}$

(b) $4(3 - x) + 3(4x + 5) = -45$

$12 - 4x + 12x + 15 = -45$

$-4x + 12x + 12 + 15 = -45$

$8x + 27 = -45$

$8x = -45 - 27$

$8x = -72$

$x = -9$

(c) $0.3(4x - 1) = 0.8 + x$

$1.2x - 0.3 = 0.8 + x$

$1.2x - x = 0.8 + 0.3$

$0.2x = 1.1$

$\frac{0.2x}{0.2} = \frac{1.1}{0.2}$

$x = 5.5$

(d) $3(5x + 2) - 7(3 - x) = (19 + 5x) + (20 - x)$

$15x + 6 - 21 + 7x = 19 + 20 + 5x - x$

$15x + 7x - 15 = 39 + 4x$

$22x - 15 = 39 + 4x$

$22x - 4x = 39 + 15$

$18x = 54$

$x = 3$

(e) $2x - [3 + 5(x - 5)] = 10$

$2x - [3 + 5x - 25] = 10$

$2x - [5x - 22] = 10$

$2x - 5x + 22 = 10$

$-3x = 10 - 22$

$-3x = -12$

$x = 4$

(f) $3x - [3 - 2(3x - 7)] = 37$

$3x - [3 - 6x + 14] = 37$

$3x - [17 - 6x] = 37$

$3x - 17 + 6x = 37$

$3x + 6x = 37 + 17$

$9x = 54$

$x = 6$

16. (a) $\frac{2(x-1)}{3} + \frac{3x}{4} = 0$

$12 \times \frac{2(x-1)}{3} + \frac{3x}{4} = 12 \times 0$

$8(x - 1) + 9x = 0$

$8x - 8 + 9x = 0$

$8x + 9x = 8$

$17x = 8$

$x = \frac{8}{17}$

(b) $\frac{6x+1}{7} - \frac{2x-7}{3} = 4$

$21 \times \left(\frac{6x+1}{7} - \frac{2x-7}{3}\right) = 21 \times 4$

$3(6x + 1) - 7(2x - 7) = 84$

$18x + 3 - 14x + 49 = 84$

$4x + 52 = 84$

$4x = 84 - 52$

$4x = 32$

$x = 8$

(c) $2x - \frac{x}{4} + \frac{3x}{5} = 14 + \frac{7x}{3}$

$2x - \frac{x}{4} + \frac{3x}{5} - \frac{7x}{3} = 14$

$\frac{x}{60} = 14$

$60 \times \frac{x}{60} = 60 \times 14$

$x = 840$

(d) $5x - 1\frac{3}{4} = 6 + 1\frac{2}{3}x - \frac{5}{6}$

$5x - 1\frac{3}{4}x = 5\frac{1}{6} + 1\frac{2}{3}x$

$5x - 1\frac{2}{3}x = 5\frac{1}{6} + 1\frac{3}{4}$

$3\frac{1}{3}x = 6\frac{11}{12}$

$x = 2\frac{3}{40}$

(e) $\frac{x}{4} = \frac{x+12}{10} + 0.6$

$\frac{x}{4} = \frac{x}{10} + \frac{12}{10} + 0.6$

$\frac{x}{4} - \frac{x}{10} = 1.2 + 0.6$

$\frac{3x}{20} = 1.8$

$x = 12$

(f) $\frac{3x-4}{6} - \frac{2x+3}{8} = \frac{2x-7}{24}$

$24 \times \left(\frac{3x-4}{6} - \frac{2x+3}{8}\right) = 24 \times \frac{2x-7}{24}$

$4(3x-4) - 3(2x+3) = 2x - 7$

$12x - 16 - 6x - 9 = 2x - 7$

$6x - 25 = 2x - 7$

$6x - 2x = -7 + 25$

$4x = 18$

$x = 4\frac{1}{2}$

(g) $\frac{5x-1}{8} - \frac{5-7x}{2} = \frac{3(6-x)}{6}$

$24 \times \left(\frac{5x-1}{8} - \frac{5-7x}{2}\right) = 24 \times \frac{3(6-x)}{6}$

$3(5x-1) - 12(5-7x) = 12(6-x)$

$15x - 3 - 60 + 84x = 72 - 12x$

$99x - 64 = 72 - 12x$

$99x + 12x = 72 + 63$

$111x = 135$

$x = 1\frac{8}{37}$

(h) $\frac{5x+2}{7} = \frac{x-3}{5} + x + 1.5$

$35 \times \frac{5x+2}{7} = 35 \times \left(\frac{x-3}{5} + x + 1.5\right)$

$5(5x+2) = 7(x-3) + 35x + 52.5$

$25x + 10 = 7x - 21 + 35x + 52.5$

$25x + 10 = 42x + 31.5x$

$25x - 42x = 31.5 - 10$

$-17x = 21.5$

$17x = -21.5$

$x = -1\frac{9}{34}$

(i) $\frac{x}{3} - \frac{7(x-2)}{9} = 4 - \frac{2x-5}{6}$

$18 \times \left(\frac{x}{3} - \frac{7(x-2)}{9}\right) = 18 \times \left(4 - \frac{2x-5}{6}\right)$

$6(x) - 14(x-2) = 72 - 3(2x-5)$

$6x - 14x + 28 = 72 - 6x + 15$

$-8x + 28 = 87 - 6x$

$-8x + 6x = 87 - 28$

$-2x = 59$

$x = -29.5$

(j) $0.5x + 2 = \frac{1}{4} + \frac{x-1}{2} + \frac{x}{4} - \frac{1}{6}$

$0.5x + 2 = \frac{1}{4} + \frac{x}{2} - \frac{1}{2} + \frac{x}{4} - \frac{1}{6}$

$0.5x - \frac{x}{2} - \frac{x}{4} = \frac{1}{4} - \frac{1}{2} - \frac{1}{6} - 2$

$-\frac{x}{4} = -2\frac{5}{12}$

$x = 9\frac{2}{3}$

(k) $4x + 1 - \frac{1}{2}(3x-2) - \frac{1}{3}(4x-1) = 0$

$6 \times \left(4x + 1 - \frac{1}{2}(3x-1) - \frac{1}{3}(4x-1)\right) = 6 \times 0$

$24x + 6 - 3(3x-2) - 2(4x-1) = 0$

$24x + 6 - 9x + 6 - 8x + 2 = 0$

$24x - 9x - 8x + 6 + 6 + 2 = 0$

$7x + 14 = 0$

$7x = -14$

$x = -2$

(l) $\frac{1}{2}\left(2x - \frac{1}{2}\right) = \frac{1}{3}\left(3x - \frac{1}{4}\right) + \frac{1}{4}(4x-3)$

$x - \frac{1}{4} = x - \frac{1}{12} + x - \frac{3}{4}$

$x - x - x = -\frac{1}{12} - \frac{3}{4} + \frac{1}{4}$

$-x = -\frac{7}{12}$

$x = \frac{7}{12}$

17. (a) $\frac{3}{x} + \frac{4}{x} = 5$

$\frac{7}{x} = 5$

$x \times \frac{7}{x} = x \times 5$

$7 = 5x$

$x = \frac{7}{5} = 1\frac{2}{5}$

(b) $\frac{5}{2x} - \frac{7}{5x} = \frac{2}{3}$

$10x \times \left(\frac{5}{2x} - \frac{7}{5x}\right) = 10x \times \frac{2}{3}$

$25 - 14 = 6\frac{2}{3}x$

$11 = 6\frac{2}{3}x$

$x = 1\frac{13}{20}$

(c) $\frac{7}{2x} + \frac{5}{3x} = 1\frac{5}{6}$

$$6x \times \left(\frac{7}{2x} + \frac{5}{3x}\right) = 6x \times 1\frac{5}{6}$$

$$21 + 10 = 11x$$

$$31 = 11x$$

$$x = 2\frac{9}{11}$$

(d) $\frac{5}{x+2} - \frac{4}{2x+4} = 6$

$$\frac{5}{x+2} - \frac{4}{2(x+2)} = 6$$

$$\frac{10}{2(x+2)} - \frac{4}{2(x+2)} = 6$$

$$\frac{6}{2(x+2)} = 6$$

$$12(x+2) = 6$$

$$12x + 24 = 6$$

$$12x = 6 - 24$$

$$12x = -18$$

$$x = -1\frac{1}{2}$$

(e) $1 - \frac{x+1}{3x+5} = \frac{1}{2}$

$$\frac{x+1}{3x+5} = 1 - \frac{1}{2}$$

$$\frac{x+1}{3x+5} = \frac{1}{2}$$

$$2(x+1) = 3x + 5$$

$$2x + 2 = 3x + 5$$

$$3x - 2x = 2 - 5$$

$$x = -3$$

18. $5(2x - 3) - 3(x - 2) = 0$

$$10x - 15 - 3x + 6 = 0$$

$$10x - 3x - 15 + 6 = 0$$

$$7x - 9 = 0$$

$$7x - 9 + 11 = 0 + 11$$

$$7x + 2 = 11$$

19. When $x = 4$,

LHS

$$= -2 - \frac{2 \times 4}{5} + \frac{3 \times 4}{2}$$

$$= -2 - \frac{8}{5} + \frac{12}{2} = 2\frac{2}{5} \neq 4\frac{3}{5} \text{ (RHS)}$$

$\therefore$ No, $x = 4$ is not a solution of the equation.

20. When $y = 2$, $p = 5$ and $q = 6$,

$$x - 2 = \frac{x(2)}{5 - 6}$$

$$x - 2 = \frac{2x}{-1}$$

$$x - 2 = -2x$$

$$x + 2x = 2$$

$$3x = 2$$

$$x = \frac{2}{3}$$

21. When $y = 8$ and $z = 2$,

$$\frac{x-1}{8+3} - \frac{x}{8} = \frac{1}{2}$$

$$\frac{x-1}{11} - \frac{x}{8} = \frac{1}{2}$$

$$\frac{x}{11} - \frac{1}{11} - \frac{x}{8} = \frac{1}{2}$$

$$\frac{x}{11} - \frac{x}{8} = \frac{1}{2} + \frac{1}{11}$$

$$-\frac{3}{88}x = \frac{13}{22}$$

$$x = -17\frac{1}{3}$$

22. $v^2 = u^2 + 2as$

When $u = 15$, $a = 9.81$, $s = 14.45$,

$$v^2 = (15)^2 + 2(9.81)(14.45)$$

$$= 508.509$$

$$v = \pm\sqrt{508.509}$$

$\therefore v = \pm 22.6$ (to 3 s.f.)

23. When $a = 3\frac{1}{2}$, $h = 10$ and $k = 15$,

$$\frac{1}{x} = \left(3\frac{1}{2} - 2\right)\left(\frac{1}{10} + \frac{1}{15}\right)$$

$$= \left(1\frac{1}{2}\right)\left(\frac{1}{6}\right)$$

$$= \frac{1}{4}$$

$\therefore x = 4$

24. When $y = 6$ and $z = -\frac{1}{2}$,

$$\frac{3x + 2(6) - 5\left(-\frac{1}{2}\right)}{6 - 4\left(-\frac{1}{2}\right)} = \frac{x}{3(6)}$$

$$\frac{3x + 12 + 2\frac{1}{2}}{8} = \frac{x}{18}$$

$$\frac{3x + 14\frac{1}{2}}{8} = \frac{x}{18}$$

$$8 \times \frac{3x + 14\frac{1}{2}}{8} = 8 \times \frac{x}{18}$$

$$3x + 14\frac{1}{2} = \frac{4x}{9}$$

$$3x - \frac{4x}{9} = -14\frac{1}{2}$$

$$2\frac{5}{9}x = -14\frac{1}{2}$$

$$x = -5\frac{31}{46}$$

25. When $p = 3$, $q = -2$,

$$\frac{5(3) - 3(-2)}{r} = \frac{3(-2) - 5(3)}{3 + (-2)}$$

$$\frac{15 + 6}{r} = \frac{-6 - 15}{1}$$

$$\frac{21}{r} = \frac{-21}{1}$$

$$21 = -21r$$

$$r = -1$$

26. $A = P + \frac{PRT}{100}$

(a) When $P = 5000$, $R = 5$ and $T = 3$,

$$A = 5000 + \frac{(5000)(5)(3)}{100}$$
$$= 5750$$

(b) When $A = 6500$, $R = 5$ and $T = 1\frac{2}{3}$,

$$6500 = P + \frac{P(5)\left(1\frac{2}{3}\right)}{100}$$

$$6500 = P + \frac{1}{12}P$$

$$6500 = 1\frac{1}{12}P$$

$$1\frac{1}{12}P = 6500$$

$$P = 6500 \div 1\frac{1}{12} = 6000$$

27. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

(a) When $u = 5$ and $v = 7$,

$$\frac{1}{f} = \frac{1}{5} + \frac{1}{7}$$

$$\frac{1}{f} = \frac{12}{35}$$

$$12f = 35$$

$$\therefore f = \frac{35}{12} = 2\frac{11}{12}$$

(b) When $f = 4$ and $v = 5$,

$$\frac{1}{4} = \frac{1}{u} + \frac{1}{5}$$

$$\frac{1}{u} = \frac{1}{4} - \frac{1}{5}$$

$$\frac{1}{u} = \frac{1}{20}$$

$$\therefore u = 20$$

28. (a) (i) Let the first number be x.
Then the second number is mx.
Then the third number is $mx - n$.
Sum of the three numbers $S = x + mx + mx - n$
$= x + 2mx - n$

(ii) When $S = 109$, $m = 4$, $n = 8$,

$$109 = x + 2(4)x - 8$$
$$109 = x + 8x - 8$$
$$9x = 109 + 8$$
$$9x = 117$$
$$\therefore x = 13$$

The three numbers are 13, 4(13) = 52 and 52 – 8 = 44.

(b) (i) The cost of the pair of shoes is $C.
Amount of money Nora has after buying the pair of shoes = $\$(p - C)$
Amount of money Priya has after buying the pair of shoes = $\$(q - C)$

$$p - C = 2(q - C)$$
$$p - C = 2q - 2C$$
$$2C - C = 2q - p$$
$$C = 2q - p$$

(ii) When $p = 42$, $q = 30$,
cost of the pair of shoes $= 2 \times 30 - 42$
$= \$18$

29. (i) Let the number Lixin is thinking of be x.

$2x + 14 = 4x - 8$

(ii) $2x + 14 = 4x - 8$

$14 + 8 = 4x - 2x$

$22 = 2x$

$x = 11$

(iii) The result is $2x + 14 = 2(11) + 14 = 36$.

30. Let the denominator of the fraction be x.

Then the numerator is $x - 1$.

$$\frac{x-1+1}{x+2} = \frac{3}{4}$$

$$\frac{x}{x+2} = \frac{3}{4}$$

$4x = 3(x + 2)$

$4x = 3x + 6$

$4x - 3x = 6$

$x = 6$

Then the numerator is $6 - 1 = 5$.

The original fraction is $\frac{5}{6}$.

31. (i) The woman's present age is $8x$ years old

(ii) Michael's age two years ago was $(x - 2)$ years old.

(iii) The woman's age two years ago was

$= (8x - 2)$ years old

$8x - 2 = 15(x - 2)$

$8x - 2 = 15x - 30$

$8x - 15x = -30 + 2$

$-7x = -28$

$7x = 28$

$x = 4$

(iv) The woman's present age $= 8 \times 4 = 32$ years old.

The woman's age in 5 years' time

$= 32 + 5$

$= 37$ years old

32. (i) Amount of time spent cycling $= \frac{x}{9}$ hours

(ii) Amount of time spent taking the train

$$= \frac{28}{60} - \frac{x}{9} - \frac{3}{60} - \frac{\frac{1}{2}}{6}$$

$$= \frac{7}{15} - \frac{x}{9} - \frac{3}{60} - \frac{1}{12}$$

$$= \left(\frac{1}{3} - \frac{x}{9}\right) \text{ hours}$$

Distance travelled by Ethan on the MRT train

$$= 60\left(\frac{1}{3} - \frac{x}{9}\right)$$

$$= \left(20 - 6\frac{2}{3}x\right) \text{ km}$$

(iii) $x + 20 - 6\frac{2}{3}x + \frac{1}{2} = 12$

$6\frac{2}{3}x - x = 20 + \frac{1}{2} - 12$

$5\frac{2}{3}x = 8\frac{1}{2}$

$x = 1\frac{1}{2}$

33. Let the number of apples bought be x.

Then the number of oranges bought is $2x$.

Then the number of pears bought is $(x - 5)$.

(i) Amount spent on the fruits = \$77

$x(0.40) + 2x(0.30) + (x - 5)(0.80) = 77$

$0.4x + 0.6x + 0.8x - 4 = 77$

$1.8x - 4 = 77$

$1.8x = 77 + 4$

$1.8x = 81$

$x = 45$

(ii) Amount of money spent on buying the pears

$= (x - 5)(0.80)$

$= (45 - 5)(0.80)$

$= (40)(0.80)$

$= \$32$

He spent \$32 on buying the pears.

34. Let the number of ducks bought be x.

Then the number of chicken bought is $3x$.

The number of geese bought is $0.5x$.

Total cost = \$607.20

$x(7.5) + 3x(3.8) + 0.5x(12.8) = 607.2$

$7.5x + 11.4x + 6.4x = 607.2$

$25.3x = 607.2$

$x = 24$

The number of geese bought is $0.5 \times 24 = 12$.

35. (i) Amount of money the salesman earned in a week

$$= 90 + \frac{12(580)}{100}$$

$= 90 + 69.60$

$= \$159.60$

(ii) To find the number of articles sold, make n the subject.

$$A = 90 + \frac{12n}{100}$$

$$A - 90 = \frac{12n}{100}$$

$12n = 100(A - 90)$

$$n = \frac{100(A-90)}{12}$$

$$= \frac{100(190.80 - 90)}{12}$$

$$= \frac{100(100.80)}{12}$$

$= 840$

(iii) $A = 80 + \frac{16n}{100}$

(iv) For the same amount of money earned before and after

$$90 + \frac{12n}{100} = 80 + \frac{16n}{100}$$
$$90 - 80 = \frac{16n}{100} - \frac{12n}{100}$$
$$\frac{n}{25} = 10$$
$$n = 250$$

The number of articles the salesman must sell in a week to earn the same amount of money before and after the adjustments is 250.

36. (i) Rui Feng's brother's age is $0.5 \times 4x = 2x$ years old.
Sum of their present ages = $4x + 2x = 6x$ years old

(ii) In 8 years' time,
Rui Feng is $(4x + 8)$ years old and his brother is $(2x + 8)$ years old.
Sum of their ages in 8 years' time
$= (4x + 8) + (2x + 8)$
$= 4x + 2x + 8 + 8$
$= (6x + 16)$ years old

37. Let the second number be x.
Then the first number is $(x + 5)$.
Then the third number is $0.5x$.
The fourth number is $3[(x + 5) + x] = 3(2x + 5)$.
The total of the four numbers is $56 \times 4 = 224$.

$$(x + 5) + x + 0.5x + 3(2x + 5) = 224$$
$$x + 5 + x + 0.5x + 6x + 15 = 224$$
$$x + x + 0.5x + 6x + 5 + 15 = 224$$
$$8.5x + 20 = 224$$
$$8.5x = 224 - 20$$
$$8.5x = 204$$
$$x = 24$$

The numbers are $24 + 5 = 29$, 24, $0.5(24) = 12$ and $3(2 \times 24 + 5) = 159$.

38. $2x \leqslant 7$
$$x \leqslant 3\frac{1}{2}$$
The largest rational number is $3\frac{1}{2}$.

39. Let $\$x$ be the amount of money each student will get.
$$32x \leqslant 4385$$
$$\frac{32x}{32} \leqslant \frac{4385}{32}$$
$$x \leqslant 137.031\ 25$$
Each student will get a maximum amount of \$135 (to the nearest \$5).

40. Let the number of concert tickets be x.
$$25x \leqslant 115$$
$$x \leqslant 4\frac{3}{5}$$
∴ The maximum number of tickets that can be purchased is 4.

41. Let the number of cakes be x.
$$4x \leqslant 39$$
$$x \leqslant 9\frac{3}{4}$$
∴ The maximum number of cakes that can be bought is 9.

42. Let the age of the woman be x years old.
Then her husband is $(x + 3)$ years old.
$$x + (x + 3) \leqslant 55$$
$$2x + 3 \leqslant 55$$
$$2x \leqslant 52$$
$$x \leqslant 26$$
∴ The maximum possible age of the woman is 26.

43. Let the first number be x.
Then the second number is $x + 1$ and the third number is $x + 2$.
$$x + x + 1 + x + 2 < 80$$
$$3x + 3 < 80$$
$$3x < 77$$
$$x < 25\frac{2}{3}$$
∴ The largest possible value of the largest integer is 27.

Advanced

44. $x^3 + 6x^2$
$= x(x^2 + 5x) + x^2$
$= x(5) + x^2$
$= x^2 + 5x$
$= 5$

45. $$\frac{x^2 - 3xy}{y^2 - 2z} = \frac{5y}{3}$$
When $y = 2$ and $z = -5$;
$$\frac{x^2 - 3x(2)}{(2)^2 - 2(-5)} = \frac{5(2)}{3}$$
$$\frac{x^2 - 6x}{14} = \frac{10}{3}$$
$$14 \times \frac{x^2 - 6x}{14} = 14 \times \frac{10}{3}$$
$$x^2 - 6x = 46\frac{2}{3}$$
By trial and error, x is approximately 10.45.
∴ $x = 10.5$ (to 3 s.f.)

46. **(a)** $5(x-2)^2 = 35$

$5(x-2)^2 \div 5 = 35 \div 5$

$(x-2)^2 = 7$

$x - 2 = \pm\sqrt{7}$

$x = 2 \pm \sqrt{7}$

$x = 4.65$ (to 2 d.p.) or $x = -0.65$ (to 2 d.p.)

(b) $\dfrac{\dfrac{2x-3}{4} - 2}{x} = 3\dfrac{1}{2}$

$\dfrac{\dfrac{2x-3}{4} - 2}{x} = \dfrac{7}{2}$

$2\left(\dfrac{2x-3}{4} - 2\right) = 7x$

$\dfrac{2x-3}{2} - 4 = 7x$

$\dfrac{2x-3}{2} = 7x + 4$

$2 \times \dfrac{2x-3}{2} = 2 \times (7x + 4)$

$2x - 3 = 14x + 8$

$2x - 14x = 8 + 3$

$-12x = 11$

$12x = -11$

$x = -\dfrac{11}{12}$

47. Let the first number be x.

Let the second number be $84 - x$.

$\dfrac{1}{2}x - \dfrac{1}{3}(84 - x) = 2$

$\dfrac{1}{2}x + \dfrac{1}{3}x - 28 = 2$

$\dfrac{5}{6}x = 2 + 28$

$\dfrac{5}{6}x = 30$

$5x = 180$

$x = 36$

The two numbers are 36 and 48.

48. In 1 hour, Raj can complete $\dfrac{1}{3}$ of the task.

In 1 hour, Farhan can complete $\dfrac{1}{2}$ of the task.

In 1 hour, when they work together, they can complete $\dfrac{1}{2} + \dfrac{1}{3} = \dfrac{5}{6}$ of the task

$\therefore$ It takes them $\dfrac{6}{5}$ hours = 1 hour and 12 minutes to complete the task.

49. Let the first number be x.

Then the second number is $x + 2$.

$x + x + 2 < 15$

$2x < 13$

$x < 6\dfrac{1}{2}$

$\therefore$ The largest possible value of the smaller integer is 5.

New Trend

50. $\dfrac{2x-1}{3} - \dfrac{3x-4}{5} = \dfrac{4}{7}$

$15 \times \left(\dfrac{2x-1}{3} - \dfrac{3x-4}{5}\right) = 15 \times \dfrac{4}{7}$

$5(2x-1) - 3(3x-4) = 8\dfrac{4}{7}$

$10x - 5 - 9x + 12 = 8\dfrac{4}{7}$

$x + 7 = 8\dfrac{4}{7}$

$x = 8\dfrac{4}{7} - 7$

$x = 1\dfrac{4}{7}$

51. $\dfrac{3}{2x+5} = \dfrac{4}{1-3x}$

$3(1-3x) = 4(2x+5)$

$3 - 9x = 8x + 20$

$9x + 8x = 3 - 20$

$17x = -17$

$x = -1$

52. $5(2-3x) - (1+7x) = 5(3-6x)$

$10 - 15x - 1 - 7x = 15 - 30x$

$9 - 22x = 15 - 30x$

$-22x + 30x = 15 - 9$

$8x = 6$

$x = \dfrac{6}{8}$

$= \dfrac{3}{4}$

53. $\dfrac{3x+2}{4} = \dfrac{2x-1}{3}$

$12 \times \dfrac{3x+2}{4} = 12 \times \dfrac{2x-1}{3}$

$3(3x+2) = 4(2x-1)$

$9x + 6 = 8x - 4$

$9x - 8x = -4 - 6$

$x = -4 - 6$

$= -10$

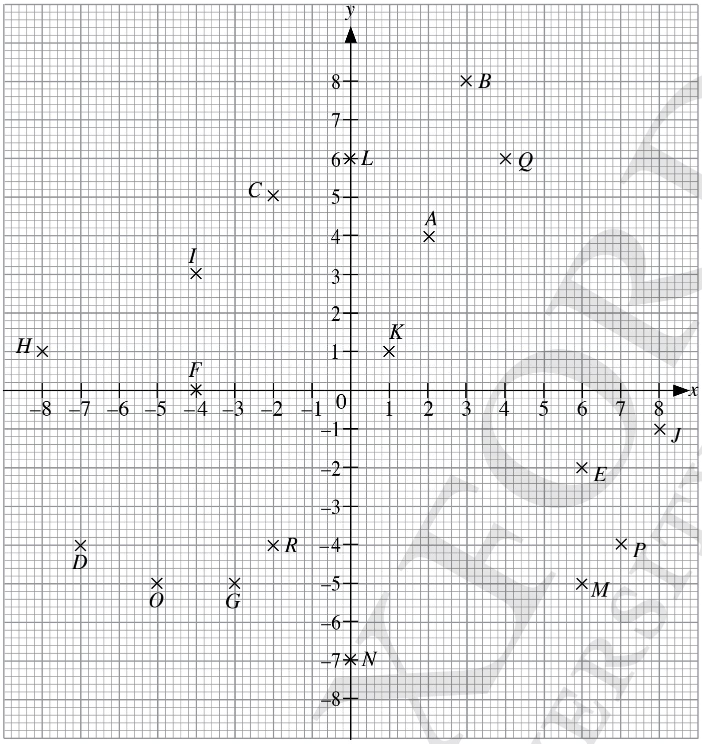
Chapter 6 Functions and Linear Graphs

Basic

1. We can find the coordinates from the graph. Each ordered pair determines the points A to R.

$A(1, 2)$	$B(7, 1)$	$C(-2, -3)$
$D(-4, 5)$	$E(6, 6)$	$F(3, -2)$
$G(-6, -2)$	$H(5, 0)$	$I(0, -5)$
$J(-7, 4)$	$L(-3, 0)$	$M(0, 3)$
$N(-5, 2)$	$O(0, 0)$	$P(6, -4)$
$Q(-3, -6)$	$R(4, -6)$	

2.



3. **(i)** When $x = 2$, $y = 5 \times 2 + 7 = 17$.

(ii) When $x = -3$, $y = 5 \times (-3) + 7 = -8$.

4. **(i)** When $y = -8$,

$-8 = 10 - 9x$

$9x = 10 + 8 = 18$

$x = 2$

(ii) When $y = -26$,

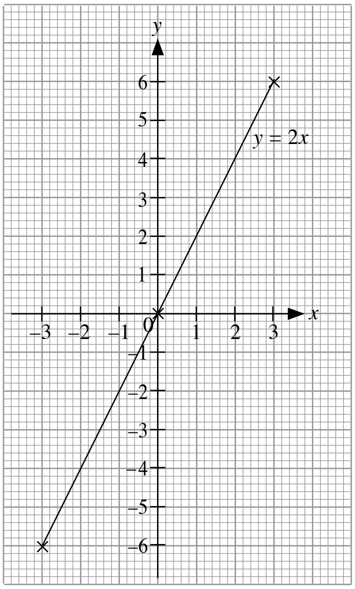
$-26 = 10 - 9x$

$9x = 10 + 26 = 36$

$x = 4$

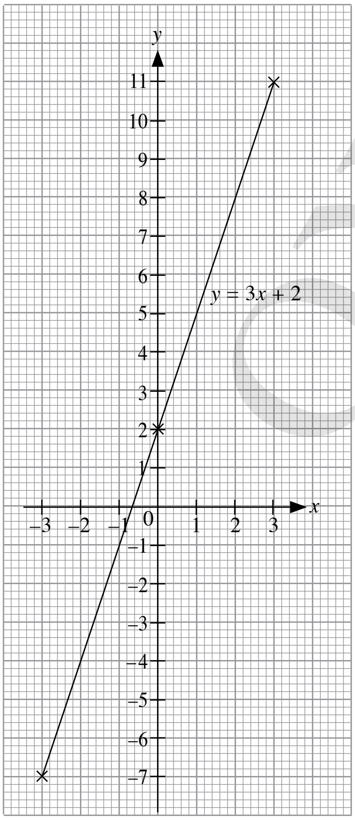
5. **(a)** $y = 2x$

x	–3	0	3
$y = 2x$	–6	0	6



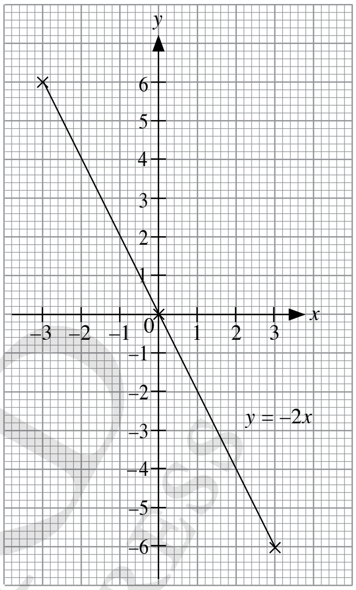
(b) $y = 3x + 2$

x	–3	0	3
$y = 3x + 2$	–7	2	11



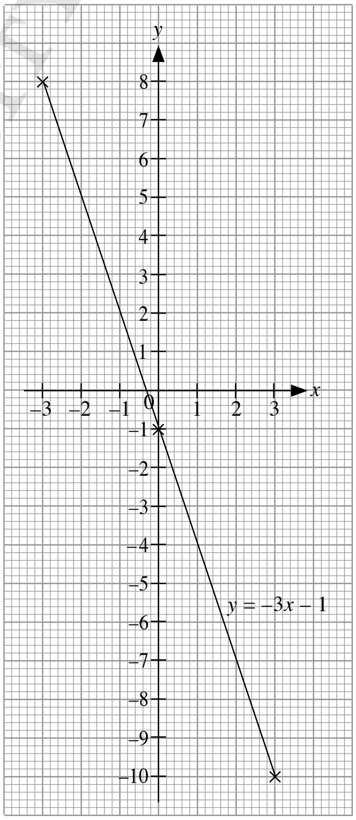
(c) $y = -2x$

x	–3	0	3
$y = -2x$	6	0	–6



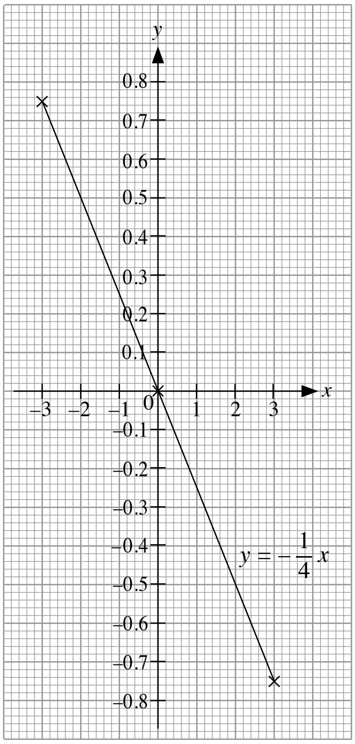
(d) $y = -3x - 1$

x	–3	0	3
$y = -3x - 1$	8	–1	–10



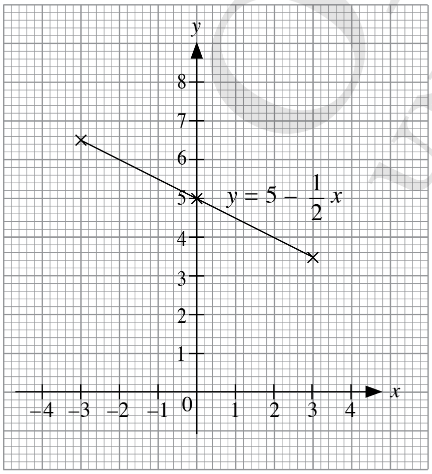
(e) $y = -\frac{1}{4}x$

x	–3	0	3
$y = -\frac{1}{4}x$	$\frac{3}{4}$	0	$-\frac{3}{4}$

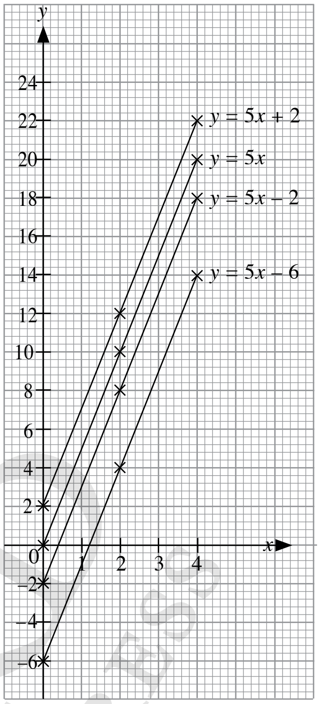


(f) $y = 5 - \frac{1}{2}x$

x	–3	0	3
$y = 5 - \frac{1}{2}x$	$6\frac{1}{2}$	5	$3\frac{1}{2}$

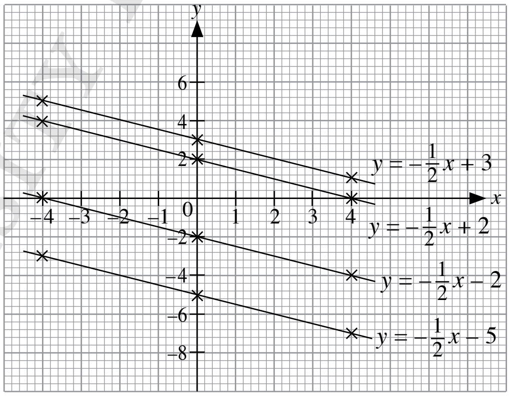


6. **(a)**



(b) They are parallel lines.

7. **(a)**



(b) They are parallel lines.

8. **(a)** **(i)** Amount of money left after 2 days
= 30 – 5 × 2 = \$20

(ii) Amount of money left after 3 days
= 30 – 5 × 3 = \$15

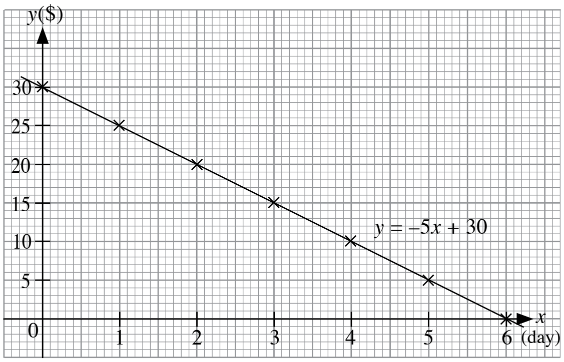
(iii) Amount of money left after 4 days
= 30 – 5 × 4 = \$10

(iv) Amount of money left after 5 days
= 30 – 5 × 5 = \$5

(b)

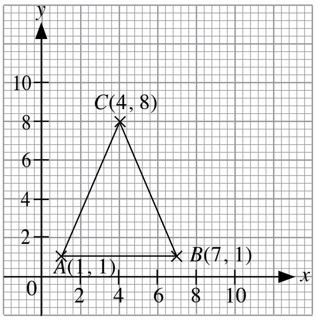
x	0	1	2	3	4	5	6
y	30	25	20	15	10	5	0

(c)



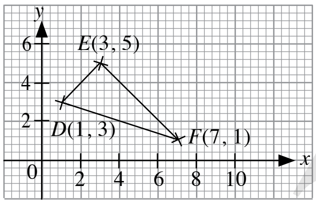
Intermediate

9. **(a)**



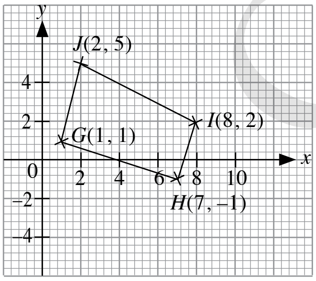
Isosceles triangle

(b)



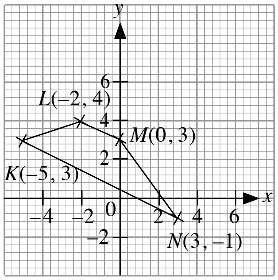
Right-angled triangle

(c)



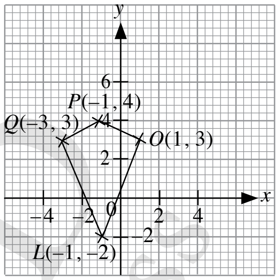
Quadrilateral

(d)



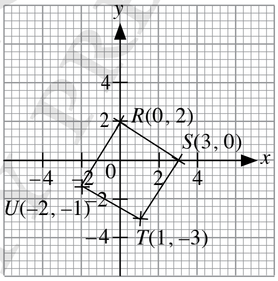
Trapezium

(e)



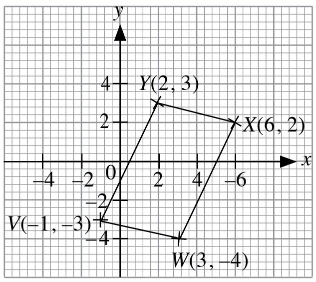
Kite

(f)



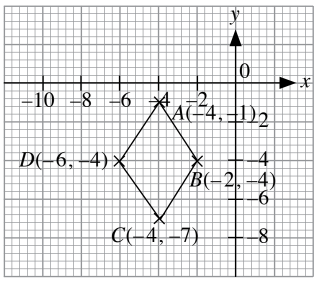
Square

(g)



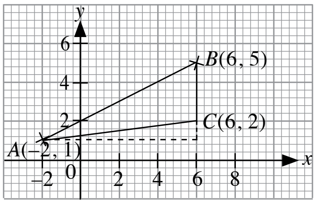
Parallelogram

(h)



Rhombus

10.

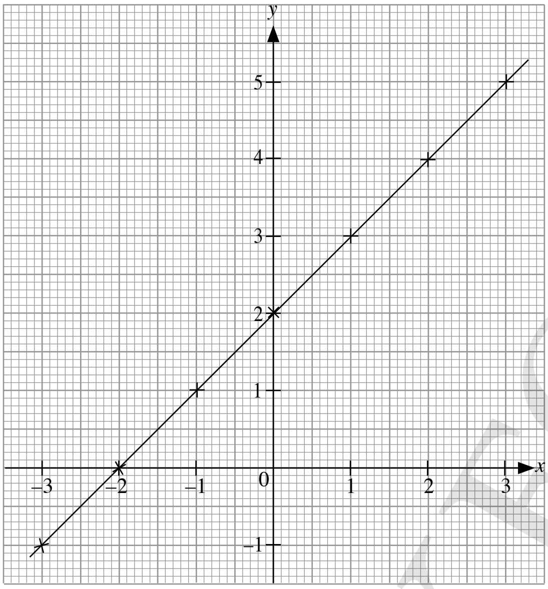


Area of triangle $ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 3 \times 8$

$= 12$ square units

11.



The points can be joined to form a straight line that slopes upwards from left to right.

12. **(i)** When $x = 5$, $y = \frac{3}{5} \times 5 - \frac{1}{2} = 2\frac{1}{2}$.

(ii) When $x = 3\frac{1}{2}$, $y = \frac{3}{5} \times 3\frac{1}{2} - \frac{1}{2} = 1\frac{3}{5}$.

(iii) When $x = -\frac{2}{3}$, $y = \frac{3}{5} \times \left(-\frac{2}{3}\right) - \frac{1}{2} = -\frac{9}{10}$.

13. **(i)** When $y = 12$,

$12 = 15 - \frac{3}{4}x$

$\frac{3}{4}x = 15 - 12$

$\frac{3}{4}x = 3$

$3x = 12$

$x = 4$

(ii) When $y = 21$,

$21 = 15 - \frac{3}{4}x$

$\frac{3}{4}x = 15 - 21$

$\frac{3}{4}x = -6$

$3x = -24$

$x = -8$

(iii) When $y = -60$,

$-60 = 15 - \frac{3}{4}x$

$\frac{3}{4}x = 15 + 60$

$\frac{3}{4}x = 75$

$3x = 300$

$x = 100$

14. For the line $y = 3x + 2$,

(a) When $x = 1$, $y = 5$, then
LHS = 5
RHS = $3 \times 1 + 2 = 5$
Since LHS = RHS, then $A(1, 5)$ lies on the line.

(b) When $x = 3$, $y = 12$, then
LHS = 12
RHS = $3 \times 3 + 2 = 11$
Since LHS ≠ RHS, then $B(3, 12)$ does not lie on the line.

(c) When $x = 0$, $y = 2$, then
LHS = 2
RHS = $3 \times 0 + 2 = 2$
Since LHS = RHS, then $C(0, 2)$ lies on the line.

(d) When $x = -2$, $y = 4$, then
LHS = 4
RHS = $3 \times (-2) + 2 = -4$
Since LHS ≠ RHS, then $D(-2, 4)$ does not lie on the line.

(e) When $x = -\frac{1}{3}$, $y = 1$, then
LHS = 1
RHS = $3 \times \left(-\frac{1}{3}\right) + 2 = 1$
Since LHS = RHS, then $E\left(-\frac{1}{3}, 1\right)$ lies on the line.

15. For the line $y = -\frac{1}{2}x - 2$,

(a) When $x = 2$, $y = -1$, then

LHS $= -1$

RHS $= -\frac{1}{2} \times 2 - 2 = -3$

Since LHS ≠ RHS, then $A(2, -1)$ does not lie on the line.

(b) When $x = -4$, $y = 0$, then

LHS $= 0$

RHS $= -\frac{1}{2} \times (-4) - 2 = 0$

Since LHS = RHS, then $B(-4, 0)$ lies on the line.

(c) When $x = \frac{2}{3}$ and $y = -\frac{7}{3}$, then

LHS $= -\frac{7}{3}$

RHS $= -\frac{1}{2} \times \frac{2}{3} - 2 = -\frac{7}{3}$

Since LHS = RHS, then $C\left(\frac{2}{3}, -\frac{7}{3}\right)$ lies on the line.

(d) When $x = -\frac{1}{2}$, $y = -\frac{7}{4}$, then

LHS $= -\frac{7}{4}$

RHS $= -\frac{1}{2} \times \left(-\frac{1}{2}\right) - 2 = -\frac{7}{4}$

Since LHS = RHS, then $D\left(-\frac{1}{2}, -\frac{7}{4}\right)$ lies on the line.

(e) When $x = 10$, $y = -3$, then

LHS $= -3$

RHS $= -\frac{1}{2} \times 10 - 2 = -7$

Since LHS ≠ RHS, then $E(10, -3)$ does not lie on the line.

16. (a) When $x = -3$,

$y = \frac{3 \times (-3) + 7}{2} = \frac{-2}{2} = -1$

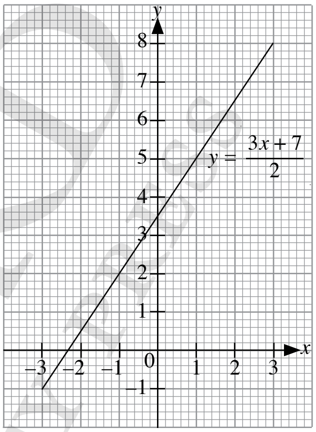
When $x = 0$,

$y = \frac{3 \times 0 + 7}{2} = \frac{7}{2}$

When $x = 3$,

$y = \frac{3 \times 3 + 7}{2} = \frac{16}{2} = 8$

x	-3	0	3
y	-1	$\frac{7}{2}$	8



(b) When $x = -3$,

$y = \frac{-2 \times (-3) + 1}{2} = \frac{7}{2}$

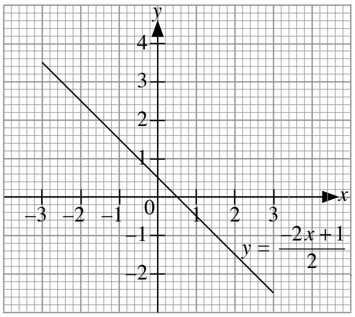
When $x = 0$,

$y = \frac{-2 \times 0 + 1}{2} = \frac{1}{2}$

When $x = 3$,

$y = \frac{-2 \times 3 + 1}{2} = -\frac{5}{2}$

x	-3	0	3
y	$\frac{7}{2}$	$\frac{1}{2}$	$-\frac{5}{2}$



(c) When $x = -3$,

$2y + 3 \times (-3) = 4$

$2y - 9 = 4$

$2y = 4 + 9$

$2y = 13$

$y = \frac{13}{2}$

When $x = 0$,

$2y + 3 \times 0 = 4$

$2y = 4$

$y = 2$

When $x = 3$,

$2y + 3 \times 3 = 4$

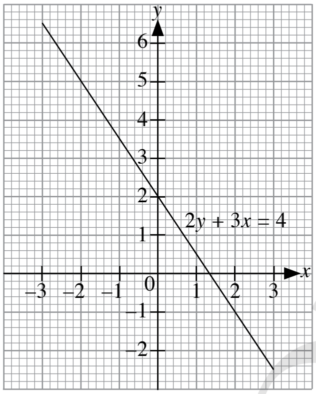
$2y + 9 = 4$

$2y = 4 - 9$

$2y = -5$

$y = -\frac{5}{2}$

x	–3	0	3
y	$\frac{13}{2}$	2	$-\frac{5}{2}$



(d) When $x = -3$,

$5y - 2 \times (-3) = 8$

$5y + 6 = 8$

$5y = 8 - 6$

$5y = 2$

$y = \frac{2}{5}$

When $x = 0$,

$5y - 2 \times 0 = 8$

$5y = 8$

$y = \frac{8}{5}$

When $x = 3$,

$5y - 2 \times 3 = 8$

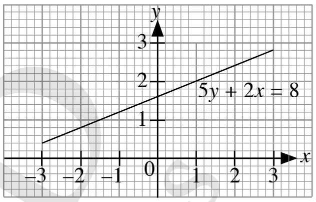
$5y - 6 = 8$

$5y = 8 + 6$

$5y = 14$

$y = \frac{14}{5}$

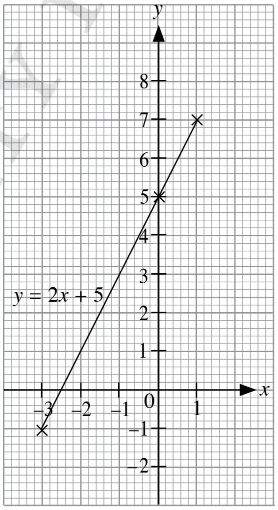
x	–3	0	3
y	$\frac{2}{5}$	$\frac{8}{5}$	$\frac{14}{5}$



17. (a)

x	–3	0	1
$y = 2x + 5$	$y = 2 \times (-3) + 5$ $= -1$	$y = 2 \times 0 + 5$ $= 5$	$y = 2 \times 1 + 5$ $= 7$

(b)

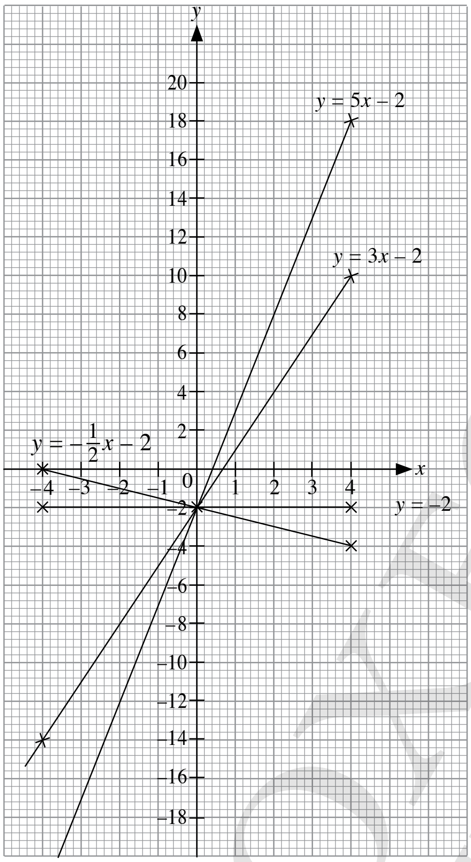


(c) From the graph, the value of x is about $-5\frac{3}{4}$.

(Note: It is necessary to extrapolate the graph so that we can find the value of x when y is less than –6.)

18. (a)

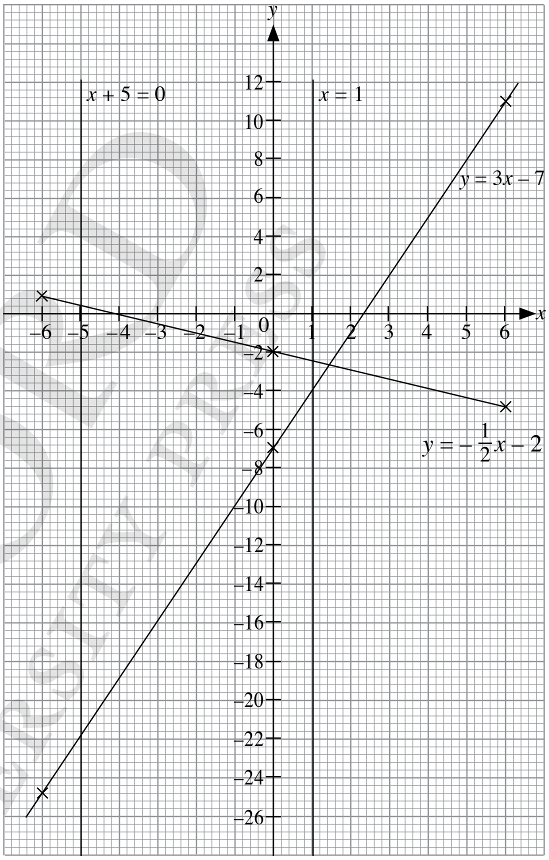
x	-4	0	4
$y = 3x - 2$	$y = 3 \times (-4) - 2$ $= -14$	$y = 3 \times 0 - 2$ $= -2$	$y = 3 \times 4 - 2 = 10$
$y = 5x - 2$	$y = 5 \times (-4) - 2$ $= -22$	$y = 5 \times 0 - 2$ $= -2$	$y = 5 \times 4 - 2 = 18$
$y = -\frac{1}{2}x - 2$	$y = -\frac{1}{2} \times (-4) - 2$ $= 0$	$y = -\frac{1}{2} \times 0 - 2$ $= -2$	$y = -\frac{1}{2} \times 4 - 2$ $= -4$
$y = -2$	$y = -2$	$y = -2$	$y = -2$



(b) All the lines pass through the point (0, –2).

19. (a)

x	-6	0	6
$x = 1$	N.A.	N.A.	N.A.
$x + 5 = 0$	N.A.	N.A.	N.A.
$y = -\frac{1}{2}x - 2$	$y = -\frac{1}{2} \times (-6) - 2$ $= 1$	$y = -\frac{1}{2} \times 0 - 2$ $= -2$	$y = -\frac{1}{2} \times 6 - 2$ $= -5$
$y = 3x - 7$	$y = 3 \times (-6) - 7$ $= -25$	$y = 3 \times 0 - 7$ $= -7$	$y = 3 \times 6 - 7$ $= 11$



(b) The shape of the figure formed by the lines is a trapezium.

(c) In order to find the area bounded by the lines, locate the coordinates of the points of intersection of the lines.

From the graph, the coordinates of points of intersections of the lines are

(1, –2.5), (1, –4), (–5, 0.5) and (–5, –22)

Area bounded by the lines

$= \frac{1}{2} \times (1.5 + 22.5) \times 6$

= 72 square units

20. (a) (i) Amount of money the house owner has to pay
$= 35 + 1 \times 15 = \$50$

(ii) Amount of money the house owner has to pay
$= 35 + 2 \times 15 = \$65$

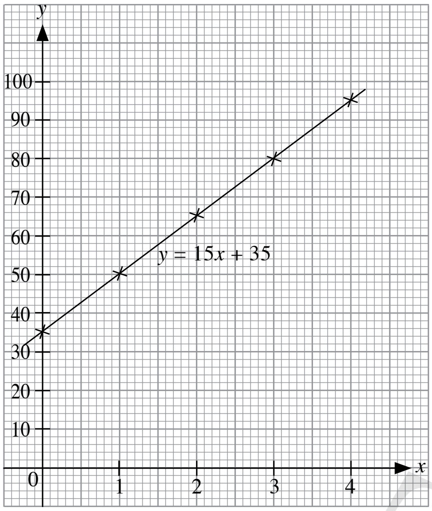
(iii) Amount of money the house owner has to pay
$= 35 + 3 \times 15 = \$80$

(iv) Amount of money the house owner has to pay
$= 35 + 4 \times 15 = \$95$

(b)

x	0	1	2	3	4
y	35	50	65	80	95

(c)



(d) (i) From the graph, the amount of money charged if he spends 2.5 hours on the job
$= \$72.50$

(ii) From the graph, the number of hours that the electrician spends on the job
≈ 3.65 hours

21. (a) From the graph, the values of C can be obtained.

N	50	100	150	200
C	150	200	250	300

(b) (i) From the graph, the value of m is 100.

(ii) He has to pay $\$m$ for the operation cost of printing the newsletters.

(c) From the graph, the amount of money Raj has to pay is \$175.

(d) From the graph, the maximum number of newsletters he can print is 155.

Chapter 7 Number Patterns

Basic

1. **(a)** Rule: Add 5 to each term to get the next term. The next two terms are 26 and 31.
   (b) Rule: Subtract 3 from each term to get the next term. The next two terms are 19 and 16.
   (c) Rule: Multiply each term by 10 to get the next term. The next two terms are 10 000 and 100 000.
   (d) Rule: Multiply each term by 5 to get the next term. The next two terms are 250 and 1250.
   (e) Rule: Multiply the previous term by the term number to get the next term. The next two terms are $24 \times 5 = 120$ and $120 \times 6 = 720$.
   (f) Rule: Take the cube of each term number to get the next term. The next two terms are $5^3 = 125$ and $6^3 = 216$.
   (g) Rule: Subtract 5 from each term to get the next term. The next two terms are 32 and 27.
   (h) Rule: Denote $64 = 8^2$ as the first term. Subtract 1 from the base of each term and square it to get the next term. The next two terms are $4^2 = 16$ and $3^2 = 9$.
   (i) Rule: Add the previous term by its term number to get the next term. The next two terms are $12 + 5 = 17$ and $17 + 6 = 23$.
   (j) Rule: Add the square of the term number to each term to get the next term. The next two terms are $34 + 5^2 = 59$ and $59 + 6^2 = 95$.
   (k) Rule: Add the term number to the previous term to get the next term. The next two terms are $30 + 5 = 35$ and $35 + 6 = 41$.
   (l) Denote 7 as the zero term.
   Rule: Add each term by 2 to the power of its term number to get the next term. The next two terms are $22 + 2^4 = 38$ and $38 + 2^5 = 70$.
   (m) Denote 90 as the first term.
   Rule 1: Subtract 10 from each odd term to get the next odd term.
   Rule 2: Add 10 to each even term to get the next even term. The next two terms are 60 and 40.
   (n) Rule: Denote $1024 = 2^{10}$ as the first term. Subtract 1 from the power of each term to get the next term. The next two terms are $2^5 = 32$ and $2^4 = 16$.

2. **(a)** Since the common difference is 5, $T_n = 5n + ?$.
   The term before T_1 is $c = T_0 = 12 - 5 = 7$.
   $\therefore$ General term of the sequence, $T_n = 5n + 7$
   (b) Since the common difference is –6, $T_n = -6n + ?$.
   The term before T_1 is $c = T_0 = 83 + 6 = 89$.
   $\therefore$ General term of the sequence, $T_n = -6n + 89$.
   (c) Since the common difference is 7, $T_n = 7n + ?$.
   The term before T_1 is $c = T_0 = 2 - 7 = -5$.
   $\therefore$ General term of the sequence, $T_n = 7n - 5$.
   (d) Since the common difference is 6, $T_n = 6n + ?$.
   The term before T_1 is $c = T_0 = 7 - 6 = 1$.
   $\therefore$ General term of the sequence, $T_n = 6n + 1$.
   (e) Since the common difference is –4, $T_n = -4n + ?$.
   The term before T_1 is $c = T_0 = 39 + 4 = 43$.
   $\therefore$ General term of the sequence, $T_n = -4n + 43$.
   (f) To find the formula, consider the following:
   1, 2, 4, 8, 16, …
   as $2^0, 2^1, 2^2, 2^3, 2^4, \ldots$
   $\therefore$ General term of the sequence, $T_n = 2^{n-1}$, $n = 1, 2, 3, \ldots$
   (g) To find the formula, consider the following:
   2, 6, 18, 54, 162, …
   as $2 \times 3^0, 2 \times 3^1, 2 \times 3^2, 2 \times 3^3, 2 \times 3^4, \ldots$
   $\therefore$ General term of the sequence, $T_n = 2 \times 3^{n-1}$, $n = 1, 2, 3, \ldots$
   (h) To find the formula, consider the following:
   12, 36, 108, 324, 972, …
   as $4 \times 3, 4 \times 3^2, 4 \times 3^3, 4 \times 3^4, 4 \times 3^5, \ldots$
   $\therefore$ General term of the sequence, $T_n = 4 \times 3^n$, $n = 1, 2, 3, \ldots$
   (i) To find the formula, consider the following:
   2000, 1000, 500, 250, 125, …
   as $\frac{4000}{2}, \frac{4000}{2^2}, \frac{4000}{2^3}, \frac{4000}{2^4}, \frac{4000}{2^5}, \ldots$
   $\therefore$ General term of the sequence, $T_n = \frac{4000}{2^n}$, $n = 1, 2, 3, \ldots$

3. **(i)** The next three terms of the sequence are 48, 96 and 192.
   (ii) The next three terms of the sequence are 52, 100, 196.
   Add 4 to the sequence in part **(i)**.

4. **(i)** The next two terms of the sequence are 96 and 192.
   (ii) To find the formula, consider the following:
   $3, 3 \times 2, 6 \times 2, 12 \times 2, 24 \times 2, \ldots$
   $3 \times 2^0, 3 \times 2, 3 \times 2^2, 3 \times 2^3, 3 \times 2^4, \ldots$
   $\therefore$ General term of the sequence, $T_n = 3 \times 2^{n-1}$
   (iii) Let $3 \times 2^{m-1} = 1536$
   $$2^{m-1} = \frac{1536}{3} = 512$$
   By trial and error, $2^9 = 512$
   $\therefore m - 1 = 9$
   $m = 9 + 1 = 10$

5. (i) The next two terms of the sequence are $\frac{1}{5^4} = \frac{1}{625}$ and $\frac{1}{6^5} = \frac{1}{7776}$.

(ii) To find the formula, consider the following:

$\frac{1}{1^0}, \frac{1}{2^1}, \frac{1}{3^2}, \frac{1}{4^3}, \ldots$

$\therefore$ General term of the sequence, $T_n = \frac{1}{n^{n-1}}$, $n = 1, 2, 3, \ldots$

(iii) When $n = 10$, $T_{10} = \frac{1}{10^{10-1}} = \frac{1}{10^9}$.

6. (a) When $n = 1$, $3(1) + 1 = 4$

When $n = 2$, $3(2) + 1 = 7$

When $n = 3$, $3(3) + 1 = 10$

The first three terms are 4, 7 and 10.

(b) When $n = 1$, $2(1) - 7 = -5$

When $n = 2$, $2(2) - 7 = -3$

When $n = 3$, $2(3) - 7 = -1$

The first three terms are -5, -3 and -1.

(c) When $n = 1$, $(1)^2 - 1 = 0$

When $n = 2$, $(2)^2 - 2 = 2$

When $n = 3$, $(3)^2 - 3 = 6$

The first three terms are 0, 2 and 6.

(d) When $n = 1$, $2(1)^2 - 3(1) + 5 = 4$

When $n = 2$, $2(2)^2 - 3(2) + 5 = 7$

When $n = 3$, $2(3)^2 - 3(3) + 5 = 14$

The first three terms are 4, 7 and 14.

(e) When $n = 1$, $\frac{(1)(1-1)}{2} = \frac{(1)(0)}{2} = 0$

When $n = 2$, $\frac{(2)(2-1)}{2} = \frac{(2)(1)}{2} = 1$

When $n = 3$, $\frac{(3)(3-1)}{2} = \frac{(3)(2)}{2} = 3$

The first three terms are 0, 1 and 3.

(f) When $n = 1$, $\frac{2}{1+1} = 1$

When $n = 2$, $\frac{2}{2+1} = \frac{2}{3}$

When $n = 3$, $\frac{2}{3+1} = \frac{2}{4} = \frac{1}{2}$

The first three terms are 1, $\frac{2}{3}$ and $\frac{1}{2}$.

7. (i)

E
D E
D E
D E
D E
D E
D E
D E
E

(ii)

Letter	Number of Letters
A	$2(1) - 1 = 1$
B	$2(2) - 1 = 3$
C	$2(3) - 1 = 5$
D	$2(4) - 1 = 7$
E	$2(5) - 1 = 9$
⋮	⋮
n^{th} letter	T_n

(iii) For the letter J, $2(10) - 1 = 19$.

(iv) Since the common difference is 2, $T_n = 2n + ?$.

The term before T_1 is $c = T_0 = 1 - 2 = -1$.

$\therefore$ General term of the sequence, $T_n = 2n - 1$.

$2n - 1 = 29$

$2n = 29 + 1$

$2n = 30$

$n = 15$

When $n = 15$, it is the letter O.

Intermediate

8. (a) 18, 24

(b) 9, 16

(c) 250, 50

(d) 16, 23

(e) 3, 5

(f) $\frac{16}{17}$, $\frac{22}{23}$

(g) $\frac{17}{1}$, $\frac{1}{23}$

9. (a) The next three terms are 39, 51 and 65.

(b) The prime numbers are 11 and 29.

(c) For the two numbers to have HCF as 13, the two numbers must have a common factor 13 and the other factor less than 13. The other factor must be different for the two numbers.

The possible numbers are 13, 26, 39, 52, 65, …

$\therefore$ The two numbers whose HCF is 13 from this sequence are 39 and 65.

(d) By prime factorisation, $195 = 3 \times 5 \times 13$.

Thus the 3 numbers whose LCM is 195 may be 3×5, 3×13 and 5×13.

$\therefore$ The three numbers whose LCM is 195 from this sequence are 15, 39 and 65.

10. For the sequence 2, 5, 8, 11, … the next few terms are 14, 17, 20, 23, 26, 29, 32, 35, 38, …
For the sequence 3, 8, 13, 18, … the next few terms are 23, 28, 33, 38, 43, …
By listing, the next two numbers which will occur in both sequences are 23 and 38.

11. (i) The next two terms of the sequence are 642 and 621.
(ii) Since the common difference is –21,
$T_n = -21n + ?$.
The term before T_1 is $c = T_0 = 747 + 21 = 768$.
∴ General term of the sequence, $T_n = 768 - 21n$.
(iii) $768 - 21r = 390$
$21r = 768 - 390$
$= 378$
$r = 18$

12. (i) $a = 26 + 9 = 37$, $b = 37 + 13 = 50$ and
$c = 50 + 3 = 53$
(ii) To find the formula, consider the following:
2, 5, 10, 17, 26, …
1+1, 4 + 1, 9 + 1, 16 + 1, 25 + 1, …
$1^2 + 1, 2^2 + 1, 3^2 + 1, 4^2 + 1, 5^2 + 1, \dots$
∴ General term of the sequence, $T_n = n^2 + 1$.
(iii) Add 3 to the odd number terms of sequence A to get the corresponding odd number term in sequence B.
Subtract 1 from the even number terms of sequence A to obtain the corresponding even number terms of sequence B.

13. (a) When $n = 1$, $2(1)^2 - 3(1) + 5 = 4$
When $n = 2$, $2(2)^2 - 3(2) + 5 = 7$
When $n = 3$, $2(3)^2 - 3(3) + 5 = 14$
When $n = 4$, $2(4)^2 - 3(4) + 5 = 25$
The first four terms of the sequence are 4, 7, 14 and 25.
(b) (i) Comparing the two sequences, the common difference between two sequences is –3.
Since the formula for the sequence in part **(a)** is $2n^2 - 3n + 5$, then the formula for the sequence is $2n^2 - 3n + 5 - 3 = 2n^2 - 3n + 2$.
(ii) When $n = 385$,
$2(385)^2 - 3(385) + 2$
$= 295\,297$.

14. (i) 5^{th} line: $n = 5$, $6 \times 5 - 10 = 20$
(ii) Note that the product is the value of n and the value of 1 more than n.
∴ $a = 29$
The value of b is an even number and it is the product of n and 2.
∴ $b = 28 \times 2 = 56$
The value of c is $29 \times 28 - 56 = 756$.
(iii) When $n = 50$,
$51 \times 50 - 50 \times 2 = 2450$

15. (i) 6^{th} line: $\dfrac{1}{6 \times 7} = \dfrac{1}{6} - \dfrac{1}{7}$
7^{th} line: $\dfrac{1}{7 \times 8} = \dfrac{1}{7} - \dfrac{1}{8}$
(ii) $272 = p \times q$
Notice that q is 1 more than p.
By trial and error, $16 \times 17 = 272$
∴ $p = 16$ and $q = 17$
(iii) $\dfrac{1}{100} - \dfrac{1}{101} = \dfrac{1}{100 \times 101} = \dfrac{1}{10\,100}$

16. (i) $1 + 2 + 3 + 4 + \dots + 99 + 100 + 99 + \dots + 3 + 2 + 1$
$= (100)^2$
$= 10\,000$
(ii) $1 + 2 + 3 + \dots + (n - 1) + n + (n - 1) + \dots + 3 + 2 + 1 = 7056$
$n^2 = 7056$
$n = 84$

17. (i) 7^{th} line: $7^3 - 7 = 336 = (7 - 1) \times 7 \times (7 + 1)$
(ii) 1320 is divisible by 10. Thus the factors of 1320 are 10, 11 and 12.
$1320 = (11 - 1) \times 11 \times (11 + 1)$
∴ $n = 11$
(iii) $19^3 - 19 = (19 - 1) \times 19 \times (19 + 1) = 6840$

18. (a) (i) The next four terms are $15 + 6 = 21$,
$21 + 7 = 28$, $28 + 8 = 36$ and $36 + 9 = 45$.
(ii) The next four terms are $35 + 21 = 56$,
$56 + 28 = 84$, $84 + 36 = 120$, and
$120 + 45 = 165$.
(b) (i) 9^{th} line: $9^3 - 9 = 720 = 6 \times 120$
10^{th} line: $10^3 - 10 = 990 = 6 \times 165$
(ii) $k = 6$
$p = 6 \times 84 = 504$
Notice that the number of terms follow the number of terms for the sequence 0, 1, 4, 10, …, 84. Since the 8^{th} term is 84, then
$8^3 - 8 = 504 = 6 \times 84$.
∴ $m = 8$

19. (a) 3^{rd} line: $(1 + 2 + 3)^2 = 36 = (1)^3 + (2)^3 + (3)^3$

4^{th} line: $(1 + 2 + 3 + 4)^2$

$= 100 = (1)^3 + (2)^3 + (3)^3 + (4)^3$

(b) (i) When $l = 7$, $(1)^3 + (2)^3 + (3)^3 + (4)^3 + (5)^3 + (6)^3 + (7)^3$

$= (1 + 2 + 3 + 4 + 5 + 6 + 7)^2$

$= (28)^2$

$= 784$

(ii) When $1 = 19$,

$(1)^3 + (2)^3 + (3)^3 + (4)^3 + (5)^3 + (6)^3 + \ldots + (19)^3$

$= (1 + 2 + 3 + 4 + 5 + 6 + \ldots + 19)^2$

$= (190)^2$

$= 36\ 100$

(c) $(1 + 2 + 3 + \ldots + n)^2 = 2025 = (45)^2$

We observe that

$45 = 40 + 5 = 4 \times 10 + 5$

$(1 + 9) + (2 + 8) + (3 + 7) + (4 + 6) + 5$

$= 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$

$\therefore n = 9$

(d) $(1)^3 + (2)^3 + (3)^3 + \ldots + (m)^3 = 78^2$

$(1 + 2 + 3 + 4 + 5 + \ldots + m)^3 = 78^2$

$1 + 2 + 3 + 4 + 5 + \ldots + m = 78$

Consider

78

$= 6 \times 13$

$= (1 + 12) + (2 + 11) + (3 + 10) + (4 + 9) + (5 + 8) + (6 + 7)$

$= 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12$

$\therefore m = 12$

20. $1^2 + 1^2 + 2^2 + 3^2 = 3 \times 5$

$1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 5 \times 8$

Notice that numbers along this column follow the sequence 1, 1, 2, 3, 5, 8, 13, …

(i) 7^{th} line: $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 13^2$

$= 13 \times 21$

(ii) $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + \ldots + l^2 + m^2 = 55 \times n$

Since the left-hand side follows the given sequence, then $l = 34$ and $m = 55$.

The right-hand side of the equation follows the given sequence too.

$\therefore n = 89$

21. (a)

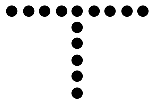


Figure 4

(b)

Figure Number (n)	1	2	3	4	5	…	n
Number of Buttons	5	8	11	14	17	…	…

(c) Since the common difference is 3, $T_n = 3n + ?$.

The term before T_1 is $c = T_0 = 5 - 3 = 2$.

$\therefore$ General term of the sequence, $T_n = 3n + 2$.

(d) (i) When $n = 38$, $T_{38} = 3(38) + 2 = 116$

$\therefore$ The number of buttons needed to form Figure 38 is 116.

(ii) $3n + 2 = 470$

$3n = 470 - 2$

$3n = 468$

$n = 156$

$\therefore$ Figure 156 is made up of 470 buttons.

(e) $3n + 2 = 594$

$3n = 594 - 2$

$3n = 592$

$n = 197\frac{1}{3}$

Since $197\frac{1}{3}$ is not a positive integer, it is not possible for a figure in the sequence to be made up of 594 buttons.

22. (a)

Figure Number	**Number of Dots**	**Number of Small Right-Angled Triangles**
1	4	2
2	9	8
3	16	18
4	25	32
⋮	⋮	⋮
10	121	200
⋮	⋮	⋮
19	400	722
⋮	⋮	⋮
n	x	y

(b) (i) $x = (n + 1)^2$

(ii) $y = 2n^2$

23. (a)

110	209	308	407	506	605	704	803	902
121	220	319	418	517	616	715	814	913
132	231	330	429	528	627	726	825	924
143	242	341	440	539	638	737	836	935
154	253	352	451	550	649	748	847	946
165	264	363	462	561	660	759	858	957
176	275	374	473	572	671	770	869	968
187	286	385	484	583	682	781	880	979
198	297	396	495	594	693	792	891	990

(b) These are some of the possible patterns.
For each column, from top cell to bottom cell, add 11 to each term to get the next term.
For each row, from left cell to right cell, add 99 to each term to get the next term.
For each diagonal, from left to right, add 10 to each term to get the next term.

(c) From the table,
$550 \div 11 = 50 = 5^2 + 5^2 + 0^2$
$803 \div 11 = 73 = 8^2 + 0^2 + 3^2$
$\therefore$ The two multiples are 550 and 803.

24. (a)

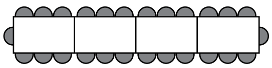


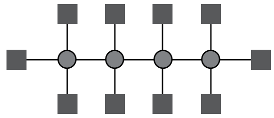
Figure 4

(b) Figure 1 8
Figure 2 14
Figure 3 20
Figure 4 26
Since the common difference is 6, $T_n = 6n + ?$.
The term before T_1 is $c = T_0 = 14 - 6 = 8$.
$\therefore$ General term of the sequence, $T_n = 6n + 8$, $n = 0, 1, 2, \ldots$
$\therefore p = 8 + 6n, n = 0, 1, 2, \ldots$

(c) When $n = 45$, $T_{45} = 6(45) + 8 = 278$
$\therefore$ 278 people can be seated when 45 tables are placed together.

(d) $6n + 8 = 245$
$6n = 245 - 8 = 237$
$n = 39\frac{1}{2}$
Since $n = 39\frac{1}{2}$ is not a positive integer, Kate is not able to follow the arrangement in part **(b)** with all the seats fully occupied.

25. (a)



Design 4

(b)

Figure Number n	Number of Circles	Number of Squares	Number of Straight Cines
1	1	4	4
2	2	6	7
3	3	8	10
4	4	10	13
5	5	12	16
⋮	⋮	⋮	⋮
10	10	22	31
⋮	⋮	⋮	⋮
23	23	48	70

(c) (i) n

(ii) Since the common difference is 2,
$T_n = 2n + ?$.
The term before T_1 is $c = T_0 = 4 - 2 = 2$.
$\therefore$ General term of the sequence of the number of squares, $T_n = 2n + 2 = 2(n + 1)$.

(iii) Since the common difference is 3,
$T_n = 3n + ?$.
The term before T_1 is $c = T_0 = 4 - 3 = 1$.
$\therefore$ General term of the sequence of the number of straight lines, $T_n = 3n + 1$.

(d) (i) When $n = 105$, $3(105) + 1 = 316$
$\therefore$ 316 straight lines are needed to form Figure 105.

(ii) $2n + 2 = 30$
$2n = 30 - 2$
$2n = 28$
$n = 14$
Figure 14 is formed using 30 squares.

(e) $2n + 2 = 50$
$2n = 50 - 2 = 48$
Since 48 is divisible by 2, there is a figure formed using 50 squares.
$3n + 1 = 75$
$3n = 75 - 1 = 74$
Since 74 is not divisible by 3, there is no figure formed using 75 straight lines.

26. (i)

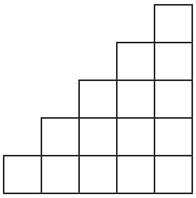


Figure 5

(ii) When $n = 5$,

Height of figure = 5

Number of squares = 5 + 4 + 3 + 2 + 1

$= \dfrac{5(5+1)}{2} = 15$

When $n = 6$,

Height of figure = 6

Number of squares = 6 + 5 + 4 + 3 + 2 + 1

$= \dfrac{6(6+1)}{2} = 21$

When $n = n$,

Number of squares $= n + (n - 1) + (n - 2) + \ldots + 3 + 2 + 1$

$= \dfrac{n(n+1)}{2}$

Advanced

27. (a) Observe that the pattern is

$\sqrt{676} = 26$, $\sqrt{625} = 25$, $\sqrt{576} = 24$,
$\sqrt{529} = 23$, $\sqrt{484} = 22$, $\sqrt{441} = 21$

$\therefore$ The missing terms are $\sqrt{529}$ and $\sqrt{484}$.

(b) Observe that the pattern is

$\sqrt[3]{3375} = 15, \ldots, \sqrt[3]{729} = 9, \sqrt[3]{343} = 7,$
$\sqrt[3]{125} = 5$

This follows that the cube root of a number gives cold numbers.

$\therefore$ The missing terms are $\sqrt[3]{13^3} = \sqrt[3]{2197}$ and $\sqrt[3]{11^3} = \sqrt[3]{1331}$.

(c) Observe that the pattern is alternate cube and square of the numbers in the sequence.

Add the term number to the previous term.

$\therefore$ The missing terms are $(23 + 7)^3 = 30^3$ and $(30 + 8)^2 = 38^2$.

(d) Observe the pattern as taking the cube of prime numbers.

$\therefore$ The missing terms are 5^3, 7^3 and 17^3.

(e) Observe that the pattern is taking the square of the prime numbers.

$\therefore$ The missing terms are 19^2, 17^2 and 11^2.

28. (a) Since the common difference is 11, $T_n = 11n + ?$.

The term before T_1 is $c = T_0 = 5 - 11 = -6$.

$\therefore$ General term of the sequence, $T_n = 11n - 6$.

$11n - 6 = 100$
$11n = 106$
$n \approx 9.6$

Thus the largest two-digit number occurs when $n = 9$.

When $n = 9$, $11(9) - 6 = 93$.

(b) To find the formula of the general term, consider the following:

8 , 27, 64, 125, …

$2^3, 3^3, 4^3, 5^3, \ldots$

$\therefore$ General term of the sequence $= n^3$,

$n = 2, 3, 4, 5, \ldots.$

$n^3 = 1000$

$n = 10$

Thus the first four-digit number occurs when $n = 10$. When $n = 9$, the largest three-digit number occurs.

When $n = 9$, $9^3 = 729$.

29. (a) To find the formula of the general term, consider the following:

4, 9, 16, 25, …

$2^2, 3^2, 4^2, 5^2, \ldots$

$\therefore$ General term of the sequence $= n^2$,

$n = 2, 3, 4, 5, \ldots.$

$n^2 = 100$

$n = 10$

The smallest three-digit number is 100.

(b) To find the formula of the general term, consider the following

$2 = 2 \times 3^0$
$6 = 2 \times 3^1$
$18 = 2 \times 3^2$
$54 = 2 \times 3^3$

$\therefore$ General term $= 2(3)^{n-1}$

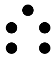
Let $2(3)^{n-1} = 1000$

$3^{n-1} = 500$

By trial and error, $3^5 = 243$ and $3^6 = 729$ and so $n - 1 = 6$

So the smallest four-digit number is $2(3)^6 = 1458$.

30. (a) 5 points



Set 5

(b) When $n = 4$,
number of lines formed = 6
When $n = 5$,
number of lines formed = 10
When $n = 10$,
number of lines formed = 45
When $n = 23$,
number of lines formed = 253

(c) (i) By observation, the number of points is the same as the set number, n.
Thus the number of points needed to form Set $n = n$.

(ii) To find the formula of the number of lines, consider the following:

When $n = 1$, $\quad 0 = \dfrac{1(1-1)}{2}$

When $n = 2$, $\quad 1 = \dfrac{2(2-1)}{2}$

When $n = 3$, $\quad 3 = \dfrac{3(3-1)}{2}$

When $n = 4$, $\quad 6 = \dfrac{4(4-1)}{2}$

When $n = 5$, $\quad 10 = \dfrac{5(5-1)}{2}$

$\therefore$ General term of the number of lines
$= \dfrac{n(n-1)}{2}$

(d) (i) When $n = 16$, number of lines $= \dfrac{16(16-1)}{2}$
$= 120$

(ii) When $n = 35$, number of lines $= \dfrac{35(35-1)}{2}$
$= 595$

(e) (i) Let $\dfrac{n(n-1)}{2} = 190$
By trial and error, n must be 17, 18, 19, 20, …
$\therefore n = 20$

(ii) Let $\dfrac{n(n-1)}{2} = 1000$

When $n = 45$, number of lines $= \dfrac{45(45-1)}{2}$
$= 990$

When $n = 46$, number of lines $= \dfrac{46(46-1)}{2}$
$= 1035$

$\therefore$ There is no set of points having 1000 lines formed.

31. (a) Term is obtained by adding the two terms immediately above.

(b) (i) The next two rows are the 6^{th} and 7^{th} rows.
6^{th} row: 1 5 10 10 5 1
7^{th} row: 1 6 15 20 15 6 1

(ii) Sum of the terms in row 1 = 1
Sum of the terms in row 2 = 1 + 1 = 2
Sum of the terms in row 3 = 1 + 2 + 1 = 4
Sum of the terms in row 4 = 1 + 3 + 3 + 1 = 8
Sum of the terms in row 5
= 1 + 4 + 6 + 4 + 1 = 16
Sum of the terms in row 6
= 1 + 5 + 10 + 10 + 5 + 1 = 32
Sum of the terms in row 7
= 1 + 6 + 15 + 20 + 15 + 6 + 1
= 64
Yes, these sums form a pattern equal to 2^{n-1}, where n is the n^{th} row.

(c) (i) Sum of terms in 11^{th} row $= 2^{10} = 1024$

(ii) Sum of terms in k^{th} row $= 2^{k-1}$

(d) (i) Number of terms in k^{th} row $= k$

(ii) The first two terms in the k^{th} row are 1 and $k - 1$.

New Trend

32. (a) (i) To find the formula of the general term, consider the following:
$81 = 88 - 7(1)$
$74 = 88 - 7(2)$
$67 = 88 - 7(3)$
$60 = 88 - 7(4)$
$\therefore$ General term, $T_n = 88 - 7n$

(ii) $T_{99} = 88 - 7(9)$
$= -605$

(b) (i) $n + 13$

(ii) $(n + 1)(n + 12) - n(n + 13)$
$= n^2 + 13n + 12 - n^2 - 13n$
$= 12$ (shown)

(iii) Sum of the numbers in the square
$= n + (n + 1) + (n + 12) + (n + 13)$
$= 4n + 26$
When $4n + 26 = 520$
$4n = 494$
$n = 123.5$
Since 123.5 is not an integer, the sum of the four numbers in the square cannot be 520.

33. (a) Common difference $= \dfrac{77 - 35}{3}$

$= 14$

$p = 35 - 14$

$= 21$

$q = 35 + 14$

$= 49$

$r = 77 - 14$

$= 63$

(b) Since the common difference is 14, $T_n = 14n + ?$.

The term before T_1 is $c = T_0 = 21 - 14 = 7$.

$\therefore$ General term of the sequence, $T_n = 14n + 7$.

(c) When $14n + 7 = 170$

$14n = 163$

$n = 11\frac{9}{14}$

Since $11\frac{9}{14}$ is not an integer, 170 is not in the sequence.

34. (a) (i) Next line is the 6th line: $6^2 - 6 = 30$.

(ii) 8th line: $8^2 - 8 = 56$

(iii) From the number pattern, we observe that

$1^2 - 1 = 1(1 - 1)$

$2^2 - 2 = 2(2 - 1)$

$3^2 - 3 = 3(3 - 1)$

$4^2 - 4 = 4(4 - 1)$

$5^2 - 5 = 5(5 - 1)$

$\vdots$

n^{th} line: $n^2 - n = n(n - 1)$

(b) $139^2 - 139 = 139(139 - 1) = 19\ 182$

35. (i) The next two terms of the sequence are 39 and 46.

(ii) Since the common difference is 7, $T_n = 7n + ?$.

The term before T_1 is $c = T_0 = 4 - 7 = -3$.

$\therefore$ General term of the sequence, $T_n = 7n - 3$.

(iii) When $n = 101$, $T_{101} = 7(101) - 3 = 704$.

(iv) $7n - 3 = 158$.

$7n = 158 + 3$

$= 161$

$n = 23$

Revision Test B1

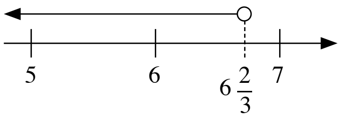
1. **(a)**
$$19 - 6x = 21 - 9x$$
$$-6x + 9x = 21 - 19$$
$$9x - 6x = 2$$
$$3x = 2$$
$$x = \frac{2}{3}$$

(b)
$$2\frac{1}{3}x = 14$$
$$\frac{7}{3}x = 14$$
$$3 \times \frac{7}{3}x = 3 \times 14$$
$$7x = 42$$
$$x = 6$$

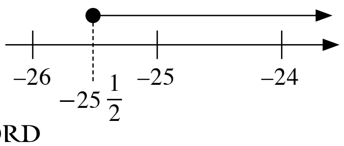
(c)
$$13 - 3x - 10 = 3x + 8 - 8x$$
$$3x - 3x = 8 - 5x$$
$$-3x + 5x = 8 - 3$$
$$2x = 5$$
$$x = \frac{5}{2}$$
$$= 2\frac{1}{2}$$

(d)
$$\frac{x-3}{4} - \frac{2x-5}{2} = \frac{1}{4}$$
$$\frac{x-3}{4} - \frac{2(2x-5)}{4} = \frac{1}{4}$$
$$\frac{x-3-2(2x-5)}{4} = \frac{1}{4}$$
$$x - 3 - 2(2x - 5) = 1$$
$$x - 3 - 4x + 10 = 1$$
$$7 - 3x = 1$$
$$6 = 3x$$
$$3x = 6$$
$$x = 2$$

2. **(a)**
$$3x < 20$$
$$x < \frac{20}{3}$$
$$x < 6\frac{2}{3}$$



(b)
$$2x \geqslant -51$$
$$x \geqslant -\frac{51}{2}$$
$$x \geqslant -25\frac{1}{2}$$



3. **(a)**
$$3x < 43$$
$$x < 14\frac{1}{3}$$

$\therefore$ The largest possible integer value of x is 14.

(b)
$$13x > 49$$
$$x > 3\frac{10}{13}$$

$\therefore$ The smallest prime value of x is 5.

4. **(a)** **(i)** When $a = 4$,
$$4x = 15 - 3x$$
$$4x + 3x = 15$$
$$7x = 15$$
$$x = \frac{15}{7} = 2\frac{1}{7}$$

(ii) When $x = 4$,
$$4a = 15 - 3(4)$$
$$4a = 15 - 12$$
$$4a = 3$$
$$a = \frac{3}{4}$$

(b)
$$8p - 9q = 7q + 3p$$
$$8p - 3p = 7q + 9q$$
$$5p = 16q$$
$$\frac{p}{q} = \frac{16}{5}$$
$$\frac{2}{5} \times \frac{p}{q} = \frac{2}{5} \times \frac{16}{5}$$
$$\frac{2p}{5q} = 1\frac{7}{25}$$

5. **(a)**
$$(2a + 15) \div 7 = 11$$
$$\frac{2a + 15}{7} = 11$$
$$2a + 15 = 77$$
$$2a = 77 - 15$$
$$2a = 62$$
$$a = 31$$

(b) Let the first even number be n.

Then the second even number is $(n + 2)$.
$$n + 2 + 4n = 72$$
$$5n = 72 - 2$$
$$5n = 70$$
$$n = 14$$

$\therefore$ The two numbers are 14 and 14 + 2 = 16.

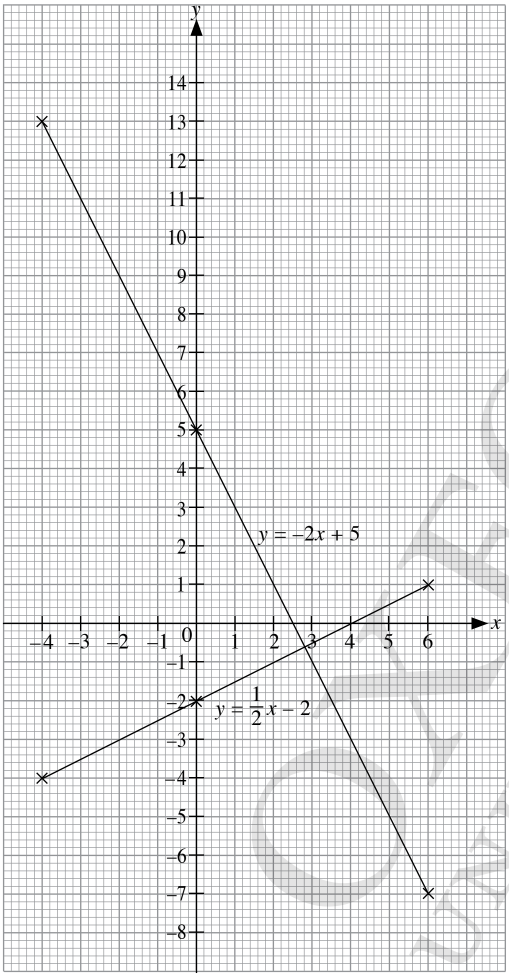
(c) Let Ethan's age be x years.

Then Mr Lin's age is $(38 - x)$ years.
In three years' time,
Ethan is $(x + 3)$ years old.
Then Mr Lin is $(38 - x + 3) = (41 - x)$ years old.

$41 - x = 3(x + 3)$
$41 - x = 3x + 9$
$3x + x = 41 - 9$
$4x = 32$
$x = 8$

$\therefore$ Mr Lin is 30 years old and his son is 8 years old.

6.



7. **(i)** The next two terms of the sequence are $8 + 8 = 16$ and $16 + 16 = 32$.

(ii) The next two terms of the sequence are $552 + 552 = 1104$ and $1104 + 1104 = 2208$.

Revision Test B2

1. (a) $\frac{1}{4}(x-2)-(x+3)=7(x-1)$

$$(x-2)-4(x+3)=28(x-1)$$
$$x-2-4x-12=28x-28$$
$$28x+4x-x=-2-12+28$$
$$31x=14$$
$$x=\frac{14}{31}$$

(b) $\frac{2(x-3)}{3}-\frac{5(3x-1)}{6}=\frac{1}{12}$

$$\frac{4(x-3)}{6}-\frac{5(3x-1)}{6}=\frac{1}{12}$$
$$\frac{4(x-3)-5(3x-1)}{6}=\frac{1}{12}$$
$$\frac{4x-12-15x+5}{6}=\frac{1}{12}$$
$$12(-11x-7)=6$$
$$2(-11x-7)=1$$
$$-22x-14=1$$
$$22x=-14-1$$
$$22x=-15$$
$$x=-\frac{15}{22}$$

(c) $\frac{2}{x-2}=\frac{5}{x+5}$

$$2(x+5)=5(x-2)$$
$$2x+10=5x-10$$
$$5x-2x=10+10$$
$$3x=20$$
$$x=\frac{20}{3}$$
$$=6\frac{2}{3}$$

(d) $\frac{5}{11-3x}=\frac{2}{3x-1}$

$$5(3x-1)=2(11-3x)$$
$$15x-5=22-6x$$
$$15x+6x=22+5$$
$$21x=27$$
$$x=\frac{27}{21}$$
$$=1\frac{2}{7}$$

2. (a) $2x \leqslant 18$

$$x \leqslant 9$$

(b) $\frac{2x}{5} \geqslant -3$

$$5\times\frac{2x}{5} \geqslant -3\times 5$$
$$2x \geqslant -15$$
$$x \geqslant -\frac{15}{2}$$
$$x \geqslant -7\frac{1}{2}$$

3. $\frac{3f-g}{f+2g}=\frac{4}{5}$

$$5(3f-g)=4(f+2g)$$
$$15f-5g=4f+8g$$
$$15f-4f=8g+5g$$
$$11f=13g$$
$$\frac{f}{g}=\frac{13}{11}$$
$$\frac{1}{39}\times\frac{f}{g}=\frac{1}{{}_3\cancel{39}}\times\frac{\cancel{13}^1}{11}$$
$$\frac{f}{39g}=\frac{1}{33}$$

4. (i) $V=\frac{1}{3}\pi r^2h$

When $r=2$, $h=5$, $\pi=3.14$,

$$V=\frac{1}{3}(3.14)(2)^2(5)$$
$$=20.9 \text{ (to 3 s.f.)}$$

(ii) $V=\frac{1}{3}\pi r^2h$

When $V=75$, $r=3$, $\pi=3.14$,

$$75=\frac{1}{3}(3.14)(3)^2h$$
$$\frac{471}{50}h=75$$
$$h=\frac{75\times 50}{471}$$
$$=7.96 \text{ (to 3 s.f.)}$$

5. $6x \geqslant 70$

$$x \geqslant \frac{70}{6}$$
$$x \geqslant 11\frac{2}{3}$$

If x is a prime number, the smallest value of x is 13.

6. (a) Perimeter of square = 52 cm

$4(2x + 5) = 52$

$8x + 20 = 52$

$8x = 52 - 20$

$8x = 32$

$x = 4$

The length of the square is $(2 \times 4 + 5) = 13$ cm.

Area of the square

$= 13^2$

$= 169 \text{ cm}^2$

(b) Let the number of type *A* eggs be *x*.

Then the number of type *B* eggs is $60 - x$.

$x(0.11) + (60 - x)(0.13) = 7$

$0.11x + 7.8 - 0.13x = 7$

$7.8 - 7 = 0.13x - 0.11x$

$0.02x = 0.8$

$x = 40$

∴ She bought 40 type *A* eggs and 20 type *B* eggs.

(c) Let the length of the field be *x* m.

Perimeter of fence = $2(x + 35)$ m

$160 = 2(x + 35)$

$80 = x + 35$

$x = 80 - 35 = 45$

∴ The length of the field is 45 m.

(d) Let the number of paperback books be *p*.

Then the number of hardcover books is $(50 - p)$.

$4p + 1\frac{1}{2}(4)(50 - p) = 256$

$4p + 6(50 - p) = 256$

$4p + 300 - 6p = 256$

$6p - 4p = 300 - 256$

$2p = 44$

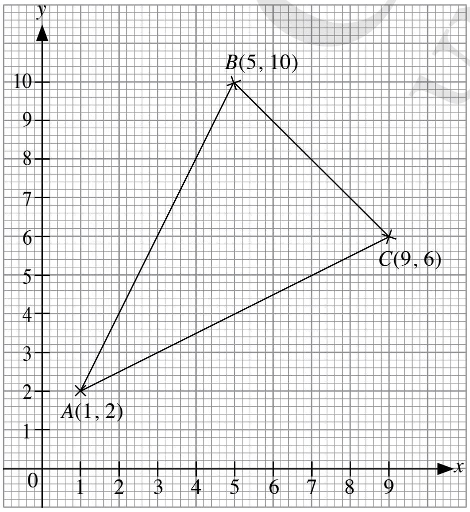
$p = 22$

Number of hardcover books

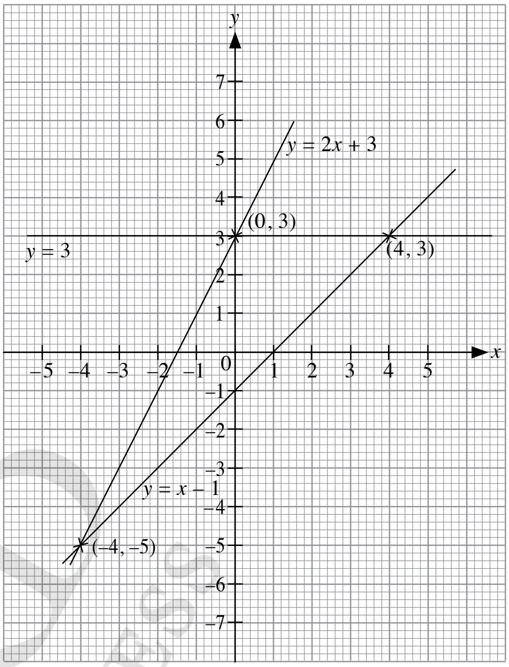
$= 50 - 22$

$= 28$

7.



8. (a)



(b) Area of triangle

$= \frac{1}{2} \times 4 \times (3 + 5)$

= 16 square units

9. 3, 7, 13, 21, …

3, 3 + 4, 3 + 4 + 6, 3 + 4 + 6 + 8, …

The next two terms of the sequence are

21 + 10 = 31 and 31 + 12 = 43

Mid-Year Examination Specimen Paper A

Part I

1. **(a)** False; 2.3 is a rational number.

(b) False; $189 \div 3 = 63$ shows that 189 does not satisfy the definition of prime numbers.

(c) True

2. **(a)** **(i)** $27.049 = 27$ (to 2 s.f.)

(ii) $27.049 = 27.0$ (to 1 d.p.)

(b) Using a calculator, $\dfrac{10}{13} = 0.769\,230\,769$

$= 0.769$ (to 3 d.p.)

3. **(a)** $\{[(-4) - (-2) \times 7] + 9 \times 5\} \div 11$

$= \{[(-4) - (-14)] + 9 \times 5\} \div 11$

$= \{[(-4) + 14] + 9 \times 5\} \div 11$

$= \{10 + 9 \times 5\} \div 11$

$= \{10 + 45\} \div 11$

$= 55 \div 11$

$= 5$

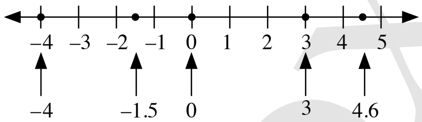
(b) $\dfrac{\left(3\frac{1}{2} - 2\frac{1}{3}\right)^2}{3\frac{1}{2} + 2\frac{1}{3}}$

$= \dfrac{\left(1\frac{1}{6}\right)^2}{5\frac{5}{6}}$

$= \dfrac{7}{30}$

4. **(a)** $\dfrac{5}{6}$, $\dfrac{11}{13}$ and $\dfrac{8}{9}$

(b)



$\therefore$ The ordering is 0, 3, –4, –1.5, 4.6 is –4, –1.5, 0, 3 and 4.6.

5. When $a = 2$, $b = 0$, $c = 1$ and $d = -3$,

(a) $\dfrac{abc - bcd}{acd + abd}$

$= \dfrac{(2)(0)(1) - (0)(1)(-3)}{(2)(1)(-3) + (2)(0)(-3)}$

$= \dfrac{0}{-6}$

$= 0$

(b) $\dfrac{ab}{c} - \dfrac{bc}{d} + \dfrac{ad}{2c}$

$= \dfrac{(2)(0)}{(1)} - \dfrac{(0)(1)}{(-3)} + \dfrac{(2)(-3)}{2(1)}$

$= -3$

6. **(a)** **(i)** $3(x - 2y) - 7[x - 3(2y - 7x)]$

$= 3(x - 2y) - 7[x - 6y + 21x]$

$= 3(x - 2y) - 7[x + 21x - 6y]$

$= 3x - 6y - 7[22x - 6y]$

$= 3x - 6y - 154x + 42y$

$= 3x - 154x - 6y + 42y$

$= -151x + 36y$

(ii) $\dfrac{2}{3}(x - 4) - \dfrac{2}{5}(x + 3)$

$= \dfrac{2}{3}x - \dfrac{8}{3} - \dfrac{2}{5}x - \dfrac{6}{5}$

$= \dfrac{2}{3}x - \dfrac{2}{5}x - \dfrac{8}{3} - \dfrac{6}{5}$

$= \dfrac{4}{15}x - 3\dfrac{13}{15}$

(b) $3x + 18y + 27z$

$= 3(x + 6y + 9z)$

7. **(a)** $2(x - 3) + 5(2x - 3) = 3$

$2x - 6 + 10x - 15 = 3$

$2x + 10x - 6 - 15 = 3$

$12x - 21 = 3$

$12x = 3 + 21$

$12x = 24$

$x = 2$

(b) $\dfrac{3}{4}(2x - 1) = \dfrac{1}{2} + \dfrac{7}{8}x$

$\dfrac{3}{2}x - \dfrac{3}{4} = \dfrac{1}{2} + \dfrac{7}{8}x$

$\dfrac{3}{2}x - \dfrac{7}{8}x = \dfrac{1}{2} + \dfrac{3}{4}$

$\dfrac{5}{8}x = 1\dfrac{1}{4}$

$x = 1\dfrac{1}{4} \div \dfrac{5}{8}$

$x = 2$

8. **(a)** Cost price of T-shirts

$= s \times \$p$

$= \$ps$

Selling price of T-shirts

$= s \times \$q$

$= \$qs$

Profit

= selling price – cost price

$= \$(qs - ps)$

$= \$s(q - p)$

(b) Let the present age of Kate's brother be x years old.

Then Kate is $(x + 10)$ years old.

In 3 years' time,

$$\begin{aligned}(x + 10) + 3 &= 2(x + 3)\\ x + 13 &= 2x + 6\\ 2x - x &= 13 - 6\\ x &= 7\end{aligned}$$

$\therefore$ Kate is 17 years old and her brother is 7 years old.

9. **(a)** Let the first number be y.

Then the second number is $(y + 8)$.

$$\begin{aligned}y + (y + 8) &= 230\\ 2y + 8 &= 230\\ 2y &= 230 - 8\\ 2y &= 222\\ y &= 111\end{aligned}$$

$\therefore$ The two numbers are 111 and 111 + 8 = 119.

(b)

$$\begin{aligned}12 &= 2^2 \times 3\\ 28 &= 2^2 \quad\quad \times 7\\ 112 &= 2^4 \quad\quad \times 7\end{aligned}$$

HCF of 12, 28 and 112 $= 2^2$

$= 4$

LCM of 12, 28 and 112 $= 2^4 \times 3 \times 7$

$= 336$

Part II

Section A

1. **(a)** **(i)** $2\frac{2}{9} - \frac{7}{15} \div 4\frac{1}{5} + \frac{1}{3}$

$= 2\frac{2}{9} - \frac{1}{9} + \frac{1}{3}$

$= 2\frac{1}{9} + \frac{1}{3}$

$= 2\frac{4}{9}$

(ii) $4\frac{1}{2} + 4\frac{1}{2} \times \frac{2}{3} - \frac{5}{9}$

$= 4\frac{1}{2} + 3 - \frac{5}{9}$

$= 7\frac{1}{2} - \frac{5}{9}$

$= 6\frac{17}{18}$

(b) Express all the numbers in decimals.

$\frac{3}{7} = 0.428\ 571\ 428$, 0.42, 0.428 282 828…,

0.424 242 42…, 0.428 428 428…

Arrange the numbers in ascending order.

0.42, 0.424 242 42…, 0.428 282 828…,

0.428 428 428…, $\frac{3}{7} = 0.428\ 571\ 428$

$\therefore 0.\dot{4}\dot{2},\ 0.42\dot{8}\dot{2}$ …

2. **(a)** **(i)** $\sqrt{12.57} + 3.89^3$

$= 62.409\ 288\ 58$

$= 62.4$ (to 3 s.f.)

(ii) $15.76^2 - \frac{1}{0.026} \times 76.8$

$= 15.76^2 - 2953.846\ 154$

$= -2705.468\ldots$

$= -2710$ (to 3 s.f.)

(b) Fraction remaining after the man saves part of his salary

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$

Fraction of the salary spent on rental

$= \frac{1}{4} \times \frac{5}{6}$

$= \frac{5}{24}$

Fraction of salary spent on food and other necessities

$= 1 - \frac{1}{6} - \frac{5}{24}$

$= \frac{5}{8}$

3. **(a)** $9a - \{3a - 2[3a(2a + 1) - 2a(3a - 1)]\}$

$= 9a - \{3a - 2[6a^2 + 3a - 6a^2 + 2a]\}$

$= 9a - \{3a - 2[6a^2 - 6a^2 + 3a + 2a]\}$

$= 9a - \{3a - 2[5a]\}$

$= 9a - \{3a - 10a\}$

$= 9a - \{-7a\}$

$= 9a + 7a$

$= 16a$

(b) $2ax - ay + 6ab - 3a$

$= a(2x - y + 6b - 3)$

4. **(a)** **(i)**
$$\begin{aligned}2x + [7 - 3(x + 5)] &= 4\\ 2x + [7 - 3x - 15] &= 4\\ 2x + [7 - 15 - 3x] &= 4\\ 2x + [-8 - 3x] &= 4\\ 2x - 8 - 3x &= 4\\ 3x - 2x &= -8 - 4\\ x &= -12\end{aligned}$$

(ii)
$$\begin{aligned}\frac{3}{7-5x} &= \frac{5}{3-2x}\\ 3(3 - 2x) &= 5(7 - 5x)\\ 9 - 6x &= 35 - 25x\\ 25x - 6x &= 35 - 9\\ 19x &= 26\\ x &= \frac{26}{19}\\ &= 1\frac{7}{19}\end{aligned}$$

(b) **(i)** $6x \geqslant 15$

$x \geqslant 2\frac{1}{2}$

(ii) $11x \leqslant -65$

$x \leqslant -5\frac{10}{11}$

Section B

5. **(i)** $120k = 2^3 \times 3 \times 5 \times k$

For $120k$ to be a perfect cube,

then the smallest value of $120k$

$= (2 \times 3 \times 5)^3$

$= (2^3 \times 3 \times 5) \times 3^2 \times 5^2$

$= 120 \times 3^2 \times 5^2$

$\therefore\ k = 3^2 \times 5^2 = 225$

(ii) $120 = 2^3 \times 3 \times 5$

$2800 = 2^4 \times 5^2 \times 7$

HCF of 120 and 2800 $= 2^3 \times 5$

$= 40$

LCM of 120 and 2800 $= 2^4 \times 3 \times 5^2 \times 7$

$= 8400$

(iii) $\sqrt[3]{120k} = \sqrt[3]{120 \times 225} = 30$

$30 = 2 \times 3 \times 5$

$2800 = 2^4 \times 5^2 \times 7$

HCF of 30 and 2800 $= 2 \times 5 = 10$

LCM of 30 and 2800 $= 2^4 \times 3 \times 5^2 \times 7$

$= 8400$

6. **(a)**
$$\begin{aligned}-2 - \frac{3}{x+3} &= \frac{4}{x+3} + 5\\ \frac{4}{x+3} + \frac{3}{x+3} &= -2 - 5\\ \frac{7}{x+3} &= -7\\ -7(x + 3) &= 7\\ x + 3 &= -1\\ x &= -4\end{aligned}$$

(b) Let the price of the printer be $\$y$.

$y + 5\frac{1}{2}y = 2210$

$6\frac{1}{2}y = 2210$

$y = 340$

$\therefore$ The printer costs \$340 and the desktop computer costs $5\frac{1}{2} \times 340 = \$1870$.

(c) Let the number of students who failed the test be n.

Then the number of students who passed the test will be $3n$.

$3n + n = 44$

$4n = 44$

$n = 11$

$\therefore$ The number of students who passed the test is $3 \times 11 = 33$.

(d) Let the first number be x.

Then the second number is $x + 9$.

$x + (x + 9) = 63$

$2x + 9 = 63$

$2x = 63 - 9$

$2x = 54$

$x = 27$

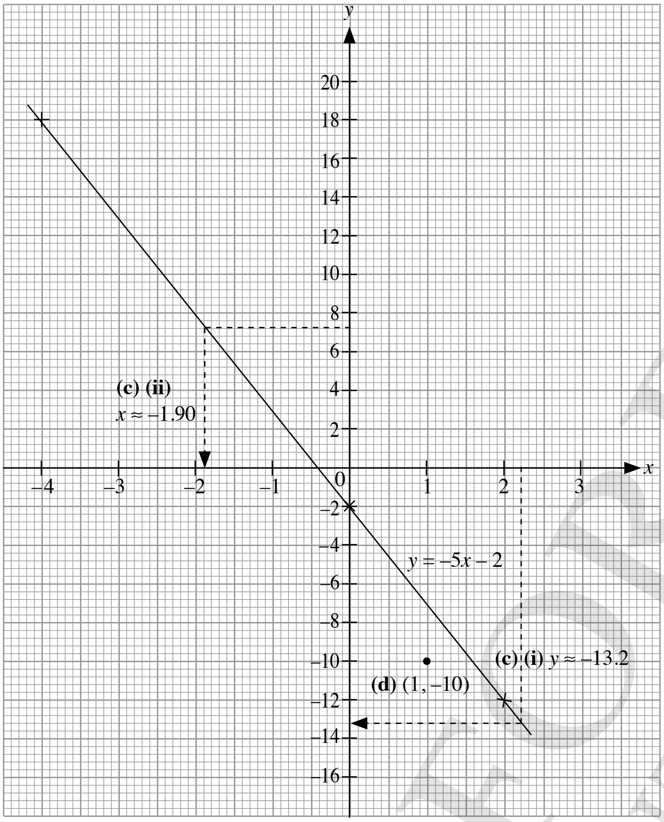
The two numbers are 27 and $27 + 9 = 36$.

7. (a) When $x = 2$,

$y = -5(2) - 2 = -12$.

The value of p is -12.

(b)



(c) (i) From the graph, $y \approx -13.2$.

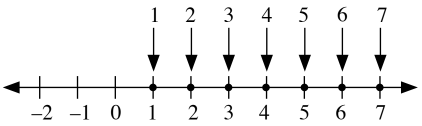
(ii) From the graph, $x \approx -1.90$.

(d) The point $(1, -10)$ is not a solution of the equation $y = -5x - 2$ as the point does not lie on the line.

Mid-Year Examination Specimen Paper B

Part I

1.



2. $25x > 52$

$x > 2\frac{2}{25}$

$\therefore$ Smallest prime value of x is 3.

3. **(a)** $5 \times 12 - 25 \div 5 \times 3$
$= 60 - 5 \times 3$
$= 60 - 15$
$= 45$

(b) $(-3)^2 \times (-2)^3 \div (-3) \times (-4)$
$= 9 \times (-8) \div (-3) \times (-4)$
$= -72 \div (-3) \times (-4)$
$= 24 \times (-4)$
$= -96$

(c) $15 - 2\{[12 - 7 \times 10 \div 2 + 3]\}$
$= 15 - 2\{[12 - 70 \div 2 + 3]\}$
$= 15 - 2\{[12 - 35 + 3]\}$
$= 15 - 2\{-20\}$
$= 15 + 40$
$= 55$

4. **(a)** Express all the numbers in decimal.

$\frac{13}{11} = 1.181\ 818\ 182$, 1.188 888, 1.818 181 81,

1.1832, $\frac{11}{9} = 1.222\ 22\ldots$

Arrange the numbers in descending order.

$\frac{11}{9} = 1.222\ 22\ldots$, 1.818 181 81, 1.188 888, 1.1832,

$\frac{13}{11} = 1.181\ 818\ 182$

$\therefore \frac{11}{9}$, $1.\dot{8}\dot{1}$, $1.1\dot{8}$, 1.1832, $\frac{13}{11}$

(b) **(i)** 24.056 = 24.06 (to 4 s.f.)

(ii) 0.000 142 254 = 0.000 142 25 (to 5 s.f.)

(iii) 150 000 (to 2 s.f.)

5. **(a)** **(i)**

2	2646
3	1323
3	441
3	147
7	49
7	7
	1

$\therefore 2646 = 2 \times 3^3 \times 7^2$

(ii) When $k = 54$, $\sqrt{\frac{2646}{54}} = \sqrt{49} = 7$

$\therefore$ The largest prime number of $\sqrt{\frac{2646}{k}}$ is 7 when $k = 54$.

(b) $42 = 2 \times 3 \times 7$
$54 = 2 \times 3^3$
$2646 = 2 \times 3^3 \times 7^2$

So, the number that gives LCM 2646 must be divisible by $7^2 = 49$. Given that $n > 54$, n is either 2×7^2 or 3×7^2. The next smallest number greater than 54 and gives the LCM 2646 is $2 \times 7^2 = 98$.

6. When $a = -1$, $b = 3$, $c = -4$,

(a) $(ab)^2 - 4ca$
$= [(-1)(3)]^2 - 4(-4)(-1)$
$= (-3)^2 - 16$
$= 9 - 16$
$= -7$

(b) $\frac{a}{b-c} + \frac{ab}{ac} - \frac{b}{a-b}$
$= \frac{(-1)}{3-(-4)} + \frac{(-1)(3)}{(-1)(-4)} - \frac{3}{(-1)-3}$
$= \frac{-1}{7} - \frac{3}{4} + \frac{3}{4}$
$= -\frac{1}{7}$

7. **(a)** $5(2x - 3y) - 3[-3(y - x) + 2y]$
$= 5(2x - 3y) - 3[-3y + 3x + 2y]$
$= 5(2x - 3y) - 3[-3y + 2y + 3x]$
$= 5(2x - 3y) - 3[-y + 3x]$
$= 10x - 15y + 3y - 9x$
$= 10x - 9x - 15y + 3y$
$= x - 12y$

(b) $\frac{1}{5}(-3x - 5) - \frac{4}{5}(-x - 3) + (x - 1)$
$= -\frac{3}{5}x - 1 + \frac{4}{5}x + \frac{12}{5} + x - 1$
$= -\frac{3}{5}x + \frac{4}{5}x + x - 1 + \frac{12}{5} - 1$
$= \frac{6}{5}x + \frac{2}{5}$

8. **(a)** $-mn - 5mnp + 3m$
$= m(-n - 5np + 3)$

(b) $3ax - 2bx - 10cx + 5dx$
$= x(3a - 2b - 10c + 5d)$

(c) $12pq - 2pr + 6pqr - 2p$
$= 2p(6q - r + 3qr - 1)$

9. **(a)** $4(2x + 3) = 2(x - 3)$

$2(2x + 3) = x - 3$

$4x + 6 = x - 3$

$4x - x = -3 - 6$

$3x = -9$

$x = -3$

(b) $\frac{4}{5}(-2x - 3) = \frac{4}{3} - \frac{17x}{15}$

$-\frac{8}{5}x - \frac{12}{5} = \frac{4}{3} - \frac{17}{15}x$

$\frac{17}{15}x - \frac{8}{5}x = \frac{4}{3} + \frac{12}{5}$

$-\frac{7}{15}x = 3\frac{11}{15}$

$x = -8$

(c) $\frac{5}{2 - x} = \frac{4}{x + 3}$

$5(x + 3) = 4(2 - x)$

$5x + 15 = 8 - 4x$

$5x + 4x = 8 - 15$

$9x = -7$

$x = -\frac{7}{9}$

10. **(a)** Let the first number be y.

Then the next consecutive number is $(y + 1)$.

$(y + 1) + 2y = 70$

$y + 2y + 1 = 70$

$3y = 70 - 1$

$3y = 69$

$y = 23$

$\therefore$ The two numbers are 23 and 23 + 1 = 24.

(b) Let Michael's brother's present age be n years.

Then Michael's present age is $1\frac{1}{2}n$ years.

Six years ago,

Michael was $\left(1\frac{1}{2}n - 6\right)$ years old and his brother was $(n - 6)$ years old.

$1\frac{1}{2}n - 6 = 2(n - 6)$

$1\frac{1}{2}n - 6 = 2n - 12$

$2n - 1\frac{1}{2}n = 12 - 6$

$\frac{1}{2}n = 6$

$n = 12$

$\therefore$ Michael is $1\frac{1}{2}(12) = 18$ years old and his brother is 12 years old.

11. **(a)** **(i)** Since the common difference is 3, $T_n = 3n + ?$.

The term before T_1 is $c = T_0 = -2 - 3 = -5$.

$\therefore$ General term of the sequence, $T_n = 3n - 5$.

(ii) $3k - 5 = 304$.

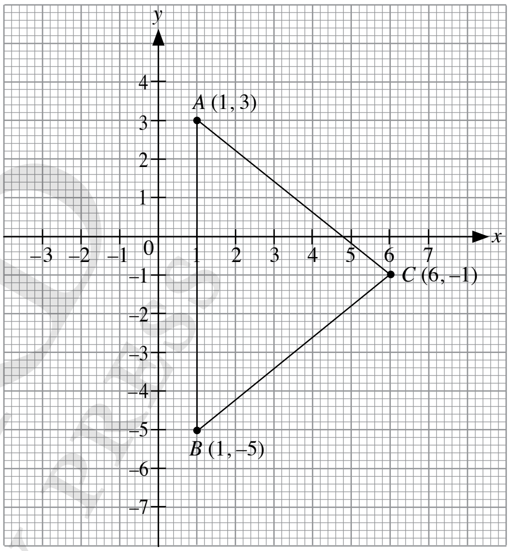
$3k = 304 + 5$

$3k = 309$

$k = 103$

$\therefore$ The value of k is 103.

(b)



Length of base of $\triangle ABC$ = length AB

$= 3 - (-5) = 8$

Perpendicular height from $C = 6 - 1 = 5$

Area of $\triangle ABC$

$= \frac{1}{2} \times$ base length $\times$ perpendicular height

$= \frac{1}{2} \times 8 \times 5$

= 20 square units

Part II

Section A

1. **(a)** $0.36 \div [0.36 - (2.16 \div 6 - 0.01 \div 0.25)] \times 0.9$
$= 0.36 \div [0.36 - (0.36 - 0.04)] \times 0.9$
$= 0.36 \div [0.36 - 0.32] \times 0.9$
$= 0.36 \div 0.04 \times 0.9$
$= 9 \times 0.9$
$= 8.1$

(b) $\frac{4.72 - 3.8 + 1.04}{12.5 - 12.43} - \frac{6.33 - 5.15 \times 0.84}{0.167}$
$= \frac{1.96}{0.07} - \frac{6.33 - 4.326}{0.167}$
$= 28 - 12$
$= 16$

(c) $\left[1.8 + 1\frac{9}{10} \times (5.9 - 3.8)\right] \div \left(1.41 - 1\frac{2}{5}\right)$
$= \left[1.8 + 1\frac{9}{10} \times 2.1\right] \div \left(1.41 - 1\frac{2}{5}\right)$
$= [1.8 + 3.99] \div \left(1.41 - 1\frac{2}{5}\right)$
$= 5.79 \div 0.01$
$= 579$

2. **(a)** Height of water when it is at high tide = +2.8 m
Height of water when it is at low tide = –1.5 m
Difference between high tide and low tide
$= +2.8 - (-1.5)$
$= 4.3$ m

(b) $\frac{1}{2}$ m^3 weighs $\frac{5}{16}$ tonnes
1 m^3 weighs $\frac{5}{16} \times 2 = \frac{5}{8}$ tonnes
$3\frac{1}{5}$ m^3 weighs $\frac{5}{8} \times 3\frac{1}{5} = 2$ tonnes

(c) Length of ribbon being cut off $= 3 \times 2\frac{5}{12}$
$= 7\frac{1}{4}$ m

Length of ribbon remaining
$= 10\frac{3}{8} - 7\frac{1}{4}$
$= 3\frac{1}{8}$ m

3. **(a)**
$$0.3(2x - 3) = \frac{1}{5}(0.7 + x) - 0.65x$$
$$0.3(2x - 3) = 0.2(0.7 + x) - 0.65x$$
$$0.6x - 0.9 = 0.14 + 0.2x - 0.65x$$
$$0.6x + 0.65x - 0.2x = 0.14 + 0.9$$
$$1.05x = 1.04$$
$$x = 0.990 \text{ (to 3 s.f.)}$$

(b)
$$-5 + \frac{3}{x-4} = \frac{5}{x-4} - 11$$
$$\frac{5}{x-4} - \frac{3}{x-4} = -5 + 11$$
$$\frac{2}{x-4} = 6$$
$$2 = 6(x - 4)$$
$$1 = 3(x - 4)$$
$$1 = 3x - 12$$
$$3x = 1 + 12$$
$$3x = 13$$
$$x = \frac{13}{3}$$
$$= 4\frac{1}{3}$$

4. **(a)** When $p = -1$, $q = 2$ and $r = 8$,
$$\frac{p}{q} = \sqrt{\frac{p(3q^2 - 2z + 5)}{2r}} - q^2$$
$$\frac{(-1)}{2} = \sqrt{\frac{(-1)(3(2)^2 - 2z + 5)}{2(8)}} - (2)^2$$
$$-\frac{1}{2} = \sqrt{\frac{-(12 - 2z + 5)}{16}} - 4$$
$$-\frac{1}{2} + 4 = \sqrt{\frac{-(12 - 2z + 5)}{16}}$$
$$3\frac{1}{2} = \sqrt{\frac{-(12 - 2z + 5)}{16}}$$
$$12\frac{1}{4} = \frac{-12 + 2z - 5}{16}$$
$$16 \times 12\frac{1}{4} = 16 \times \frac{-12 + 2z - 5}{16}$$
$$196 = -12 - 5 + 2z$$
$$196 = -17 + 2z$$
$$2z = 196 + 17$$
$$2z = 213$$
$$z = 106\frac{1}{2}$$

(b) $3xa - 3a - 3xb + 3b + 2ya - 2yb$
$= 3xa - 3xb - 3a + 3b + 2ya - 2yb$
$= 3x(a - b) - 3(a - b) + 2y(a - b)$
$= (a - b)(3x - 3 + 2y)$

Section B

5. $40 = 2^3 \times 5$

$98 = 2 \times 7^2$

$500 = 2^2 \times 5^3$

(i) The greatest whole number that will divide 40, 98 and 500 exactly means the HCF of 40, 98 and 500.

HCF of 40, 98 and 500 = 2

(ii) The smallest whole that is divisible by 40, 98 and 500 means the LCM of 40, 98 and 500.

LCM of 40, 98 and 500 = $2^3 \times 5^3 \times 7^2 = 49\,000$

6. **(a)** Let the breadth of the rectangular field be x m.

Then the length of the field is $2x$ m.

Perimeter of field = $2(2x + x)$ m

$360 = 2(2x + x)$

$360 = 2(3x)$

$6x = 360$

$\therefore x = 60$

The breadth is 60 m and the length of the field is $60 \times 2 = 120$ m.

Area of field = 120×60

$= 7200 \text{ m}^2$

(b) **(i)** Perimeter of $ABCD$ = Perimeter of PQR

$2[(4x - 3) + (6x - 7)]$

$= 2x + (6x - 3) + (4x + 3)$

$2[4x - 3 + 6x - 7] = 2x + 6x + 4x - 3 + 3$

$2[4x + 6x - 3 - 7] = 12x$

$10x - 10 = 6x$

$10x - 6x = 10$

$4x = 10$

$x = 2.5$

(ii) Length of rectangle $ABCD = (6 \times 2.5 - 7)$

$= 8$ cm

Breadth of rectangle $ABCD = (4 \times 2.5 - 3)$

$= 7$ cm

Area of rectangle $= 8 \times 7$

$= 56 \text{ cm}^2$

Length of base of $\triangle PQR = 2 \times 2.5$

$= 5$ cm

Perpendicular height of $\triangle PQR = 6 \times 2.5 - 3$

$= 12$ cm

Area of $\triangle PQR = \frac{1}{2}(12)(5)$

$= 30 \text{ cm}^2$

(iii) No, even though the perimeter of the two figures are the same.

7. **(a)** 1, 1, 2, 3, 5, 8, 13,...

+1 +1 +2 +3 +5 +8

The rule:

The next term can be obtained by adding the previous two terms.

The next five terms are

$13 + 8 = 21$

$21 + 13 = 34$

$34 + 21 = 55$

$55 + 34 = 89$

$89 + 55 = 144$

(b) **(i)** 5th line: $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = 8 \times 13$

(ii) 6th line: $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 13^2 = 13 \times 21$

7th line: $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 13^2 + 21^2 = 21 \times 34$

8th line: $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 13^2 + 21^2 + 34^2 = 34 \times 55$

9th line: $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 13^2 + 21^2 + 34^2 + 55^2 = 55 \times 89$

(iii) Adding the squares of the terms in the sequence is the same as taking the product of the last term in the sum, on the LHS, and the next term in the sequence.

8. **(a)** When $x = -4$,

$4p + 2(-4) = -1$

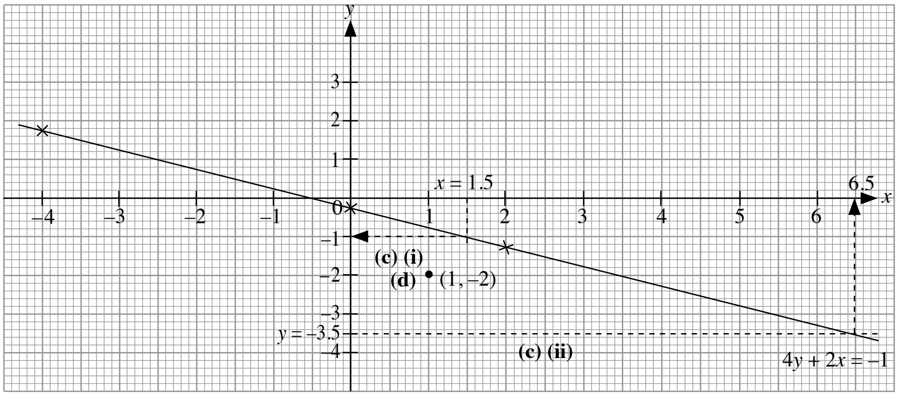
$4p - 8 = -1$

$4p = -1 + 8$

$4p = 7$

$p = 1.75$

(b)



(c) **(i)** From the graph, $y \approx -1$.

(ii) Note: In this case, extrapolation is needed to obtain the answer.

After extrapolating the graph, we find that $x = 6.5$.

(d) No. The point (1, –2) does not lie on the line with equation $4y + 2x = -1$.

Chapter 8 Percentage

Basic

1. **(a)** $18\% = \frac{18}{100}$

$= \frac{9}{50}$

(b) $85\% = \frac{85}{100}$

$= \frac{17}{20}$

(c) $125\% = \frac{125}{100}$

$= 1\frac{1}{4}$

(d) $210\% = \frac{210}{100}$

$= 2\frac{1}{10}$

(e) $0.25\% = \frac{0.25}{100}$

$= \frac{0.25 \times 100}{100 \times 100}$

$= \frac{25}{10\,000}$

$= \frac{1}{400}$

(f) $4.8\% = \frac{4.8}{100}$

$= \frac{4.8 \times 10}{100 \times 10}$

$= \frac{48}{1000}$

$= \frac{6}{125}$

(g) $1\frac{1}{3}\% = \frac{4}{3}\%$

$= \frac{4}{3} \div 100$

$= \frac{4}{3} \times \frac{1}{100}$

$= \frac{4}{300}$

$= \frac{1}{75}$

(h) $12\frac{1}{2}\% = \frac{25}{2}\%$

$= \frac{25}{2} \div 100$

$= \frac{25}{2} \times \frac{1}{100}$

$= \frac{25}{200}$

$= \frac{1}{8}$

2. **(a)** $9\% = \frac{9}{100}$

$= 0.09$

(b) $99\% = \frac{99}{100}$

$= 0.99$

(c) $156\% = \frac{156}{100}$

$= 1.56$

(d) $0.05\% = \frac{0.05}{100}$

$= 0.0005$

(e) $0.68\% = \frac{0.68}{100}$

$= 0.0068$

(f) $1.002\% = \frac{1.002}{100}$

$= 0.010\,02$

(g) $2.4\% = \frac{2.4}{100}$

$= 0.024$

(h) $14\frac{2}{5}\% = \frac{72}{5}\%$

$= \frac{72}{5} \div 100$

$= \frac{72}{5} \times \frac{1}{100}$

$= \frac{72}{500}$

$= 0.144$

3. **(a)** $\frac{4}{625} = \frac{4}{625} \times 100\%$

$= 0.64\%$

(b) $\frac{9}{125} = \frac{9}{125} \times 100\%$

$= 7.2\%$

(c) $\frac{6}{25} = \frac{6}{25} \times 100\%$

$= 24\%$

(d) $\frac{3}{4} = \frac{3}{4} \times 100\%$
$= 75\%$

(e) $\frac{19}{20} = \frac{19}{20} \times 100\%$
$= 95\%$

(f) $\frac{9}{8} = \frac{9}{8} \times 100\%$
$= 112.5\%$

(g) $\frac{7}{5} = \frac{7}{5} \times 100\%$
$= 140\%$

(h) $\frac{33}{8} = \frac{33}{8} \times 100\%$
$= 412.5\%$

4. **(a)** $0.0034 = 0.0034 \times 100\%$
$= 0.34\%$

(b) $0.027 = 0.027 \times 100\%$
$= 2.7\%$

(c) $0.05 = 0.05 \times 100\%$
$= 5\%$

(d) $0.14 = 0.14 \times 100\%$
$= 14\%$

(e) $0.5218 = 0.5218 \times 100\%$
$= 52.18\%$

(f) $6.325 = 6.325 \times 100\%$
$= 632.5\%$

(g) $16.8 = 16.8 \times 100\%$
$= 1680\%$

(h) $332 = 332 \times 100$
$= 33\,200\%$

5. **(a)** Convert 1*l* to m*l*.
1*l* = 1000 m*l*
$\frac{175}{1000} \times 100\% = 17.5\%$

(b) Convert 1 day to hours.
1 day = 24 hours
$\frac{6}{24} \times 100\% = 25\%$

(c) Convert 1 hour to minutes.
1 hour = 60 minutes
$\frac{20}{60} \times 100\% = 33\frac{1}{3}\%$

(d) Convert \$1.44 to cents.
\$1.44 = 144 cents
$\frac{80}{144} \times 100\% = 55\frac{5}{9}\%$

(e) Convert 20 cm to mm.
20 cm = 20 × 10 = 200 mm
$\frac{225}{200} \times 100\% = 112.5\%$

(f) Convert 45 kg to g.
45 kg = 45 × 1000 = 45 000 g
$\frac{45\,000}{36\,000} \times 100\% = 125\%$

(g) Convert 2 years to months.
2 years = 2 × 12 = 24 months
$\frac{24}{18} \times 100\% = 133\frac{1}{3}\%$

(h) Convert \$4.40 to cents.
\$4.40 = 440 cents
$\frac{440}{99} \times 100\% = 444\frac{4}{9}\%$

6. Total amount of mixture = 8 + 42 = 50*l*

(i) Percentage of milk in the mixture
$= \frac{42}{50} \times 100\%$
$= 84\%$

(ii) Percentage of water in the mixture
$= \frac{8}{50} \times 100\%$
$= 16\%$

7. Percentage of latecomers in school *A*
$= \frac{25}{1500} \times 100\%$
$= 1\frac{2}{3}\%$ or 1.67% (to 3 s.f.)

Percentage of latecomers in school *B*
$= \frac{25}{1800} \times 100\%$
$= 1\frac{7}{18}\%$ or 1.39% (to 3 s.f.)

School *A* has 1.67% of students coming late whereas school *B* has 1.39% of students coming late. Thus, school *B* has a lower percentage of latecomers.

8. **(a)** 0.25% of 4000
$= \frac{0.25}{100} \times 4000$
$= 0.25 \times 40$
$= 10$

(b) 6% of 200 $= \frac{6}{100} \times 200$
$= 12$

(c) 7.5% of \$2500 $= \frac{7.5}{100} \times 2500$
$= 7.5 \times 25$
$= \$187.50$

(d) 8% of 130 g $= \frac{8}{100} \times 130$
$= 10.4$ g

(e) 20.6% of 15 000 people

$= \frac{20.6}{100} \times 15\,000$

$= 20.6 \times 150$

= 3090 people

(f) $37\frac{1}{2}$ % of 56 cm

$= \frac{75}{2}$ % of 56 cm

$= \frac{75}{2} \times \frac{1}{100} \times 56$

= 21 cm

(g) 45% of 4 kg

$= \frac{45}{100} \times 4$

= 1.8 kg

(h) $66\frac{2}{3}$ % of 72 litres

$= \frac{200}{3}$ % of 72 litres

$= \frac{200}{3} \times \frac{1}{100} \times 72$

= 48 litres

(i) $112\frac{1}{2}$ % of 200 m

$= \frac{225}{2}$ % of 200 m

$= \frac{225}{2} \times \frac{1}{100} \times 200$

= 225 m

(j) 180% of 320

$= \frac{180}{100} \times 320$

= 576

9. **Method 1**

Number of kilograms of zinc = 25% of 60

$= \frac{25}{100} \times 60$

= 15

Number of kilograms of copper

= 60 – 15

= 45

The ingot of copper contains 45 kg of copper.

Method 2

Percentage of copper in ingot = 100% – 25% = 75%

Number of kilograms of copper in ingot = 75% of 60

$= \frac{75}{100} \times 60$

= 45

The ingot of brass contains 45 kg of copper.

10. **(a)** Required value = 110% of \$60

$= \frac{110}{100} \times 60$

= \$66

(b) Required value = 128% of 69 *l*

$= \frac{128}{100} \times 69$

= 88.32 *l*

(c) Required value = 225% of 50 m

$= \frac{225}{100} \times 50$

= 112.5 m

(d) Required value = 400% of 24 kg

$= \frac{400}{100} \times 24$

= 96 kg

(e) Required value = $112\frac{1}{2}$ % of 32 g

$= \frac{225}{2}$ % × 32

$= \left(\frac{225}{2} \div 100\right) \times 32$

$= \frac{225}{2} \times \frac{1}{100} \times 32$

= 36 g

(f) Required value = 100.03% of \$400

$= \frac{100.03}{100} \times 400$

$= 100.03 \times 4$

= \$400.12

(g) Required value = 100.5% of \$4000

$= \frac{100.5}{100} \times 4000$

$= 100.5 \times 40$

= \$4020

(h) Required value = 2600% of \$1.50

$= \frac{2600}{100} \times 1.50$

= \$39

11. (a) Required value

$= 99.4\% \text{ of } 1.25 \text{ km}$

$= \frac{99.4}{100} \times 1.25$

$= 1.2425 \text{ km}$

(b) Required value

$= 95\% \text{ of } \$88$

$= \frac{95}{100} \times 88$

$= \$83.60$

(c) Required value

$= 93\% \text{ of } \$7500$

$= \frac{93}{100} \times 7500$

$= \$6975$

(d) Required value

$= 87\frac{1}{2}\% \text{ of } 64 \text{ g}$

$= \frac{175}{2}\% \text{ of } 64 \text{ g}$

$= \frac{175}{2} \times \frac{1}{100} \times 64$

$= 56 \text{ g}$

(e) Required value

$= 86.5\% \text{ of } 78 \text{ kg}$

$= \frac{86.5}{100} \times 78$

$= 67.47 \text{ kg}$

(f) Required value

$= 85\% \text{ of } 124\ l$

$= \frac{85}{100} \times 124$

$= 105.4\ l$

(g) Required value

$= 58\% \text{ of } 350 \text{ m}^2$

$= \frac{58}{100} \times 350$

$= 203 \text{ m}^2$

(h) Required value

$= 15\% \text{ of } 520$

$= \frac{15}{100} \times 520$

$= 78$

12. (a) Let the number be x.

$12\% \text{ of } x = 48$

$\frac{12}{100} \times x = 48$

$x = 48 \div \frac{12}{100}$

$= 48 \times \frac{100}{12}$

$x = 400$

(b) Let the number be x.

$15\frac{5}{8}\% \text{ of } x = 555$

$\frac{125}{8}\% \times x = 555$

$\left(\frac{125}{8} \div 100\right) \times x = 555$

$\frac{125}{8} \times \frac{1}{100} \times x = 555$

$\frac{5}{32} \times x = 555$

$x = 555 \div \frac{5}{32}$

$= 3552$

(c) Let the number be x.

$21\% \text{ of } x = 147$

$\frac{21}{100} \times x = 147$

$x = 147 \div \frac{21}{100}$

$= 147 \times \frac{100}{21}$

$x = 700$

(d) Let the number be x.

$77.5\% \text{ of } x = 217$

$\frac{77.5}{100} \times x = 217$

$x = 217 \div \frac{77.5}{100}$

$= 217 \times \frac{100}{77.5}$

$= 280$

(e) Let the number be x.

$124\% \text{ of } x = 155$

$\frac{124}{100} \times x = 155$

$x = 155 \div \frac{124}{100}$

$= 155 \times \frac{100}{124}$

$= 125$

13. (a) Let the number be x.

120% of $x = 48$

$$\frac{120}{100} \times x = 48$$

$$x = 48 \div \frac{120}{100}$$

$$= 48 \times \frac{100}{120}$$

$$= 40$$

(b) Let the number be x.

70% of $x = 147$

$$\frac{70}{100} \times x = 147$$

$$x = 147 \div \frac{70}{100}$$

$$= 147 \times \frac{100}{70}$$

$$= 210$$

(c) Let the number be x.

$$33\frac{1}{3}\% \text{ of } x = 432$$

$$\frac{100}{3}\% \text{ of } x = 432$$

$$\left(\frac{100}{3} \div 100\right) \times x = 432$$

$$\frac{100}{3} \times \frac{1}{100} \times x = 432$$

$$\frac{1}{3}x = 432$$

$$x = 432 \div \frac{1}{3}$$

$$= 1296$$

14. Increase in the number of buses operating

$= 1420 - 1000$

$= 420$

Percentage increase in the number of buses in operation

$$= \frac{\text{Increase}}{\text{Original value}} \times 100\%$$

$$= \frac{420}{1000} \times 100\%$$

$= 42\%$

15. Decrease in the price of MP3 player

$= \$382 - \$261.50$

$= \$120.50$

Percentage decrease in the price

$$= \frac{\text{Decrease}}{\text{Original value}} \times 100\%$$

$$= \frac{120.5}{382} \times 100\%$$

$= 31.5\%$ (to 3 s.f.)

16. 120% of Michael's income = \$120

$$\frac{120}{100} \times \text{Michael's income} = \$120$$

$$\text{Michael's income} = 120 \div \frac{120}{100}$$

$$= 120 \times \frac{100}{120}$$

$$= \$100$$

17. Price of notebook in 2013 = 70% of \$2000

$$= \frac{70}{100} \times 2000$$

$$= \$1400$$

Price of notebook in 2014 = 70% of \$1400

$$= \frac{70}{100} \times 1400$$

$$= \$980$$

Intermediate

18. Let the total number of students taking Additional Mathematics be x.

35% of $x = 42$

$$\frac{35}{100} \times x = 42$$

$$x = 42 \div \frac{35}{100}$$

$$= 42 \times \frac{100}{35}$$

$$= 120$$

Number of students taking Additional Mathematics in class C

$= 120 - 42 - 40$

$= 38$

19. Percentage of candidates who obtained grade C

$= 100\% - 18\% - 38\%$

$= 44\%$

Let the total number of candidates be x.

44% of $x = 77$

$$\frac{44}{100} \times x = 77$$

$$x = 77 \div \frac{44}{100}$$

$$= 77 \times \frac{100}{44}$$

$$= 175$$

The total number of candidates is 175.

20. Amount of milk in the solution

$= 30\%$ of $125\ l$

$= \frac{30}{100} \times 125$

$= 37.5\ l$

Let the amount of water to be added be $x\ l$.

$\frac{37.5}{125 + x} = 14\%$

$\frac{37.5}{125 + x} = \frac{7}{50}$

$875 + 7x = 1875$

$7x = 100$

$x = 142\frac{6}{7}$

Amount of water added $= 142\frac{6}{7}\ l$

21. 140% of price in first half of 2013 = \$52 640

Price in first half of 2013 $= 52\ 640 \div \frac{140}{100}$

$= 52\ 640 \times \frac{100}{140}$

$= \$37\ 600$

98% of price in 2012 = \$37 600

Price in 2012 $= 37\ 600 \div \frac{98}{100}$

$= 37\ 600 \times \frac{100}{98}$

$= \$38\ 367.35$

105% of original price of painting = \$38 367.35

Original price of painting

$= 38\ 367.35 \div \frac{105}{100}$

$= 38\ 367.35 \times \frac{100}{105}$

= \$36 540.33 (to the nearest cent)

22. Number of girls in the club = 70% of 40

$= \frac{70}{100} \times 40$

$= 28$

Number of boys in the club $= 40 - 28$

$= 12$

Let the number of new members who are girls be x and the number of new members who are boys be y.

Then $y - x = 6$.

$y = 6 + x$

New percentage of girls in the club = 60%

$\frac{28 + x}{40 + x + y} = \frac{60}{100}$

$\frac{28 + x}{40 + x + y} = \frac{3}{5}$

$5(28 + x) = 3(40 + x + y)$

$140 + 5x = 120 + 3x + 3y$

$140 - 120 + 5x - 3x = 3y$

$20 + 2x = 3y$

Substitute $y = 6 + x$:

$20 + 2x = 3(6 + x)$

$20 + 2x = 18 + 3x$

$20 - 18 = 3x - 2x$

$x = 2$

$y = 6 + 2$

$= 8$

No. of members who are boys $= 12 + 8$

$= 20$

23. **(i)** 3 parts of the length AB = 3 cm

1 part of the length AB = 1 cm

7 parts, which is the length of AB = 7 cm

(ii) $BC = 135\%$ of AB

$= \frac{135}{100} \times 7$

$= 9.45$ cm

(iii) $AC = 85\%$ of BC

$= \frac{85}{100} \times 9.45$

$= 8.0325$ cm

24. **(i)** Selling price of the flat = 115% of \$145 000

$= \frac{115}{100} \times 145\ 000$

$= \$166\ 750$

Amount gained by selling the flat

$= 166\ 750 - 145\ 000$

$= \$21\ 750$

(ii) Selling price of the car = 88% of \$50 000

$= \frac{88}{100} \times 50\ 000$

$= \$44\ 000$

Amount lost by selling his car

$= 50\ 000 - 44\ 000$

$= \$6000$

(iii) Yes, he still gained an amount of \$21 750 – \$6000 = \$15 750

Advanced

25. (a) Zhi Xiang's new monthly salary under scheme *B*

$= 104.5\%$ of $\$1500 + \$50$

$= \frac{104.5}{100} \times 1500 + 50$

$= 1567.5 + 50$

$= \$1617.50$

Zhi Xiang's new salary as a percentage of his present salary

$= \frac{1617.50}{1500} \times 100\%$

$= 108\%$ (to 3 s.f.)

(b) Tom's new monthly salary under scheme *A*

$= 106\%$ of $\$1200$

$= \frac{106}{100} \times 1200$

$= \$1272$

Tom's new monthly salary under scheme *B*

$= 104.5\%$ of $\$1200 + \$50$

$= \frac{104.5}{100} \times 1200 + 50$

$= \$1304$

$\therefore$ Since Tom's salary will be higher under scheme *B*, he should choose scheme *B*.

(c) Let Sharon's current monthly wage be $\$x$.

106% of $\$x = 104.5\%$ of $\$x + \$50$

$(106 - 104.5)\%$ of $x = 50$

1.5% of $x = 50$

$\frac{1.5}{100} \times x = 50$

$x = 50 \div \frac{1.5}{100}$

$= 50 \times \frac{100}{1.5}$

$x = 3333.33$ (to the nearest cent)

$\therefore$ Sharon's salary is $\$3333.33$.

New Trend

26. Let x be the total number of crayons.

Number of blue crayons $= \frac{4}{9}x$

Number of red crayons $= 65\% \times \frac{5}{9}x$

$= \frac{65}{100} \times \frac{5}{9}x$

$= \frac{13}{36}x$

Number of yellow crayons $= x - \frac{4}{9}x - \frac{13}{36}x$

$= \frac{7}{36}x$

$\frac{7}{36}x = 14$

$x = 72$

There are 72 crayons altogether.

27. In 2013, the value of the bracelet

$= 110\%$ of $\$12\,650$

$= \frac{110}{100} \times 12\,650$

$= \$13\,915$

In 2014, the value of the bracelet

$= 110\%$ of $\$13\,915$

$= \frac{110}{100} \times 13\,915$

$= \$15\,306.50$

In 2015, the value of the bracelet

$= 110\%$ of $\$15\,306.50$

$= \frac{110}{100} \times 15\,306.50$

$= \$16\,837.15$

$\frac{16\,837.15 - 12\,650}{12\,650} \times 100\% = 33.1\%$

The value of the bracelet in 2015 is $16 837.15 and the overall percentage increase is 33.1%.

28. 103% of original bill $= \$82.70$

$\frac{103}{100} \times$ original bill $= 82.70$

Original bill $= 82.70 \div \frac{103}{100}$

$= 82.70 \times \frac{100}{103}$

$= \$80.29$

29. (a) Convert 3.96 m to cm.

3.96 m $= (3.96 \times 100)$ cm

$= 396$ cm

$\frac{33}{396} \times 100\% = 8\frac{1}{3}\%$

(b) $15 \div 0.3 = 50$

50 glasses can be filled.

Chapter 9 Ratio, Rate and Speed

Basic

1. (a) 14 : 35

$14 \div 7 : 35 \div 7$

2 : 5

(b) 24 : 42

$24 \div 6 : 42 \div 6$

4 : 7

(c) 36 : 132

$36 \div 6 : 132 \div 6$

6 : 22

$6 \div 2 : 22 \div 2$

3 : 11

(d) 135 : 240

$135 \div 5 : 240 \div 5$

27 : 48

$27 \div 3 : 48 \div 3$

9 : 16

(e) 144 : 128

$144 \div 16 : 128 \div 16$

9 : 8

(f) 162 : 384

$162 \div 6 : 384 \div 6$

27 : 64

(g) 192 : 75

$192 \div 3 : 75 \div 3$

64 : 25

(h) 418 : 242

$418 \div 2 : 242 \div 2$

209 : 121

$209 \div 11 : 121 \div 11$

19 : 11

2. (a) $\frac{9}{20} : \frac{3}{5} = \frac{9}{20} \times 20 : \frac{3}{5} \times 20$

$= 9 : 12$

$= 3 : 4$

(b) $\frac{7}{15} : \frac{14}{9} = \frac{7}{15} \times 9 : \frac{14}{9} \times 9$

$= \frac{21}{5} : 14$

$= \frac{21}{5} \times 5 : 14 \times 5$

$= 21 : 70$

$= 3 : 10$

(c) $\frac{15}{28} : \frac{18}{7} = \frac{15}{28} \times 28 : \frac{18}{7} \times 28$

$= 15 : 72$

$= 5 : 24$

(d) $\frac{25}{44} : \frac{50}{33} = \frac{25}{44} \times 11 : \frac{50}{33} \times 11$

$= \frac{25}{4} : \frac{50}{3}$

$= \frac{25}{4} \times 12 : \frac{50}{3} \times 12$

$= 75 : 200$

$= 3 : 8$

(e) $1\frac{25}{56} : \frac{18}{21} = \frac{81}{56} : \frac{18}{21}$

$= \frac{81}{56} \times 21 : \frac{18}{21} \times 21$

$= \frac{243}{8} : 18$

$= \frac{243}{8} \times 8 : 18 \times 8$

$= 243 : 144$

$= 27 : 16$

(f) $4\frac{1}{3} : 65 = \frac{13}{3} : 65$

$= \frac{13}{3} \times 3 : 65 \times 3$

$= 13 : 195$

$= 1 : 15$

(g) $8\frac{3}{4} : 3\frac{1}{8} = \frac{35}{4} : \frac{25}{8}$

$= \frac{35}{4} \times 8 : \frac{25}{8} \times 8$

$= 70 : 25$

$= 14 : 5$

(h) $2.4 : 1\frac{1}{5} = 2\frac{4}{10} : 1\frac{1}{5}$

$= \frac{12}{5} \times 5 : \frac{6}{5} \times 5$

$= 12 : 6$

$= 2 : 1$

3. (a) 0.09 : 0.21

$0.09 \times 100 : 0.21 \times 100$

9 : 21

3 : 7

(b) 0.192 : 0.064

$0.192 \times 1000 : 0.064 \times 1000$

192 : 64

3 : 1

(c) $0.25 : 1.5$

$0.25 \times 100 : 1.5 \times 100$

$25 : 150$

$1 : 6$

(d) $0.63 : 9.45$

$0.63 \times 100 : 9.45 \times 100$

$63 : 945$

$1 : 15$

(e) $0.84 : 1.12$

$0.84 \times 100 : 1.12 \times 100$

$84 : 112$

$21 : 28$

$3 : 4$

(f) $1.26 : 0.315$

$1.26 \times 1000 : 0.315 \times 1000$

$1260 : 315$

$4 : 1$

(g) $1.44 : 0.48$

$1.44 \times 100 : 0.48 \times 100$

$144 : 48$

$3 : 1$

(h) $1.8 : 0.4$

$1.8 \times 10 : 0.4 \times 10$

$18 : 4$

$9 : 2$

4. (a) 6 parts = \$336

1 part = $\frac{336}{6}$ = \$56

5 parts = 56×5 = \$280

$\therefore$ \$56 : \$280

(b) 14 parts = \$336

1 part = $\frac{336}{14}$ = \$24

3 parts = 24×3 = \$72

11 parts = 24×11 = \$264

$\therefore$ \$72 : \$264

(c) 16 parts = \$336

1 part = $\frac{336}{16}$ = \$21

3 parts = 21×3 = \$63

13 parts = 21×13 = \$273

$\therefore$ \$63 : \$273

(d) 8 parts = \$336

1 part = $\frac{336}{8}$ = \$42

5 parts = 42×5 = \$210

3 parts = 42×3 = \$126

$\therefore$ \$210 : \$126

(e) 12 parts = \$336

1 part = $\frac{336}{12}$ = \$28

5 parts = 28×5 = \$140

7 parts = 28×7 = \$196

$\therefore$ \$140 : \$196

(f) 14 parts = \$336

1 part = $\frac{336}{14}$ = \$24

5 parts = 24×5 = \$120

9 parts = 24×9 = \$216

$\therefore$ \$120 : \$216

(g) 24 parts = \$336

1 part = $\frac{336}{24}$ = \$14

7 parts = 14×7 = \$98

17 parts = 14×17 = \$238

$\therefore$ \$98 : \$238

(h) 21 parts = \$336

1 part = $\frac{336}{21}$ = \$16

8 parts = 16×8 = \$128

13 parts = 16×13 = \$208

$\therefore$ \$128 : \$208

(i) 21 parts = \$336

1 part = $\frac{336}{21}$ = \$16

10 parts = 16×10 = \$160

11 parts = 16×11 = \$176

$\therefore$ \$160 : \$176

(j) 24 parts = \$336

1 part = $\frac{336}{24}$ = \$14

11 parts = 14×11 = \$154

13 parts = 14×13 = \$182

$\therefore$ \$154 : \$182

5. (a) Convert \$1 to cents.

\$1 = 100 cents

45 cents : 100 cents

$= \frac{45}{100}$

$= \frac{9}{20}$

$\therefore$ 45 cents : \$1 = 9 : 20

(b) Convert 1.25 m to cm.

1.25 m = 1.25 × 100 = 125 cm

25 cm : 125 cm

$= \frac{25}{125}$

$= \frac{1}{5}$

∴ 25 cm : 1.25 m = 1 : 5

(c) Convert 0.25 km to m.

0.25 km = 0.25 × 1000 = 250 m

250 m : 75 m

$= \frac{250}{75}$

$= \frac{10}{3}$

∴ 0.25 km : 75 m = 10 : 3

(d) Convert 0.2 kg to g.

0.2 kg = 0.2 × 1000 = 200 g

200 g : 40 g

$= \frac{200}{40}$

$= \frac{5}{1}$

∴ 0.2 kg : 40 g = 5 : 1

(e) Convert 1 hour to minutes.

1 hour = 60 minutes

35 min : 60 min

$= \frac{35}{60}$

$= \frac{7}{12}$

∴ 35 minutes : 1 hour = 7 : 12

(f) Convert 2 cm to mm.

2 cm = 2 × 10 = 20 mm

15 mm : 20 mm

$= \frac{15}{20}$

$= \frac{3}{4}$

∴ 15 mm : 2 cm = 3 : 4

(g) Convert 3.2 hours to minutes.

3.2 hours = 3.2 × 60 = 192 minutes

192 min : 72 min

$= \frac{192}{72}$

$= \frac{8}{3}$

∴ 3.2 hours : 72 minutes = 8 : 3

(h) Convert $\frac{7}{200}\,l$ to cm^3.

$\frac{7}{200}\,l = \frac{7}{200} \times 1000 = 35\ cm^3$

$35\ cm^3 : 105\ cm^3$

$= \frac{35}{105}$

$= \frac{1}{3}$

$\therefore \frac{7}{200}\,l : 105\ cm^3 = 1 : 3$

6. **(a)** 57 : 19 : 133

57 ÷ 19 : 19 ÷ 19 : 133 ÷ 19

3 : 1 : 7

(b) 64 : 96 : 224

64 ÷ 32 : 96 ÷ 32 : 224 ÷ 32

2 : 3 : 7

(c) 108 : 36 : 60

108 ÷ 6 : 36 ÷ 6 : 60 ÷ 6

18 : 6 : 10

18 ÷ 2 : 6 ÷ 2 : 10 ÷ 2

9 : 3 : 5

(d) 644 : 476 : 140

644 ÷ 28 : 476 ÷ 28 : 140 ÷ 28

23 : 17 : 5

(e) 665 : 1995 : 1330

665 ÷ 35 : 1995 ÷ 35 : 1330 ÷ 35

19 : 57 : 38

19 ÷ 19 : 57 ÷ 19 : 38 ÷ 19

1 : 3 : 2

(f) 1015 : 350 : 455

1015 ÷ 35 : 350 ÷ 35 : 455 ÷ 35

29 : 10 : 13

7. **(a)** $3 : 9 = 4 : a$

$\frac{3}{9} = \frac{4}{a}$ (express ratios as fractions)

$3a = 36$

$a = 12$

(b) $4 : 3 = a : 6$

$\frac{4}{3} = \frac{a}{6}$ (express ratios as fractions)

$3a = 24$

$a = 8$

(c) $5 : 11 = 10 : a$

$\frac{5}{11} = \frac{10}{a}$

$5a = 110$

$a = 22$

(d) $12 : 25 = a : 5$

$$\frac{12}{25} = \frac{a}{5}$$

$$25a = 60$$

$$a = \frac{60}{25} = 2\frac{2}{5}$$

(e) $14 : 9 = 7 : a$

$$\frac{14}{9} = \frac{7}{a}$$

$$14a = 63$$

$$a = 4.5 \text{ or } 4\frac{1}{2}$$

(f) $a : 5.7 = 8 : 12$

$$\frac{a}{5.7} = \frac{8}{12}$$

$$12a = 45.6$$

$$a = 3.8 \text{ or } 3\frac{4}{5}$$

8. **(i)** Convert 1.68 cm to cm.

1.68 m = 1.68 × 100 = 168 cm

168 cm : 105 cm

$$= \frac{168}{105}$$

$$= \frac{8}{5}$$

∴ The ratio of Rui Feng's height to his brother's height is 8 : 5.

(ii) Total height of the boys (in cm)

= 168 + 105 = 273 cm

1.68 m : 273 cm

168 cm : 273 cm

$$= \frac{168}{273}$$

$$= \frac{8}{13}$$

∴ The ratio of Rui Feng's height to the total height of both boys is 8 : 13.

9. Total number of parts = 126 + 42 = 168 parts

(i)

Total number of parts	:	Number of parts of pure gold
168	:	126
168 ÷ 42	:	126 ÷ 42
4	:	3

(ii)

Total number of parts	:	Number of parts of alloy *B*
168	:	42
168 ÷ 42	:	42 ÷ 42
4	:	1

Alloy *B*	:	Pure Gold
1	:	3

10. **(a)** For the ratio 1 : 2 : 6,

9 parts = $180

$$1 \text{ part} = \frac{180}{9} = \$20$$

6 parts = 20 × 6 = $120

∴ The smallest share is $20 and the largest share is $120.

(b) For the ratio 1 : 4 : 7,

12 parts = $180

$$1 \text{ part} = \frac{180}{12} = \$15$$

7 parts = 15 × 7 = $105

∴ The smallest share is $15 and the largest share is $105.

(c) For the ratio 2 : 3 : 5,

10 parts = $180

$$1 \text{ part} = \frac{180}{10} = \$18$$

2 parts = 18 × 2 = $36

5 parts = 18 × 5 = $90

∴ The smallest share is $36 and the largest share is $90.

(d) For the ratio 2 : 13 : 5,

20 parts = $180

$$1 \text{ part} = \frac{180}{20} = \$9$$

2 parts = 9 × 2 = $18

13 parts = 9 × 13 = $117

∴ The smallest share is $18 and the largest share is $117.

(e) For the ratio 3 : 1 : 11,

15 parts = $180

$$1 \text{ part} = \frac{180}{15} = \$12$$

11 parts = 12 × 11 = $132

∴ The smallest share is $12 and the largest share is $132.

(f) For the ratio 4 : 11 : 3,

18 parts = $180

$$1 \text{ part} = \frac{180}{18} = \$10$$

3 parts = 10 × 3 = $30

11 parts = 10 × 11 = $110

∴ The smallest share is $30 and the largest share is $110.

11. **(a)** 7 parts = \$84

$1 \text{ part} = \frac{84}{7} = \$12$

18 parts = 12 × 18 = \$216

∴ Largest part is \$216.

Total sum = (15 + 18 + 7) × 12 = \$480

(b) 7 parts = \$133

$1 \text{ part} = \frac{133}{7} = \$19$

18 parts = 19 × 18 = \$342

∴ Largest part is \$342.

Total sum = (15 + 18 + 7) × 19 = \$760

(c) 7 parts = \$301

$1 \text{ part} = \frac{301}{7} = \$43$

18 parts = 43 × 18 = \$774

∴ Largest part is \$774.

Total sum = (15 + 18 + 7) × 43 = \$1720

(d) 7 parts = \$3990

$1 \text{ part} = \frac{3990}{7} = \$570$

18 parts = 570 × 18 = \$10 260

∴ Largest part is \$10 260.

Total sum = (15 + 18 + 7) × 570 = \$22 800

12. **(a)** 11 parts = 187°

$1 \text{ part} = \frac{187}{11} = 17°$

7 parts = 17 × 7 = 119°

Angle D = 360 – 187 = 173°

Ratio of angle C to angle D = 119 : 173

(b) 11 parts = 242°

$1 \text{ part} = \frac{242}{11} = 22°$

7 parts = 22 × 7 = 154°

Angle D = 360 – 242 = 118°

Ratio of angle C to angle D = 154 : 118

(c) 11 parts = 275°

$1 \text{ part} = \frac{275}{11} = 25°$

7 parts = 25 × 7 = 175°

Angle D = 360 – 275 = 85°

Ratio of angle C to angle D = 175 : 85

= 35 : 17

13. **(a)** $\text{Rate} = \frac{350}{40} = \frac{35}{4} = 8.75 \text{ km}/l$

(b) $\text{Rate} = \frac{120}{8} = \$15/\text{hour}$

(c) $\text{Rate} = \frac{82 \times 100}{300} = \frac{82}{3} = 27\frac{1}{3}$ cents/unit

(d) $\text{Rate} = \frac{320}{8} = 40$ words/min

(e) $\text{Rate} = \frac{60}{12} = \$5/\text{tile}$

(f) $\text{Rate} = \frac{1760}{15} = 117\frac{1}{3}$ cents/min

14. **(i)** Cost of 1 m^2 of flooring = $\frac{\$36}{20} = \$1.80$

(ii) Cost of 55 m^2 of flooring = \$1.80 × 55 = \$99

(iii) Area of flooring for a cost of \$1 = $\frac{20}{36} = \frac{5}{9}$ m^2

Area of flooring for the cost of \$63

$= \frac{5}{9} \times \$63 = 35 \text{ m}^2$

15. Amount required to travel a distance of 50 km

= \$1.35 × 50 = \$67.50

Amount that each child will have to pay = $\frac{\$67.50}{54}$

= \$1.25

16. Convert 75 cm to m.

75 cm = 75 ÷ 100 = 0.75 m

Area of rectangular brass sheet = 1.5 × 0.75

= 1.125 m^2

Area of 1 kg of brass sheet = $\frac{1.125}{7.2}$

= 0.156 25 m^2

Area of 12.8 kg of brass sheet = 0.156 25 × 12.8

= 2 m^2

17. Time required for one man to finish the project

= 45 × 8

= 360 hours

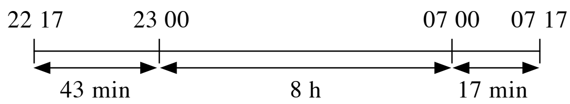
Time required for (45 – 5) = 40 men to finish the project = $\frac{360}{40}$

= 9 hours

18. 5.55 p.m. + 40 min = 6.35 p.m.

= 18 35

19.



Total time taken = 43 min + 8 h + 17 min

= 9 h

20. (a) $84 \text{ km/h} = \dfrac{84 \text{ km}}{1 \text{ h}}$

$= \dfrac{84\,000 \text{ m}}{3600 \text{ s}}$

$= 23\dfrac{1}{3}$ m/s

(b) $15 \text{ m/s} = \dfrac{15 \text{ m}}{1 \text{ s}}$

$= \dfrac{(15 \div 1000) \text{ km}}{(1 \div 3600) \text{ s}}$

= 54 km/h

(c) $\dfrac{2}{3} \text{ km/min} = \dfrac{\frac{2}{3} \text{ km}}{1 \text{ min}}$

$= \dfrac{\frac{2}{3} \text{ km}}{(1 \div 60) \text{ h}}$

= 40 km/h

(d) $120 \text{ cm/s} = \dfrac{120 \div 100}{1 \text{ s}}$

= 1.2 m/s

21. Convert 44 minutes to hours.

$44 \text{ min} = \dfrac{44}{60} = \dfrac{11}{15}$ h

Time taken to travel a distance of 1 km = $\dfrac{11}{15} \div 11$

$= \dfrac{1}{15}$ h

(a) (i) Time taken to travel a distance of 45 km

$= \dfrac{1}{15} \times 45$

= 3 h

(ii) Time taken to travel a distance of 36 km

$= \dfrac{1}{15} \times 36$

$= 2\dfrac{2}{5}$ h or 2 h 24 min

(iii) Time taken to travel a distance of 20 km

$= \dfrac{1}{15} \times 20$

$= 1\dfrac{1}{3}$ h or 1 h 20 min

(b) Speed of the cyclist

$= \dfrac{11 \times 1000}{44 \times 60}$

$= 4\dfrac{1}{6}$ m/s

22. (i) Time taken for the journey

= 50 min + 3 h 24 min + 2 h 6 min + 1 h 30 min

$= \dfrac{5}{6} \text{ h} + 3\dfrac{2}{5} \text{ h} + 2\dfrac{1}{10} \text{ h} + 1\dfrac{1}{2} \text{ h}$

$= 7\dfrac{5}{6}$ h or 7 h 50 min

(ii) Average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{687}{7\frac{5}{6}}$

= 87.7 km/h (to 3 s.f.)

23. (i) Convert 36 minutes to hours.

$36 \text{ min} = \dfrac{36}{60} = \dfrac{3}{5}$ h

Average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{27}{\frac{3}{5}}$

= 45 km/h

(ii) Time at which Michael reaches the station

= 08 37 + 36 min

= 09 13

Time at which the train arrives at the station

= 09 42 + 11 min

= 09 53

Waiting time = 09 53 – 09 13 = 40 min

24. (i) Time taken by the car for the whole journey

= 15 10 – 08 45

= 6 h 25 min

(ii) Distance = speed × time

$= 84 \times 6\dfrac{5}{12}$

= 539 km

25. (i) Convert 54 minutes to hours.

$54 \text{ min} = \dfrac{54}{60} = \dfrac{9}{10}$ h

Distance travelled for the first part of the journey

$= 70 \times \dfrac{9}{10}$

= 63 km

Distance travelled for the return journey

= 63 km

Time taken for the return journey

$= \dfrac{63}{45}$

$= 1\dfrac{2}{5}$ h or 1 h 24 min

(ii) Time at which Khairul starts to return to the original point
= 0955 + 54 min + 40 min
= 1129
Time when Khairul arrives at the starting point
= 1129 + 1 h 24 min
= 1253

Intermediate

26. **(a)** 16 parts = \$160

$1 \text{ part} = \frac{160}{16} = \$10$

9 parts = $\$10 \times 9 = \$90$

Difference between the largest share and the smallest share
= \$90 – \$10
= \$80

(b) 20 parts = \$160

$1 \text{ part} = \frac{160}{20} = \$8$

2 parts = $\$8 \times 2 = \$16$

13 parts = $\$8 \times 13 = \$104$

Difference between the largest share and the smallest share
= \$104 – \$16
= \$88

(c) 40 parts = \$160

$1 \text{ part} = \frac{160}{40} = \$4$

5 parts = $\$4 \times 5 = \$20$

22 parts = $\$4 \times 22 = \$88$

Difference between the largest share and the smallest share
= \$88 – \$20
= \$68

(d) 80 parts = \$160

$1 \text{ part} = \frac{160}{80} = \$2$

11 parts = $\$2 \times 11 = \$22$

37 parts = $\$2 \times 37 = \$74$

Difference between the largest share and the smallest share
= \$74 – \$22
= \$52

27. **(a)** $X : Y = 2 : 3 = 10 : 15$ $Y : Z = 5 : 4 = 15 : 12$

$\therefore X : Z = 10 : 12 = 5 : 6$

(b) $X : Y = 5 : 7 = 65 : 91$ $Y : Z = 13 : 10 = 91 : 70$

$\therefore X : Z = 65 : 70 = 13 : 14$

(c) $X : Y = 7 : 3 = 77 : 33$ $Y : Z = 11 : 21 = 33 : 63$

$\therefore X : Z = 77 : 63 = 11 : 9$

(d) $X : Y = 8 : 15 = 56 : 105$ $Y : Z = 21 : 32 = 105 : 160$

$\therefore X : Z = 56 : 160 = 7 : 20$

28. Rice B is sold at \$6.90 for 5 kg. Thus it is sold at \$13.80 for 10 kg.
Ratio of prices of rice A and B
= \$9.20 : \$13.80
= 920 : 1380
= 2 : 3

29. $A : B = 8 : 3 = 40 : 15$ $A : C = 5 : 12 = 40 : 96$

The ratio of salaries A, B and C
= 40 : 15 : 96

30. Height of the hall = $\frac{28}{7} \times 6 = 24$ m

Ratio of its breadth to its height
= 21 : 24
= 7 : 8

31. **(i)** Dimensions of second rectangle

$= 32 \times \frac{5}{4}$ cm by $24 \times \frac{5}{4}$ cm

= 40 cm by 30 cm

Ratio of perimeters of original rectangle and second rectangle
= 2(32 + 24) : 2(40 + 30)
= 112 : 140
= 4 : 5

(ii) Ratio of areas of original rectangle and second rectangle
$= 32 \times 24 : 40 \times 30$
= 768 : 1200
= 16 : 25

32. **(i)** Time for which the car is parked
= 16 30 – 07 45
= 8 h 45 min or $8\frac{3}{4}$ h

(ii) Parking fee
$= \$2.50 + 14 \times \$0.80 + \$0.80 + \$0.80$
= \$15.30

33. (i) Amount each tourist spends for 4 days

$= \dfrac{\$3600}{9} = \$400$

Cost of staying in the hotel for one day

$= \dfrac{\$400}{4} = \$100$

Cost of staying in the hotel for 6 days

$= \$100 \times 6 = \$600$

Amount 15 tourists spend for staying in the hotel for 6 days

$= \$600 \times 15$

$= \$9000$

(ii) Amount each tourist spends $= \dfrac{\$3000}{10} = \$300$

Number of days each tourist can stay in the hotel

$= \dfrac{\$300}{\$100} = 3$

34. (i) Charges due to the number of calls

$= 493 \times \$0.1605$

$= \$79.1265$

Total charges for the month

$= \$82.93 + \$79.1265$

$= \$162.06$ (to the nearest cent)

(ii) Charges due to calls $= \$93.523 - \$82.93$

$= \$10.593$

Number of calls made $= \dfrac{\$10.593}{\$0.1605} = 66$

She made 66 calls.

35. No. of hours 1 man will take to complete 1200 m

$= 8 \times 20 \times 50$

$= 8000$ h

No. of hours 1 man will take to complete 1800 m

$= \dfrac{1800}{1200} \times 8000$

$= 12\,000$ h

No. of men needed to complete the work on time

$= \dfrac{12\,000}{10 \times 10}$

$= 120$

Additional number of men to be employed

$= 120 - 60$

$= 70$

36. (i) Amount of time to work on the project per day

$= 8.5 \times 4$

$= 34$ h

Time required to finish the work

$= \dfrac{272}{34} = 8$ days

It will take 8 days for 4 men to finish the work.

(ii) Amount to be paid to the men per day

$= \$8.50 \times 8.5 \times 4$

$= \$289$

Total amount to be paid for the whole project

$= 8 \times \$289$

$= \$2312$

(iii) Let the number of overtime hours needed to complete the project in 4 days by each worker be x.

$5[4(8.5 + x)] = 272$

$5(34 + 4x) = 272$

$170 + 20x = 272$

$20x = 272 - 170 = 102$

$x = 5.1$

The number of overtime hours is 5.1 h.

(iv) Overtime hourly rate

$= 1.5 \times \$8.50$

$= \$12.75$

Total amount to be paid to the 4 men if the project is to be completed in 5 days

$= 5\{4[(8.5 \times \$8.50) + (5.1 \times \$12.75)]\}$

$= \$2745.50$

37. Distance travelled by the wheel $= 765 \times 2.8$

$= 2142$ m

Number of revolutions made by the wheel to travel a distance of 2142 m

$= \dfrac{2142}{1.7}$

$= 1260$ times

38. Convert 46 minutes to hours.

$46 \text{ min} = \frac{46}{60} = \frac{23}{30} \text{ h}$

Let the time taken to travel from Town Y to Z be T hours.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$80 = \frac{80 + 48}{T + \frac{23}{30}}$$

$$80\left(T + \frac{23}{30}\right) = 128$$

$$80T + 61\frac{1}{3} = 128$$

$$80T = 128 - 61\frac{1}{3}$$

$$= 66\frac{2}{3}$$

$$T = \frac{5}{6} \text{ h}$$

Speed of the driver when he is driving from Town Y to Z

$$= \frac{80}{\frac{5}{6}}$$

$= 96$ km/h

39. **(i)** Time arrived at $B = 1035 + 0019$

$= 1054$

Time arrived at $C = 1150 + (0019 - 0011)$

$= 1158$

(ii) Time to travel from Town C to D

$= 1320 - 1158$

$= 1$ h 22 min

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{123}{1\frac{22}{60}}$$

$= 90$ km/h

Advanced

40. $\frac{a - 2b}{10} = \frac{b}{6}$

$$6(a - 2b) = 10b$$

$$6a - 12b = 10b$$

$$6a = 10b + 12b$$

$$6a = 22b$$

$$\frac{6a}{b} = 22$$

$$\frac{a}{b} = \frac{22}{6} = \frac{11}{3}$$

The ratio of $a : b = 11 : 3$.

41. Let the distance travelled by the motorist be y km.

$$y = x \times 2\frac{1}{2}$$

$$= 2\frac{1}{2}x \qquad \text{— (1)}$$

$$y = (x + 4) \times \left(2\frac{1}{2} - \frac{15}{60}\right)$$

$$= 2\frac{1}{4}(x + 4) \qquad \text{— (2)}$$

Substitute (1) into (2):

$$2\frac{1}{2}x = 2\frac{1}{4}(x + 4)$$

$$2\frac{1}{2}x = 2\frac{1}{4}x + 9$$

$$2\frac{1}{2}x - 2\frac{1}{4}x = 9$$

$$\frac{1}{4}x = 9$$

$$x = 36$$

The value of x is 36.

42. Time taken for the van to travel a distance of 130 km

$$= \frac{130}{65}$$

$= 2$ h

Time taken for the car to travel a distance of 130 km

$$= 2 - \frac{35}{60}$$

$$= 1\frac{5}{12} \text{ h}$$

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{130}{1\frac{5}{12}}$$

$$= 91\frac{13}{17} \text{ km/h}$$

New Trend

43. (a) (i) $\frac{7}{3} = \frac{\text{Number of sedans}}{180}$

Number of sedans = $\frac{7}{3} \times 180 = 420$

(ii) Number of vehicles altogether

$= \frac{180}{3} \times (7 + 3 + 2)$

$= 720$

(b) Blue : Black : White

$3 \quad : \quad 5$

$\times 7 \qquad 6 \quad : \quad 7 \qquad \times 5 \quad \times 5$

$= 21 : 30 : 35$

$\therefore$ Blue sedan : black sedan : white sedan

$= 21 : 30 : 35$

44. (a) $280 \text{ km/h} = \frac{280 \text{ km}}{1 \text{ h}}$

$= \frac{280\,000 \text{ m}}{3600 \text{ s}}$

$= 77\frac{7}{9} \text{ m/s}$

(b) Time take for bullet train to pass through tunnel completely

$= \frac{(20\,500 + 250) \text{ m}}{77\frac{7}{9} \text{ m/s}}$

$= 266\frac{11}{14} \text{ s}$

= 4 min 27 s (to the nearest second)

45. Lixin gets 13 – 7 = 6 parts more than Nora.

(a) 6 parts = $78

1 part = $\frac{78}{6}$ = $13

12 parts = $13 × 12 = $156

(b) 6 parts = $126

1 part = $\frac{126}{6}$ = $21

12 parts = $21 × 12 = $252

(c) 6 parts = $360

1 part = $\frac{360}{6}$ = $60

12 parts = $60 × 12 = $720

(d) 6 parts = $540

1 part = $\frac{540}{6}$ = $90

12 parts = $90 × 12 = $1080

46. Time taken to fly from Singapore to Helsinki

$= \frac{9257}{752}$

= 12.3125 h

= 12 h 0.1325 × 60 min

= 12 h 19 min (to the nearest minute)

47. (i) Distance travelled on 1 litre of petrol

$= \frac{128}{12}$

$= 10\frac{2}{3}$ km

Distance travelled on 30 litres of petrol

$= 10\frac{2}{3} \times 30$

= 320 km

(ii) Amount of petrol required to travel a distance of 1 km

$= \frac{12}{128}$ litres

Amount of petrol required to travel a distance of 15 000 km

$= \frac{12}{128} \times 15\,000$

= 1406.25 litres

Amount the car owner has to pay

= 1406.25 × $2.03

= $2854.69 (to the nearest cent)

48. (a) 180 km → 50.4 litres

100 km → $\frac{50.4}{180} \times 100$

= 28 litres

The fuel consumption of the bus is 28 *l*/100 km.

(b) (i) 7.6 litres → 100 km

50 litres → $\frac{100}{7.6} \times 50$

= 658 km (to 3 s.f.)

(ii) 100 km → 7.6 litres

330 km → $\frac{7.6}{100} \times 330$

= 25.08 litres

1 litre → $2.07

25.08 litres → $2.07 × 25.08

= $51.92 (to the nearest cent)

The petrol will cost Fred $51.92 for a journey of 330 km.

Chapter 10 Basic Geometry

Basic

1. (a) $x° + 90° + 38° = 180°$ (adj. ∠s on a str. line)

$x° = 180° - 90° - 38°$

$= 52°$

$\therefore x = 52$

(b) $2x° + 80° = 180°$ (adj. ∠s on a str line)

$2x° = 180° - 80°$

$= 100°$

$x° = 50°$

$\therefore x = 50$

(c) $2x° + (5x - 9)° = 180°$ (adj. ∠s on a str. line)

$7x° - 9° = 180°$

$7x° = 189°$

$x° = 27°$

$\therefore x = 27$

(d) $(5x - 23)° + (7x - 13)° = 180°$ (adj. ∠s on a

$5x° + 7x° - 23° - 13° = 180°$ str. line)

$12x° - 36° = 180°$

$12x° = 180° + 36°$

$= 216°$

$x° = 18°$

$\therefore x = 18$

(e) $2x° + 90° + 3x° = 180°$ (adj. ∠s on a str. line)

$2x° + 3x° = 180° - 90°$

$5x° = 90°$

$x° = 18°$

$\therefore x = 18$

(f) $3x° + 4x° + 2x° = 180°$ (adj. ∠s on a str. line)

$9x° = 180°$

$x° = 20°$

$\therefore x = 20$

2. (a) $4x° + 3x° + 2x° = 180°$ (vert. opp. ∠s;

$9x° = 180°$ adj. ∠s on a str. line)

$x° = 20°$

$\therefore x = 20$

(b) $3x° + 49° + 62° = 180°$ (adj. ∠s on a str. line)

$3x° = 180° - 49° - 62°$

$3x° = 69°$

$x° = 23°$

$3x° + z° = 180°$ (adj. ∠s on a str. line)

$3(23°) + z° = 180°$

$69° + z° = 180°$

$z° = 180° - 69°$

$= 111°$

$y° + z° = 180°$ (adj. ∠s on a str. line)

$y° + 111° = 180°$

$y° = 180° - 111°$

$= 69°$

$\therefore x = 23, y = 69$ and $z = 111$

3. (a) $(3x + 34)° = (5x - 14)°$ (alt. ∠s, *AB* // *CD*)

$5x° - 3x° = 34° + 14°$

$2x° = 48°$

$x° = 24°$

$\therefore x = 24$

(b) $(7x - 12)° + (4x - 17)° = 180°$ (int. ∠s,

$7x° + 4x° - 12° - 17° = 180°$ *AB* // *CD*)

$11x° - 29° = 180°$

$11x° = 180° + 29°$

$11x° = 209°$

$x° = 19°$

$\therefore x = 19$

(c) $4x° + 5x° = 180°$ (alt. ∠s, adj. ∠s on a str. line)

$9x° = 180°$

$x° = 20°$

$\therefore x = 20$

(d) $(5x - 14)° + (3x - 10)° = 180°$ (alt. ∠s, adj. ∠s

$5x° + 3x° - 14° - 10° = 180°$ on a str. line)

$8x° - 24° = 180°$

$8x° = 180° + 24°$

$= 204°$

$x° = 25.5°$

$\therefore x = 25.5$

(e) $(5x - 15)° + (75 - x)° = 180°$ (vert. opp. ∠s,

$5x° - x° - 15° + 75° = 180°$ int. ∠s,

$4x° + 60° = 180°$ *AB* // *CD*)

$4x° = 180° - 60°$

$= 120°$

$x° = 30°$

$\therefore x = 30$

(f) $(3x + 40)° = (5x - 20)°$ (corr. ∠s, *AB* // *CD*)

$5x° - 3x° = 40° + 20°$

$2x° = 60°$

$x° = 30$

$(5x - 20)° = 2y°$ (vert. opp. ∠s)

$5 \times 30° - 20° = 2y°$

$2y° = 130°$

$y° = 65°$

$\therefore x = 30$ and $y = 65$

Intermediate

4. **(a)** $3x° + (7x - 21)° + (4x - 9)° = 180°$ (adj. ∠s on a str. line)

$3x° + 7x° + 4x° - 21° - 9° = 180°$

$14x° - 30° = 180°$

$14x° = 180° + 30°$

$= 210°$

$x° = 15°$

$\therefore x = 15$

(b) $\left(\frac{1}{3}x + 8\right)° + \left(\frac{3}{4}x - 18\right)° + \frac{1}{2}x°$

$= 180°$ (adj. ∠s on a str. line)

$\frac{1}{3}x° + \frac{3}{4}x° + \frac{1}{2}x° + 8° - 18° = 180°$

$1\frac{7}{12}x° = 180° + 10°$

$= 190°$

$x° = 120°$

$\therefore x = 120$

(c) $1.8x° + (2x + 12)° + x° = 180°$ (adj. ∠s on a str. line)

$1.8x° + 2x° + x° = 180° - 12°$

$4.8x° = 168°$

$x° = 35°$

$\therefore x = 35$

(d) $(0.5x + 14)° + (x + 15)° + (0.2x + 15)°$

$= 180°$ (adj. ∠s on a str. line)

$0.5x° + x° + 0.2x° + 14° + 15° + 15° = 180°$

$1.7x° + 44° = 180°$

$1.7x° = 136°$

$x° = 80°$

$\therefore x = 80$

5. **(a)** $3x° + (7x - 20)° = 180°$ (adj. ∠s on a str. line)

$3x° + 7x° = 180° + 20°$

$10x° = 200°$

$x° = 20°$

$3x° + y° = 180°$ (adj. ∠s on a str. line)

$3(20°) + y° = 180°$

$60° + y° = 180°$

$y° = 180° - 60° = 120°$

$\therefore x = 20$ and $y = 120$

(b) $(4x - 5)° + (8x - 41)° + 3x° + (3x + 10)°$

$= 360°$ (∠s at a point)

$4x° + 8x° + 3x° + 3x° - 5° - 41° + 10° = 360°$

$18x° - 36° = 360°$

$18x° = 360° + 36°$

$= 396°$

$x° = 22°$

$\therefore x = 22$

(c) $y° + 70° = 180°$ (adj. ∠s on a str. line)

$y° = 180° - 70°$

$= 110°$

$28° + (3x - 5)° + 70° = 180°$ (adj. ∠s on a str. line)

$3x° + 28° - 5° + 70° = 180°$

$3x° + 93° = 180°$

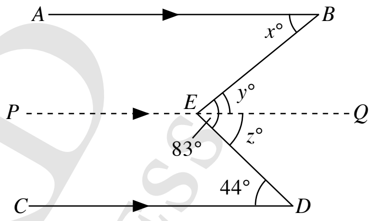
$3x° = 180° - 93°$

$= 87°$

$x° = 29°$

$\therefore x = 29$ and $y = 110$

6. **(a)** Draw a line *PQ* through *E* that is parallel to *AB* and *CD*.



$z° = 44°$ (alt. ∠s, $PQ \parallel CD$)

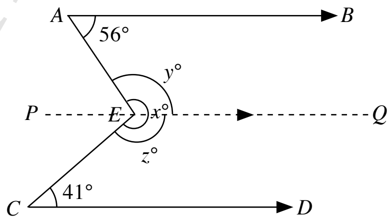
$y° = 83° - 44°$

$= 39°$

$x° = y° = 39°$ (alt. ∠s, $PQ \parallel AB$)

$\therefore x = 39$

(b) Draw a line *PQ* through *E* that is parallel to *AB* and *CD*.



$41° + z° = 180°$ (int. ∠s, $PQ \parallel CD$)

$z° = 180° - 41°$

$= 139°$

$56° + y° = 180°$ (int. ∠s, $PQ \parallel AB$)

$y° = 180° - 56°$

$= 124°$

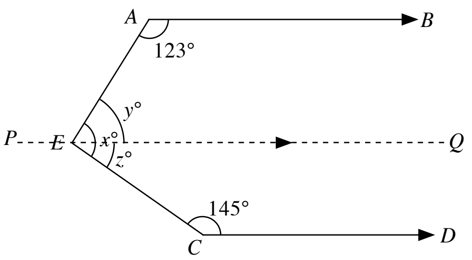
$x° = y° + z°$

$= 124° + 139°$

$= 263°$

$\therefore x = 263$

(c) Draw a line PQ through E that is parallel to AB and CD.



$123° + y° = 180°$ (int. ∠s, PQ // AB)

$y° = 180° – 123°$

$= 57°$

$145° + z° = 180°$ (int. ∠s, PQ // CD)

$z° = 180° – 145°$

$= 35°$

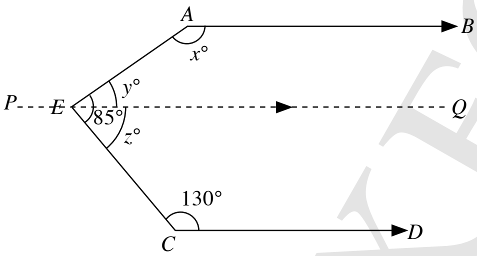
$x° = y° + z°$

$= 57° + 35°$

$= 92°$

$\therefore x = 92$

(d) Draw a line PQ through E that is parallel to AB and CD.



$130° + z° = 180°$ (int. ∠s, PQ // CD)

$z° = 180° – 130°$

$= 50°$

$z° + y° = 85°$

$y° = 85° – z°$

$= 85° – 50°$

$= 35°$

$y° + x° = 180°$ (int. ∠s, PQ // AB)

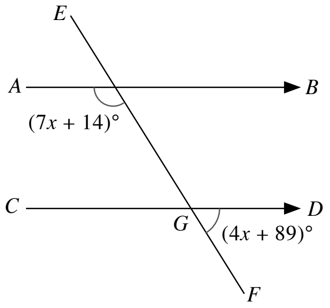
$x° = 180° – y°$

$= 180° – 35°$

$= 145°$

$\therefore x = 145$

(e)



$\angle CGE = (4x + 89)°$ (vert. opp. ∠s)

$(4x + 89)° + (7x + 14)° = 180°$ (int. ∠s, AB // CD)

$4x° + 7x° + 89° + 14° = 180°$

$11x° + 103° = 180°$

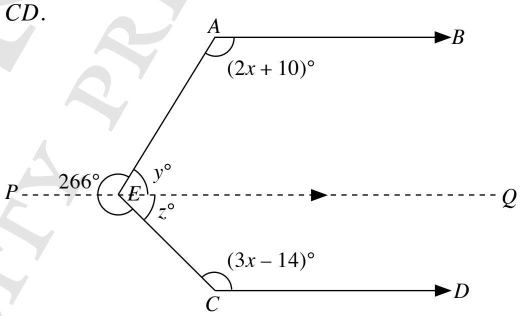
$11x° = 180° – 103°$

$= 77°$

$x° = 7°$

$\therefore x = 7$

(f) Draw a line PQ through E that is parallel to AB and CD.



$\angle AEC = 360° – 266° = 94°$ (∠s at a point)

$y° + (2x + 10)° = 180°$

$y° = 180° – (2x + 10)°$

$= 180° – 2x° – 10°$

$= 170° – 2x°$

$z° + (3x – 14)° = 180°$

$z° = 180° – (3x – 14)°$

$= 180° – 3x° + 14°$

$= 194° – 3x°$

$y° + z° = 94°$

$170° – 2x° + 194° – 3x° = 94°$

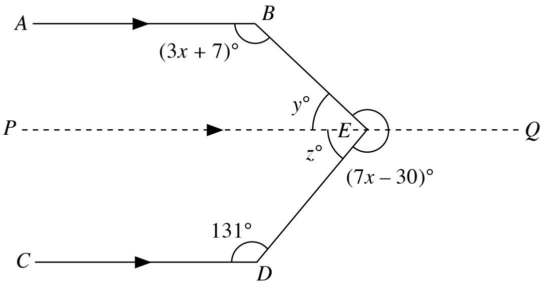
$2x° + 3x° = 170° + 194° – 94°$

$5x° = 270°$

$x° = 54°$

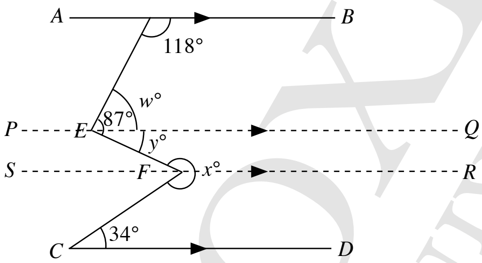
$\therefore x = 54$

(g) Draw a line PQ through E that is parallel to AB and CD.



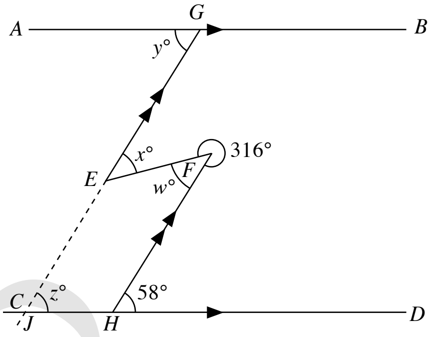
$z° + 131° = 180°$ (int. ∠s, PQ // CD)
$z° = 180° - 131°$
$= 49°$
$y° + z° + (7x - 30)° = 360°$ (∠s at a point)
$y° + 49° + (7x - 30)° = 360°$
$y° = 360° - 49° - (7x - 30)°$
$= 360° - 49° - 7x° + 30°$
$= 341° - 7x°$
$(3x + 7)° + y° = 180°$ (int. ∠s, PQ // AB)
$(3x + 7)° + 341° - 7x° = 180°$
$3x° + 7° + 341° - 7x° = 180°$
$4x° = 168°$
$x° = 42°$
$\therefore x = 42$

(h) Draw a line PQ through E, and a line SR through F, that is parallel to AB and CD.



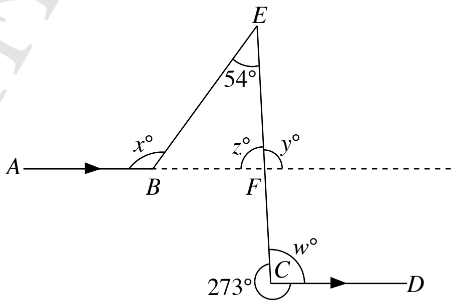
$118° + w° = 180°$ (int. ∠s, PQ // AB)
$w° = 180° - 118°$
$= 62°$
$w° + y° = 87°$
$y° = 87° - w°$
$= 87° - 62°$
$= 25°$
$\angle RFE + y° = 180°$ (int. ∠s, SR // PQ)
$\angle RFE = 180° - y°$
$= 180° - 25° = 155°$
$\angle CFR + 34° = 180°$ (int. ∠s, SR // CD)
$\angle CFR = 180° - 34°$
$= 146°$
$x° = 155° + 146° = 301°$
$\therefore x = 301$

7. (a)



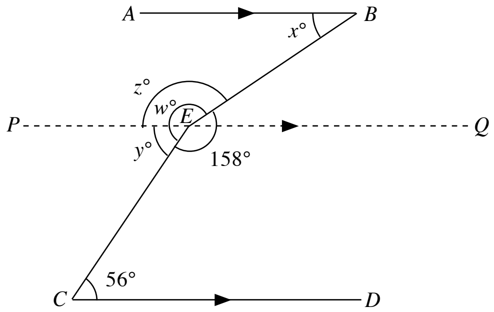
$w° + 316° = 360°$ (∠s at a point)
$w° = 360° - 316° = 44°$
$w° = x°$ (alt. ∠s, EG // HF)
$x° = 44°$
Extend the line EG to meet the line CD at J.
$z° = 58°$ (corr. ∠s, JG // HF)
$y° = 58°$ (alt. ∠s, AB // CD)
$\therefore x = 44$ and $y = 58$

(b) Extend the line AB to meet the line EC at F.



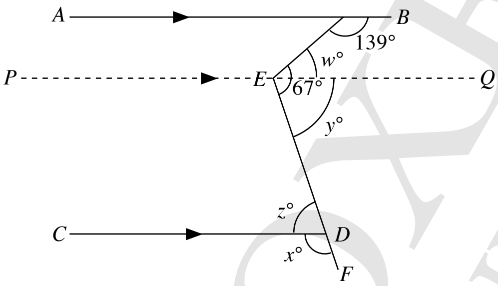
$w° + 273° = 360°$ (∠s at a point)
$w° = 360° - 273° = 87°$
$y° = w° = 87°$ (corr. ∠s, AB // CD)
$z° + y° = 180°$ (adj. ∠s on a str. line)
$z° = 180° - y°$
$= 180° - 87°$
$= 93°$
$x° = 54° + z°$ (ext. ∠ of △BEF)
$= 54° + 93°$
$= 147°$
$\therefore x = 147$

(c) Draw a line PQ through E that is parallel to AB and CD.



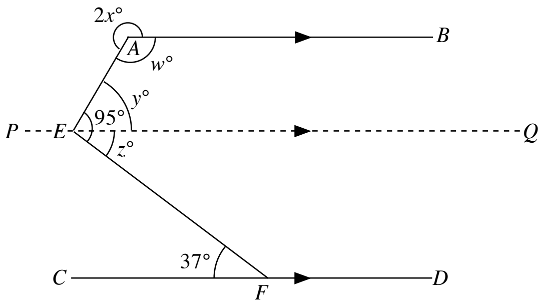
$w° + 158° = 360°$ (∠s at a point)
$w° = 360° - 158°$
$= 202°$
$y° = 56°$ (alt. ∠s, PQ // CD)
$z° + y° = w° = 202°$
$z° = 202° - y°$
$= 202° - 56°$
$= 146°$
$x° + z° = 180°$ (int. ∠s, PQ // AB)
$x° = 180° - z°$
$= 180° - 146°$
$= 34°$
$\therefore x = 34$

(d) Draw a line PQ through E that is parallel to AB and CD.



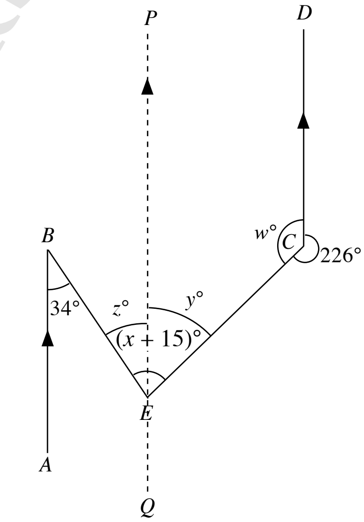
$w° + 139° = 180°$ (int. ∠s, PQ // AB)
$w° = 180° - 139°$
$= 41°$
$w° + y° = 67°$
$y° = 67° - w°$
$= 67° - 41°$
$= 26°$
$z° = y° = 26°$ (alt. ∠s, PQ // CD)
$x° + z° = 180°$ (adj. ∠s on a str. line)
$x° = 180° - z°$
$= 180° - 26°$
$= 154°$
$\therefore x = 154$

(e) Draw a line PQ through E that is parallel to AB and CD.



$z° = 37°$ (alt. ∠s, PQ // CD)
$y° + z° = 95°$
$y° = 95° - z°$
$= 95° - 37°$
$= 58°$
$w° + y° = 180°$ (int. ∠s, PQ // AB)
$w° = 180° - y°$
$= 180° - 58°$
$= 122°$
$2x° + w° = 360°$ (∠s at a point)
$2x° = 360° - w°$
$= 360° - 122°$
$= 238°$
$x° = 119°$
$\therefore x = 119$

(f) Draw a line PQ through E that is parallel to AB and CD.



$w° + 226° = 360°$ (∠s at a point)
$w° = 360° - 226°$
$= 134°$

$y° + w° = 180°$ (int. ∠s, *PQ* // *DC*)

$y° = 180° - w°$
$= 180° - 134°$
$= 46°$

$z° = 34°$ (alt. ∠s, *PQ* // *BA*)

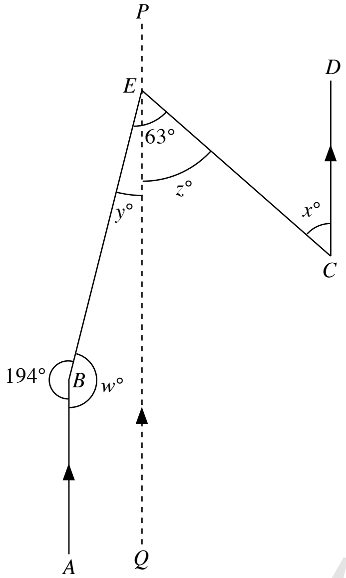
$(x + 15)° = y° + z°$

$(x + 15)° = 46° + 34° = 80°$

$x° = 80° - 15°$
$= 65°$

$\therefore x = 65$

(g) Draw a line *PQ* through *E* that is parallel to *AB* and *CD*.



$w° + 194° = 360°$ (∠s at a point)

$w° = 360° - 194°$
$= 166°$

$y° + w° = 180°$ (int. ∠s, *PQ* // *BA*)

$y° = 180° - w°$
$= 180° - 166°$
$= 14°$

$z° + y° = 63°$

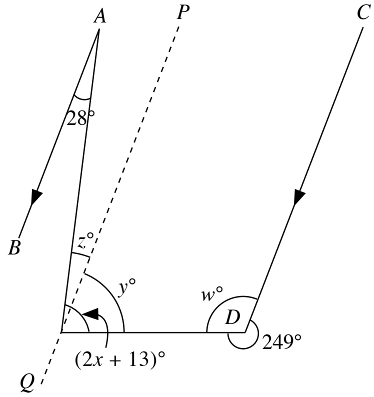
$z° = 63° - y°$
$= 63° - 14°$

f = 49°

$x° = z° = 49°$ (alt. ∠s, *PQ* // *DC*)

$\therefore x = 49$

(h) Draw a line *PQ* through *E* that is parallel to *AB* and *CD*.



$w° + 249° = 360°$ (∠s at a point)

$w° = 360° - 249°$
$= 111°$

$y° + w° = 180°$ (int. ∠s, *PQ* // *CD*)

$y° = 180° - w°$
$= 180° - 111°$
$= 69°$

$z° = 28°$ (alt. ∠s, *AB* // *PQ*)

$(2x + 13)° = y° + z°$

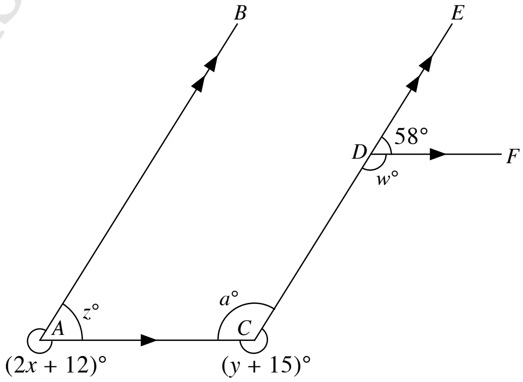
$(2x + 13)° = 69° + 28° = 97°$

$2x° = 97° - 13°$
$= 84°$

$x° = 42°$

$\therefore x = 42$

8. (a)



$w° + 58° = 180°$ (adj. ∠s on a str. line)

$w° = 180° - 58°$
$= 122°$

$a° = w° = 122°$ (alt. ∠s, *DF* // *AC*)

$(y + 15)° + a° = 360°$ (∠s at a point)

$y° + 15° + 122° = 360°$

$y° = 360° - 15° - 122°$
$= 223°$

$z° + a° = 180°$ (int. $\angle$s, AB // CE)

$z° = 180° - a°$

$= 180° - 122°$

$= 58°$

$(2x + 12)° + z° = 360°$ ($\angle$s at a point)

$2x° + 12° + 58° = 360°$

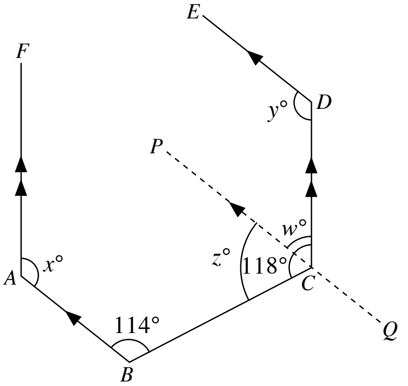
$2x° = 360° - 12° - 58°$

$= 290°$

$x° = 145°$

$\therefore x = 145$ and $y = 223$

(b) Draw a line PQ through C that is parallel to ED and AB.



$z° + 114° = 180°$ (int. $\angle$s, PQ // AB)

$z° = 180° - 114°$

$= 66°$

$w° + z° = 118°$

$w° = 118° - 66°$

$= 52°$

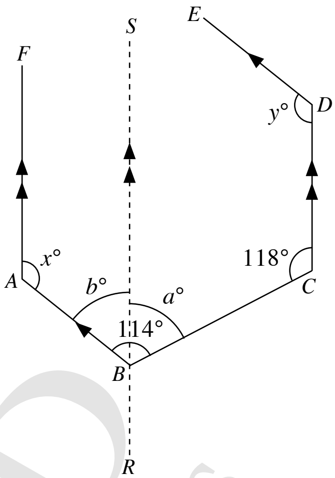
$y° + w° = 180°$ (int. $\angle$s, PQ // ED)

$y° + 52° = 180°$

$y° = 180° - 52°$

$= 128°$

Draw another line SR through B that is parallel to AF and CD.



$a° + 118° = 180°$ (int. $\angle$s, SR // DC)

$a° = 180° - 118°$

$= 62°$

$b° + a° = 114°$

$b° = 114° - a°$

$= 114° - 62°$

$= 52°$

$x° + b° = 180°$ (int. $\angle$s, SR // FA)

$x° = 180° - b°$

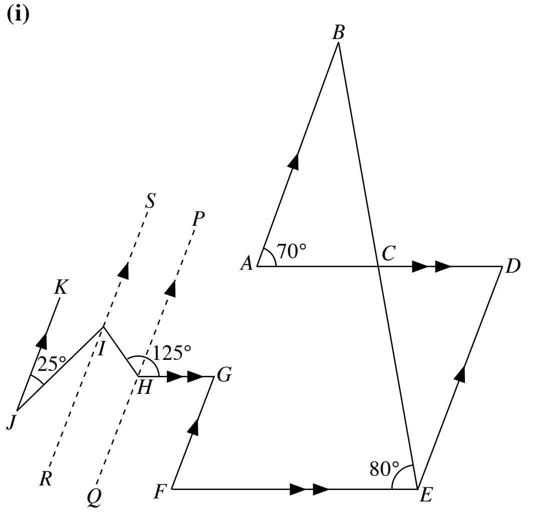
$= 180° - 52°$

$= 128°$

$\therefore x = y = 128$

Advanced

9. (i)



$\angle ADE = 70°$ (alt. $\angle$s, AB // ED)

$\angle FED + \angle ADE = 180°$ (int. $\angle$s, FE // AD)

$\angle FED = 180° - \angle ADE$

$= 180° - 70°$

$= 110°$

$\angle CED + \angle FEB = \angle FED$

$\angle CED = \angle FED - \angle FEB$

$= 110° - 80°$

$= 30°$

(ii) $\angle EFG + \angle FED = 180°$ (int. $\angle$s, FG // ED)

$\angle EFG = 180° - \angle FED$

$= 180° - 110°$

$= 70°$

(iii) $\angle HGF = \angle EFG$ (alt. $\angle$s, HG // FE)

$= 70°$

Draw a line PQ through H that is parallel to FG and KJ.

$\angle GHP = \angle HGF = 70°$ (alt. $\angle$s, PQ // FG)

$\angle IHP = 125° - 70°$

$= 55°$

Draw a line SR through I that is parallel to PQ and KJ.

$\angle HIR = \angle IHP = 55°$ (alt. $\angle$s, SR // PQ)

$\angle JIR = \angle IJK = 25°$ (alt. $\angle$s, SR // KJ)

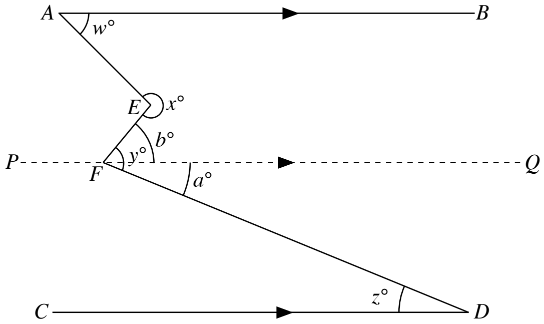
$\angle HIJ = 55° + 25°$

$= 80°$

Reflex $\angle HIJ = 360° - 80°$

$= 280°$

10. Draw a line PQ through F that is parallel to AB and CD.



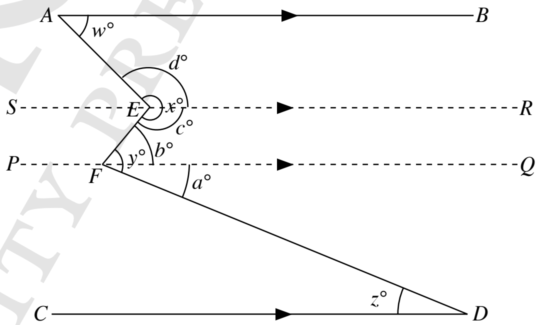
$a° = z°$ (alt. $\angle$s, PQ // CD)

$b° + a° = y°$

$b° = y° - a°$

$= y° - z°$

Draw a line SR through E that is parallel to AB, PQ and CD.



$c° + b° = 180°$ (int. $\angle$s, SR // PQ)

$c° = 180° - b°$

$= 180° - (y° - z°)$

$= 180° - y° + z°$

$d° + w° = 180°$ (int. $\angle$s, SR // AB)

$d° = 180° - w°$

$x° = d° + c°$

$= 180° - w° + 180° - y° + z°$

$= 360° - w° - y° + z°$

$\therefore x = 360 - w - y + z$

New Trend

11. (i) $W\hat{P}X = 180° - 65° - (180° - 145°)$ (vert. opp. ∠s, adj. ∠s on a str. line, ∠ sum of △)

$= 80°$

(ii) Reason 1

Converse of interior angles theorem

Since $W\hat{Y}Z + Y\hat{W}X = 180°$, then $AB \parallel CD$ (converse of int. ∠s)

Reason 2

Converse of corresponding angles postulate

$P\hat{W}X = 180° - 145°$ (adj. ∠s on a str. line)

$= 35°$

∴ Since $P\hat{W}X = W\hat{Y}Z$, then $AB \parallel CD$ (converse of corr. ∠s)

(iii) $D\hat{Z}R = B\hat{X}Z$ (corr. ∠s, $AB \parallel CD$)

$= 65°$

Chapter 11 Triangles, Quadrilaterals and Polygons

Basic

1. **(a)** $2x° + 46° + 82° = 180°$ ($\angle$ sum of $\triangle$)
$2x° = 180° - 46° - 82°$
$= 52°$
$x° = 26°$
$\therefore x = 26$

(b) $x° + 58° + 58° = 180°$ ($\angle$ sum of $\triangle$)
$x° = 180° - 58° - 58°$
$= 64°$
$\therefore x = 64$

(c) $x° + x° + 70° = 180°$
$2x° = 180° - 70°$
$= 110°$
$x° = 55°$
$\therefore x = 55$

(d) $3x° = 63°$
$x° = 21°$
$\therefore x = 21$

(e) $3y° = 48°$ (base $\angle$s of isos. $\triangle$)
$y° = 16°$
$2x° + 3y° + 48° = 180°$ ($\angle$ sum of $\triangle$)
$2x° = 180° - 3y° - 48°$
$= 180° - 3(16°) - 48°$
$= 180° - 48° - 48°$
$= 84°$
$x° = 42°$
$\therefore x = 42$ and $y = 16$

2. **(a)** $x° + 39° = 123°$ (ext. $\angle$ of $\triangle$)
$x° = 123° - 39°$
$= 84°$
$\therefore x = 84$

(b) $y° + 40° = 180°$ (adj. $\angle$s on a str. line)
$y° = 180° - 40°$
$y° = 140°$
$4x° + 3x° = y° = 140°$ (ext. $\angle$ of $\triangle$)
$7x° = 140°$
$x° = 20°$
$\therefore x = 20$ and $y = 140$

(c) $26° + 26° = x°$ (ext. $\angle$ of $\triangle$)
$x° = 52°$
$\therefore x = 52$

(d) $\angle CAD = 180° - 110°$ (adj. $\angle$s on a str. line)
$= 70°$
$\angle CDA = \angle CAD = 70°$ (base $\angle$s of isos. $\triangle ACD$)
$x° + 70° = 110°$ (ext. $\angle$ of $\triangle$)
$x° = 110° - 70°$
$= 40°$
$\therefore x = 40$

(e) $\angle EAD = x°$ (vert. opp. $\angle$s)
$x° + 72° + 50° = 180°$ ($\angle$ sum of $\triangle$)
$x° = 180° - 72° - 50°$
$= 58°$
$\therefore x = 58$

(f) $\angle DAC = 60°$ ($\angle$s of equilateral $\triangle ACD$)
$2y° + 2y° = 60°$ (base $\angle$s of isos. $\triangle ACB$,
$4y° = 60°$ ext. $\angle$ of $\triangle$)
$y° = 15°$
$\therefore y = 15$

3. **(a)** $\angle CAB = 46°$ (alt. $\angle$s, DE // AB)
$x° + 46° = 91°$ (ext. $\angle$ of $\triangle ACB$)
$x° = 91° - 46°$
$= 45°$
$\therefore x = 45$

(b) $\angle CBA = 3x°$ (alt. $\angle$s, CD // AB)
$3x° + 2x° + 55° = 180°$ ($\angle$ sum of $\triangle ACB$)
$5x° = 180° - 55°$
$= 125°$
$x° = 25°$
$\therefore x = 25$

(c) $\angle BDA = x°$ (base $\angle$s of isos. $\triangle ABD$)
$x° + x° + x° + 63° = 180°$ ($\angle$ sum of $\triangle ADC$)
$3x° = 180° - 63° = 117°$
$x° = 39°$
$\therefore x = 39$

(d) $\angle DBA = 58°$ (alt. $\angle$s, DE // AB)
$x° + 58° = 79°$ (ext. $\angle$ of $\triangle ACB$)
$x° = 79° - 58°$
$= 21°$
$y° + 79° + 3x° = 180°$ ($\angle$ sum of $\triangle ACD$)
$3x° + y° = 180° - 79° = 101°$
$y° = 101° - 3x°$
$= 101° - 3(21°)$
$= 38°$
$\therefore x = 21$ and $y = 38$

4. (a) $2x° + 62° = 134°$ (ext. ∠ of △)
$2x° = 134° - 62°$
$= 72°$
$x° = 36°$
$\angle BCE = 2x°$ (alt. ∠s, $CE // AB$)
$y° + 134° + 2x° = 360°$ (∠s at a point)
$y° = 360° - 134° - 2(36°)$
$= 360° - 134° - 72°$
$= 154°$
∴ $x = 36$ and $y = 154$

(b) $\angle ACD = 180° - 109°$ (int. ∠s, $ED // AF$)
$= 71°$
$x° + 24° = 71°$ (ext. ∠ of △ABC)
$x° = 71° - 24°$
$= 47°$
∴ $x = 47$

(c) $y° + 63° = 142°$ (ext. ∠ of △)
$y° = 142° - 63°$
$= 79°$
$\angle ADF + 63° = 180°$ (int. ∠s, $EF // AC$)
$\angle ADF = 180° - 63°$
$= 117°$
$x° = \angle ADF = 117°$ (vert. opp. ∠s)
∴ $x = 117$ and $y = 79$

(d) $\angle DEC = y°$ (alt. ∠s, $ED // AC$)
$4x° + y° = 180°$ (adj. ∠s on a str. line)
$y° = 180° - 4x°$
$\angle ECD = 180° - y° - 36°$
$= 144° - y°$
$144° - y° + 2x° = 4x°$ (ext. ∠ of △DEC)
$144° - (180° - 4x°) + 2x° = 4x°$
$2x° = 36°$
$x° = 18°$
$y° = 180° - 4(18°)$
$= 108°$
∴ $x = 18$ and $y = 108$

5. (i) $\angle BAC = 36°$ (base ∠s of isos. △ABC)
$\angle ACD = \angle ABC + \angle BAC$ (ext. ∠ of △ABC)
$= 36° + 36°$
$= 72°$
$\angle ADC = \angle ACD = 72°$ (base ∠s of isos. △ACD)
$\angle CAD = 180° - 72° - 72°$ (∠ sum of △ACD)
$= 36°$

(ii) $\angle ADE = \angle CAD + \angle ACD$ (ext. ∠ of △ACD)
$= 72° + 36°$
$= 108°$

6. (a) $x° + 29° = 90°$ (∠DAB is a right angle)
$x° = 90° - 29°$
$= 61°$
$y° = \angle BAC = 29°$ (alt. ∠s, $DC // AB$)
∴ $x = 61$ and $y = 29$

(b) $x° = \dfrac{180° - 118°}{2}$ (base ∠s of isos. △)
$= 31°$
$\angle CBD + 31° = 90°$ (∠CBA is a right angle)
$\angle CBD = 90° - 31°$
$= 59°$
$y° + 59° = 118°$ (ext. ∠ of △)
$y° = 118° - 59°$
$= 59°$
∴ $x = 31$ and $y = 59$

(c) $x° + x° = (3x - 18)°$ (ext. ∠ of △)
$2x° = 3x° - 18°$
$x° = 18°$
$y° + x° + 90° - x° + x° = 180°$ (∠ sum of △ABD)
$y° = 180° - 90° - x°$
$= 180° - 90° - 18°$
$= 72°$
∴ $x = 18$ and $y = 72$

(d) $2x° + 2x° = 180° - (162 - 3x)°$ (ext. ∠ of △)
$4x° = 180° - 162° + 3x°$
$x° = 18°$
$\angle DAC = 90° - 2(18°)$
$= 54°$
$y° + 54° = (162 - 3x)°$ (ext. ∠ of △)
$y° = (162 - 3x)° - 54°$
$= (162 - 3(18))° - 54°$
$= 108° - 54°$
$= 54°$
∴ $x = 18$ and $y = 54$

7. (a) $y° = 120°$ (opp. ∠s of //gram)
$x° + 24° + y° = 180°$ (∠ sum of △ABC)
$x° = 180° - 24° - y°$
$= 180° - 24° - 120°$
$= 36°$
∴ $x = 36$ and $y = 120$

(b) $7x° + 5x° = 180°$ (int. ∠s, $DC // AB$)
$12x° = 180°$
$x° = 15°$
$2y° = 5x°$ (opp. ∠s of //gram)
$2y° = 5(15°)$
$2y° = 75°$
$y° = 37.5°$
∴ $x = 15$ and $y = 37.5$

(c) $\angle DAB = 180° - 68°$ (int. $\angle$s, AD // BC)
$= 112°$
$x° + 112° = 139°$ (ext. $\angle$ of $\triangle$)
$x° = 139° - 112°$
$= 27°$
$y° + x° = 68°$ (opp. $\angle$s of //gram)
$y° = 68° - x°$
$= 68° - 27°$
$= 41°$
$\therefore x = 27$ and $y = 41$

8. **(a)** $y° = \dfrac{180° - 58°}{2}$ (base $\angle$s of isos. $\triangle ABC$)
$= 61°$
$x° = 180° - 31° - 31°$ (base $\angle$s of isos. $\triangle ACD$)
$= 118°$
$\therefore x = 118$ and $y = 61$

(b) $x° + 33° + 56° = 180°$ ($\angle$ sum of $\triangle ABD$)
$x° = 180° - 33° - 56°$
$= 91°$
$y° = 33°$
$\therefore x = 91$ and $y = 33$

(c) $(x + 5)° + 90° + 28° = 180°$ ($\angle$ sum of $\triangle$)
$x° + 5° + 90° + 28° = 180°$
$x° = 180° - 5° - 90° - 28°$
$= 57°$
$(y - 6)° + 47° + 90° = 180°$ ($\angle$ sum of $\triangle$)
$y° - 6° + 47° + 90° = 180°$
$y° = 180° + 6° - 47° - 90°$
$= 49°$
$\therefore x = 57$ and $y = 49$

(d) $(x - 5)° + 63° + 90° = 180°$ ($\angle$ sum of $\triangle$)
$x° - 5° + 63° + 90° = 180°$
$x° = 180° + 5° - 63° - 90°$
$= 32°$
$(2y - 3)° + 37° + 90° = 180°$ ($\angle$ sum of $\triangle$)
$2y° - 3° + 37° + 90° = 180°$
$2y° = 180° + 3° - 37° - 90°$
$= 56°$
$y° = 28°$
$\therefore x = 32$ and $y = 28$

9. **(a)** Sum of interior angles of a polygon with 7 sides
$= (n - 2) \times 180°$
$= (7 - 2) \times 180°$
$= 900°$

(b) Sum of interior angles of a polygon with 17 sides
$= (n - 2) \times 180°$
$= (17 - 2) \times 180°$
$= 2700°$

(c) Sum of interior angles of a polygon with 22 sides
$= (n - 2) \times 180°$
$= (22 - 2) \times 180°$
$= 3600°$

(d) Sum of interior angles of a polygon with 30 sides
$= (n - 2) \times 180°$
$= (30 - 2) \times 180°$
$= 5040°$

10. **(a)** Sum of interior angles of a polygon with 4 sides
$= (n - 2) \times 180°$
$= (4 - 2) \times 180°$
$= 360°$
$a° + 125° + 65° + 92° = 360°$
$a° = 360° - 125° - 65° - 92°$
$= 78°$
$\therefore a = 78$

(b) Sum of interior angles of a polygon with 4 sides
$= (n - 2) \times 180°$
$= (4 - 2) \times 180°$
$= 360°$
$2b° + 105° + 75° + b° = 360°$
$2b° + b° = 360° - 105° - 75°$
$3b° = 180°$
$b° = 60°$
$\therefore b = 60$

(c) Sum of interior angles of a polygon with 5 sides
$= (n - 2) \times 180°$
$= (5 - 2) \times 180°$
$= 540°$
$c° + (2c - 15)° + 130° + 65° + 120° = 540°$
$c° + 2c° = 540° + 15° - 130° - 65° - 120°$
$3c° = 240°$
$c° = 80°$
$\therefore c = 80$

11. Let the number of sides of the regular polygon be n.

Size of each interior angle $= \dfrac{(n - 2) \times 180°}{n}$

(a) $\dfrac{(n - 2) \times 180°}{n} = 108°$
$(n - 2) \times 180 = 108n$
$180n - 108n = 2 \times 180$
$72n = 360$
$n = 5$

(b) $\dfrac{(n - 2) \times 180°}{n} = 156°$
$(n - 2) \times 180 = 156n$
$180n - 156n = 2 \times 180$
$24n = 360$
$n = 15$

12. Let the number of sides of the regular polygon be n.

Size of each exterior angle = $\frac{360°}{n}$.

(a) $\frac{360°}{n} = 5°$

$5n = 360$

$n = 72$

(b) $\frac{360°}{n} = 6°$

$6n = 360$

$n = 60$

(c) $\frac{360°}{n} = 8°$

$8n = 360$

$n = 45$

(d) $\frac{360°}{n} = 18°$

$18n = 360$

$n = 20$

13. **(a)** Size of each exterior angle = $\frac{360°}{6} = 60°$

(b) Size of each exterior angle = $\frac{360°}{8} = 45°$

(c) Size of each exterior angle = $\frac{360°}{24} = 15°$

(d) Size of each exterior angle = $\frac{360°}{72} = 5°$

14. **(a)** The sum of interior angles of the polygon is 1620°.

i.e. $(n - 2) \times 180° = 1620°$

$\therefore$ Number of sides of the polygon

$= \frac{1620}{180} + 2 = 11$

(b) The sum of interior angles of the polygon is 3600°.

i.e. $(n - 2) \times 180° = 3600°$

$\therefore$ Number of sides of the polygon

$= \frac{3600}{180} + 2 = 22$

(c) The sum of interior angles of the polygon is 4500°.

i.e. $(n - 2) \times 180° = 4500°$

$\therefore$ Number of sides of the polygon

$= \frac{4500}{180} + 2 = 27$

(d) The sum of interior angles of the polygon is 7020°.

i.e. $(n - 2) \times 180° = 7020°$

$\therefore$ Number of sides of the polygon

$= \frac{7020}{180} + 2 = 41$

15. **(i)** The sum of exterior angles of a triangle = 360°.

$(2x + 10)° + (3x - 5)° + (2x + 40)° = 360°$

$2x° + 3x° + 2x° = 360° - 10° + 5° - 40°$

$7x° = 315°$

$x° = 45°$

$\therefore x = 45$

(ii) The largest exterior angle gives the smallest interior angle.

The largest exterior angle

$= (3x - 5)°$ or $(2x + 40)°$

$= (3 \times 45 - 5)°$ or $(2 \times 45 + 40)°$

$= 130°$

The smallest interior angle = 180° – 130° = 50°

(iii) The smallest exterior angle gives the largest interior angle.

The smallest exterior angle $= (2x + 10)°$

$= (2 \times 45 + 10)°$

$= 100°$

The largest interior angle = 180° – 100° = 80°

16. **(i)** Sum of interior angles of a quadrilateral

$= (n - 2) \times 180°$

$= (4 - 2) \times 180°$

$= 360°$

$(2x + 15)° + (2x - 5)° + (3x + 75)° + (3x - 25)°$

$= 360°$

$2x° + 2x° + 3x° + 3x° = 360° - 15° + 5° - 75° + 25°$

$10x° = 300°$

$x° = 30°$

$\therefore x = 30$

(ii) Smallest interior angle

$= (2x - 5)°$

$= (2 \times 30 - 5)°$

$= 55°$

(iii) Largest interior angle gives the smallest exterior angle

Largest interior angle

$= (3x + 75)°$

$= (3 \times 30 + 75)°$

$= 165°$

Smallest exterior angle = 180° – 165° = 15°

17. **(i)** Sum of interior angles of a hexagon

$= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 720°$

$(2x + 17)° + (3x - 25)° + (2x + 49)° + (x + 40)° +$

$(4x - 17)° + (3x - 4)° = 720°$

$2x° + 3x° + 2x° + x° + 4x° + 3x°$

$= 720° - 17° + 25° - 49° - 40° + 17° + 4°$

$15x° = 660°$

$x° = 44°$

$\therefore x = 44$

(ii) Smallest interior angle of the hexagon

$= (x + 40)°$

$= (44 + 40)°$

$= 84°$

(iii) The largest interior angle gives the smallest exterior angle.

The largest interior angle

$= (4x - 17)°$

$= (4 \times 44 - 17)°$

$= 159°$

The smallest exterior angle $= 180° - 159° = 21°$

18. **(i)** The sum of exterior angles of a pentagon $= 360°$.

$2x° + (2x + 5)° + (3x + 10)° + (3x - 15)° + (x + 30)°$

$= 360°$

$2x° + 2x° + 3x° + 3x° + x°$

$= 360° - 5° - 10° + 15° - 30°$

$11x° = 330°$

$x° = 30°$

$\therefore x = 30$

(ii) The largest exterior angle gives the smallest interior angle.

The largest exterior angle

$= (3x + 10)°$

$= (3 \times 30 + 10)°$

$= 100°$

The smallest interior angle

$= 180° - 100° = 80°$

(iii) The smallest exterior angle gives the largest interior angle.

Smallest exterior angle

$= 2x°$

$= 2(30°) = 60°$

The largest interior angle $= 180° - 60° = 120°$

19. **(i)** The sum of interior angles of a quadrilateral $= 360°$

30 parts $= 360°$

1 part $= 12°$

9 parts $= 12 \times 9 = 108°$

The largest interior angle $= 108°$.

(ii) 6 parts $= 12° \times 6 = 72°$

The smallest interior angle $= 72°$.

The largest exterior angle

$= 180° - 72°$

$= 108°$

Intermediate

20. **(a)** $y° = 61° + 59°$ (ext. $\angle$ of $\triangle$)

$= 120°$

$\angle GDE = 61°$ (vert. opp. $\angle$s)

$x° = 69° + 61°$

$= 130°$ (ext. $\angle$ of $\triangle$)

$\therefore x = 130$ and $y = 120$

(b) $x° = 110° + 40°$ (ext. $\angle$ of $\triangle$)

$= 150°$

$\angle ADB + x° = 180°$ (adj. $\angle$s on a str. line)

$\angle ADB = 180° - x°$

$= 180° - 150°$

$= 30°$

$y° = 30° + 90°$ (ext. $\angle$ of $\triangle$)

$= 120°$

$\therefore x = 150$ and $y = 120$

21. **(a)** $\angle BEF = 180° - 84° = 96°$ (adj. $\angle$s on a str. line)

Sum of angles in a quadrilateral is $360°$.

$x° + 92° + 118° + 96° = 360°$

$x° = 360° - 92° - 118° - 96°$

$= 54°$

$y° + x° + 92° = 180°$ ($\angle$ sum of $\triangle$)

$y° = 180° - x° - 92°$

$= 180° - 54° - 92°$

$= 34°$

$\therefore x = 54$ and $y = 34$

(b) $\angle EBA = 53°$ (corr. $\angle$s, CD // AB)

$y° + 53° = 360°$ ($\angle$s at a point)

$y° = 360° - 53°$

$= 307°$

$\angle FED = 53°$ (base $\angle$s of isos. $\triangle$)

$x° = 53° + 53°$ (ext. $\angle$ of $\triangle$, corr. $\angle$s)

$= 106°$

$\therefore x = 106$ and $y = 307$

(c) $x° + 25° + 121° = 180°$ (corr. ∠s, adj. ∠s on a str. line)

$x° = 180° - 25° - 121°$

$= 34°$

$y° + x° + 78° = 180°$ (∠ sum of △)

$y° = 180° - x° - 78°$

$= 180° - 34° - 78°$

$= 68°$

$\therefore x = 34$ and $y = 68$

(d) $x° + 124° = 180°$ (corr. ∠s, adj. ∠s on a str. line)

$x° = 180° - 124°$

$= 56°$

$\angle ABD = 103°$ (corr. ∠s)

$y° + 103° = 180°$ (adj. ∠s on a str. line)

$y° = 180° - 103°$

$= 77°$

$z° + y° = 124°$ (ext. ∠ of △)

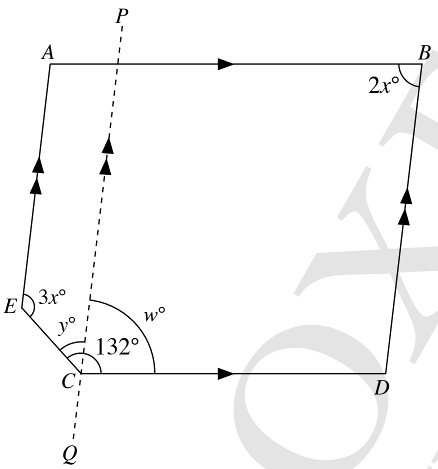
$z° = 124° - y°$

$= 124° - 77°$

$= 47°$

$\therefore x = 56, y = 77$ and $z = 47$

(e) Draw a line PQ through C that is parallel to AE and BD.



$w° = 2x°$ (opp. ∠s of //gram)

$y° = 132° - w°$

$= 132° - 2x°$

$3x° + y° = 180°$ (int. ∠s, EA // QP)

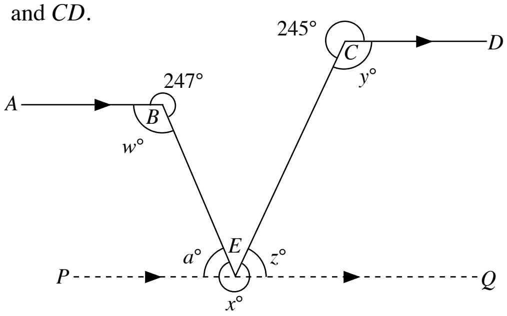
$3x° + 132 - 2x° = 180°$

$x° = 180° - 132°$

$= 48°$

$\therefore x = 48$

(f) Draw a line PQ through E that is parallel to AB and CD.



$w° + 247° = 360°$ (∠s at a point)

$w° = 360° - 247°$

$= 113°$

$a° + w° = 180°$ (int. ∠s, AB // PQ)

$a° = 180° - w°$

$= 180° - 113°$

$= 67°$

$y° + 245° = 360°$ (∠s at a point)

$y° = 360° - 245°$

$= 115°$

$z° + y° = 180°$ (int. ∠s, CD // PQ)

$z° = 180° - y°$

$= 180° - 115°$

$= 65°$

$x° = 180° + 67° + 65°$

$= 312°$

$\therefore x = 312$

22. Sum of angles in a triangle = 180°

$$(2x - 5)° + \left(3x - \frac{1}{2}\right)° + \left(30 - \frac{1}{2}x\right)° = 180°$$

$$2x° + 3x° - \frac{1}{2}x° = 180° + 5° + \frac{1}{2}° - 30°$$

$$4.5x° = 155.5°$$

$$x° = 34\frac{5}{9}°$$

$\therefore x = 34\frac{5}{9}$

23. (a) $64° + \angle ADC = 180°$ (int. ∠s, DC // AB)

$\angle ADC = 180° - 64°$

$= 116°$

$x° = \frac{1}{2} \times 116°$

$= 58°$

$y° = x°$ (alt. ∠s, DA // CB)

$= 58°$

$\therefore x = y = 58$

(b) $108° + \angle BAD = 180°$ (int. $\angle$s, BC // AD)

$\angle BAD = 180° - 108°$

$= 72°$

$x° = \frac{1}{2} \times 72°$

$= 36°$

$y° = x°$ (alt. $\angle$s, DC // AB)

$= 36°$

$\therefore x = y = 36$

(c) $(y - 5)° = 36°$ (alt. $\angle$s, DC // AB)

$y° = 36° + 5°$

$y° = 41°$

$x° = (y - 5)°$

$= (41 - 5)°$

$x° = 36°$

$\therefore x = 36$ and $y = 41$

(d) $(3x - 30)° = (2x + 15)°$ (opp. $\angle$s of //gram)

$3x° - 2x° = 15° + 30°$

$x° = 45°$

$(3x - 30)° + \angle BCD = 180°$ (int. $\angle$s, BA // CD)

$\angle BCD = 180° - (3x - 30)°$

$= 180° - (3 \times 45 - 30)$

$= 75°$

$y° = \frac{1}{2} \times 75°$

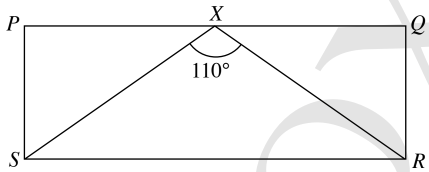
$= 37.5°$

$z° = y°$ (alt. $\angle$s, BA // CD)

$= 37.5°$

$\therefore x = 45, y = 37.5$ and $z = 37.5$

24.



(i) $\angle PXS = \frac{180° - 110°}{2}$

$= 35°$ (adj. $\angle$s on a str. line)

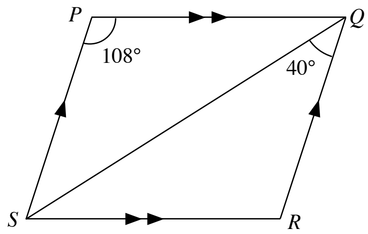
$\angle PSX = 180° - 90° - 35°$ ($\angle$sum of $\triangle$)

$= 55°$

(ii) $\angle XRS = \angle QXR$ (alt. $\angle$s PQ // SR)

$= 35°$

25.



(i) $\angle PSQ = 40°$ (alt. $\angle$s, PS // QR)

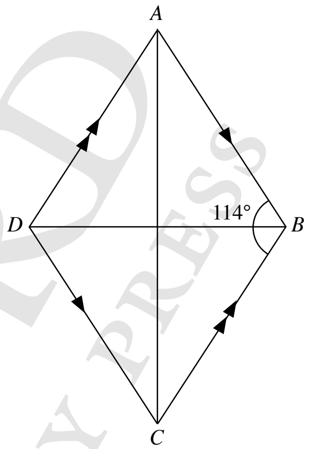
(ii) $\angle PSR = 180° - 108°$ (int. $\angle$s, PQ // SR)

$= 72°$

$\angle QSR = 72° - 40°$

$= 32°$

26.



(i) $\angle ABD = \frac{114°}{2}$

$= 57°$

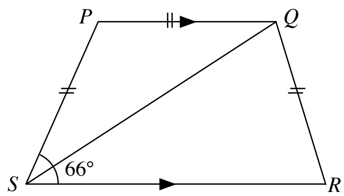
(ii) $\angle ACD = \angle ACB$

$= 180° - 90° - 57°$

(The diagonals of a rhombus bisect at 90°)

$= 33°$

27.



(i) $\angle QRS = 66°$ ($PS = QR$)

(ii) $\angle SPQ = 180° - 66°$ (int. $\angle$s, PQ // SR)

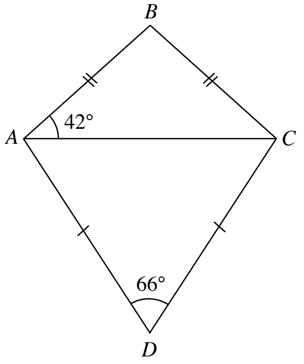
$= 114°$

$\angle PQS = \angle PSQ$

$= \frac{180° - 114°}{2}$ (base $\angle$s of isos. $\triangle PQS$)

$= 33°$

28.



(i) $\angle ACD = \dfrac{180° - 66°}{2}$ (base $\angle$s of isos. $\triangle ACD$)

$= 57°$

(ii) $\angle ABC = 180° - 42° - 42°$ (base $\angle$s of isos. $\triangle ABC$)

$= 96°$

29. (i) $115° + \angle PQR = 180°$ (int. $\angle$s, PQ // SR)

$\angle PQR = 180° - 115°$

$= 65°$

$\angle UQV = 65° - 35°$

$= 30°$

$\angle TVQ + 30° = 110°$ (ext. $\angle$ of $\triangle QUV$)

$\angle TVQ = 110° - 30°$

$= 80°$

(ii) $\angle SXY = 110°$ (corr. $\angle$s, TV // WY)

$110° + \angle SXW = 180°$ (adj. $\angle$s on a str. line)

$\angle SXW = 180° - 110°$

$= 70°$

30. (i) $66° + \angle ADC = 180°$ (int. $\angle$s, AB // DC)

$\angle ADC = 180° - 66°$

$= 114°$

$\angle CDQ + 145° + 114° = 360°$ ($\angle$s at a point)

$\angle CDQ = 360° - 145° - 114°$

$= 101°$

(ii) $114° + \angle BCD = 180°$ (int. $\angle$s, AD // BC)

$\angle BCD = 180° - 114°$

$= 66°$

(iii) $\angle DCP = \angle CDQ = 101°$ (alt. $\angle$s, PC // DQ)

$\angle PCB = 101° - 66°$

$= 35°$

31. (i) $\angle DAC = 33°$

$\angle QBC = 66°$ (corr. $\angle$s, AD // BC)

(ii) $66° + \angle ADC = 180°$ (int. $\angle$s, AB // DC)

$\angle ADC = 180° - 66°$

$= 114°$

$\angle ADB = 114° \div 2 = 57°$

$\angle DBC = \angle ADB$ (alt. $\angle$s, AD // BC)

$= 57°$

(iii) $\angle BCD = 66°$ (opp. $\angle$s in a //gram)

$72° + 66° +$ Reflex $\angle BCR = 360°$ ($\angle$s at a point)

Reflex $\angle BCR = 360° - 72° - 66°$

$= 222°$

32. (i) $\angle AEB = 180° - 53° - 90°$ ($\angle$ sum of $\triangle$)

$= 37°$

$\angle PEQ = \angle AEB$ (vert. opp. $\angle$s)

$= 37°$

(ii) $\angle QED = 90° - 37° = 53°$

$\angle EDR = 180° - 53°$ (int. $\angle$s, QE // RD)

$= 127°$

(iii) $\angle EDC = 360° - 126° - 127°$ ($\angle$s at a point)

$= 107°$

$\angle BCD + 107° = 180°$ (int. $\angle$s, ED // AC)

$\angle BCD = 180° - 107°$

$= 73°$

33. (i) $\angle ABQ = 45° - 21°$

$= 24°$

$\angle BAQ = \dfrac{180° - 24°}{2}$ (base $\angle$s of isos. $\triangle ABQ$)

$= 78°$

(ii) Since $BQ = BA$, $BQ = BC$,

$\angle ABC = 45° + 21°$

$= 66°$

$\angle BCQ = \dfrac{180° - 66°}{2}$ (base $\angle$s of isos. $\triangle QBC$)

$= 57°$

$\angle DCQ = 90° - 57°$

$= 33°$

(iii) $\angle DPC = 180° - 45° - 33°$ ($\angle$ sum of $\triangle$)

$= 102°$

$\angle QPB = 102°$ (vert. opp. $\angle$s)

34. (i) $\angle QAD = 60°$ ($\angle$s of equilateral $\triangle$)

$\angle BAD + 90° + 135° + 60° = 360°$ ($\angle$s at a point)

$\angle BAD = 360° - 90° - 135° - 60°$

$= 75°$

(ii) Sum of interior angles of a quadrilateral = 360°.

$\angle CDA + 106° + 100° + 75° = 360°$ ($\angle$s at a point)

$\angle CDA = 360° - 106° - 100° - 75°$

$= 79°$

(iii) $\angle PDQ + 60° + 79° = 180°$ (adj. $\angle$s on a str. line)

$\angle PDQ = 180° - 60° - 79°$

$= 41°$

35. (i) $\angle ABC = 180° - 68° - 68°$ (base $\angle$s of isos. $\triangle ABC$)
$= 44°$

(ii) $\angle ACD = 68°$
$\angle ADC = 180° - 68° - 68°$ ($\angle$ sum of $\triangle$)
$= 44°$
$60° + 90° + \angle ADP = 180°$ ($\angle$ sum of $\triangle$)
$\angle ADP = 180° - 90° - 60°$
$= 30°$
$\angle PDQ + \angle ADP = \angle ADC$
$\angle PDQ = \angle ADC - \angle ADP$
$= 44° - 30°$
$= 14°$

(iii) $\angle DQR = \angle PDQ + \angle DPR$
$= 14° + 90°$
$= 104°$

36. (i) $\angle TBD = 180° - 81°$ (int. $\angle$s, BD // TE)
$= 99°$
$\angle DBC = 61°$ (alt. $\angle$s, ED // BC)
$\angle ABT = 180° - 99° - 61°$ (adj. $\angle$s on a str. line)
$= 20°$

(ii) $\angle TED = 180° - 61°$ (int. $\angle$s, DB // ET)
$= 119°$

(iii) $\angle BCD = 180° - 61° - 61°$
$= 58°$

37. (i) $\angle BAD = 180° - 88°$ (int. $\angle$s, BC // AD)
$= 92°$
$\angle DAE = 180° - 92°$ (adj. $\angle$s on a str. line)
$= 88°$
$\angle AED = \dfrac{180° - 88°}{2}$ (base $\angle$s of isos. $\triangle ADE$)
$= 46°$

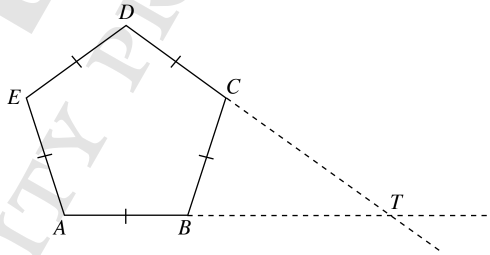
(ii) $\angle FAB = 180° - 162°$ (adj. $\angle$s on a str. line)
$= 18°$
$\angle FAD = \angle FAB + \angle BAD$
$= 18° + 92°$
$= 110°$

(iii) $\angle ADC = 88°$ (opp. $\angle$s in a //gram)
Sum of angles in a quadrilateral = 360°
$\angle FCD + 48° + 110° + 88° = 360°$
$\angle FCD = 360° - 48° - 110° - 88°$
$= 114°$
$\angle BCF = \angle FCD - \angle BCD$
$= 114° - 92°$
$= 22°$

38. Let the number of sides of the polygon be n.
Sum of interior angles = $(n - 2) \times 180°$
Sum of exterior angles = 360°
$(n - 2) \times 180° = 2 \times 360°$
$180n - 360 = 720$
$180n = 720 + 360$
$= 1080$
$n = 6$
$\therefore$ The number of sides of the polygon is 6.

39. Let the number of sides of the regular polygon be n.
Size of each interior angle = $\dfrac{(n-2)\times 180°}{n}$
Size of each exterior angle = $\dfrac{360°}{n}$
$\dfrac{(n-2)\times 180°}{n} = 35 \times \dfrac{360°}{n}$
$(n - 2) \times 180° = 35 \times 360°$
$180n - 360 = 12\,600$
$180n = 12\,600 + 360$
$= 12\,960$
$n = 72$

40.



Size of each exterior angle of the pentagon = $\dfrac{360°}{5}$
$= 72°$
$\angle CBT = \angle BCT = 72°$
$\angle BTC = 180° - 72° - 72°$ ($\angle$ sum of $\triangle$)
$= 36°$

41. (i) Size of each interior angle = $\dfrac{(12-2)\times 180°}{12}$
$= 150°$
$\therefore \angle ABC = 150°$

(ii) $\angle BCA = \dfrac{180° - 150°}{2} = 15°$
$\angle ACD + \angle BCA = \angle BCD = 150°$
$\angle ACD = 150° - \angle BCA$
$= 150° - 15°$
$= 135°$

42. **(a)** Let the number of sides of the polygon be n.

Sum of interior angles = $(n-2) \times 180°$

$$(n-2)\times 180° = 124° + (n-1)\times 142°$$
$$180n - 360 = 124 + 142n - 142$$
$$180n - 142n = 124 - 142 + 360$$
$$38n = 342$$
$$n = 9$$

$\therefore$ The number of sides of the polygon is 9.

(b) Size of each angle in a pentagon $ABCDE$

$= \dfrac{(5-2)\times 180°}{5}$

$= 108°$

Size of each angle in a hexagon $CDZYXW$

$= \dfrac{(6-2)\times 180°}{6}$

$= 120°$

(i) $\angle WCD$ = size of each angle in a hexagon
$= 120°$

(ii) $\angle BCD$ = size of each angle in a pentagon
$= 108°$

(iii) Since $CB = CW$, $\triangle BCW$ is an isosceles triangle.

$\angle BCW = 360° - 108° - 120°$ ($\angle$s at a point)
$= 132°$

$\angle CBW = \dfrac{180° - 132°}{2}$ (base $\angle$s of isos. $\triangle BCW$)
$= 24°$

43. **(i)** $\angle CBA = 180° - 18°$ (adj. $\angle$s on a str. line)
$= 162°$

(ii) Size of each interior angle = $\dfrac{(n-2)\times 180°}{n}$

$$162 = \frac{(n-2)\times 180°}{n}$$
$$162n = (n-2)\times 180°$$
$$162n = 180n - 360$$
$$180n - 162n = 360$$
$$18n = 360$$
$$n = 20$$

$\therefore$ The value of n is 20.

(iii) $\angle BCY = \dfrac{180° - 162°}{2}$
$= 9°$

$\angle CBY = (360° - 162° - 162°) \div 2$
$= 18°$

$\angle BYC = 180° - 18° - 9°$ ($\angle$ sum of $\triangle$)
$= 153°$

44. **(i)** Size of each interior angle = $\dfrac{(n-2)\times 180°}{n}$.

$$174° = \frac{(n-2)\times 180°}{n}$$
$$174n = (n-2)\times 180$$
$$174n = 180n - 360$$
$$180n - 174n = 360$$
$$6n = 360$$
$$n = 60$$

$\therefore$ The value of n is 60.

(ii) $\angle PBC = \dfrac{(5-2)\times 180°}{5}$ (base $\angle$s of isos. $\triangle PBQ$)
$= 108°$

$\angle ABP = 360° - 108° - 174°$ ($\angle$s at a point)
$= 78°$

(iii) $\angle PBQ = \dfrac{180° - 108°}{2}$
$= 36°$

$\angle QBC = 108° - 36°$
$= 72°$

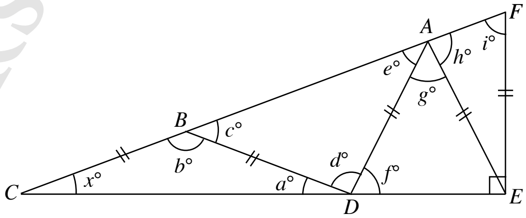
$\angle BQC = 180° - 72° - 72°$ (base $\angle$s of isos. $\triangle BQC$)
$= 36°$

(iv) $\angle DCR = 360° - 108° - 174°$ ($\angle$s at a point)
$= 78°$

$\angle CDR = \dfrac{180° - 78°}{2}$ (base $\angle$s of isos. $\triangle CDR$)
$= 51°$

Advanced

45.



(i) For $\triangle BCD$,

$a° = x°$ (base $\angle$s of isos. $\triangle BCD$)

$b° = 180° - x° - x°$ ($\angle$ sum of $\triangle BCD$)
$= 180° - 2x°$

For $\triangle ADB$,

$c° = 180° - b°$ (adj. $\angle$s on a str. line)
$= 180° - (180° - 2x°)$
$= 180° - 180° + 2x°$
$= 2x°$

$e° = c°$ (base $\angle$s of isos. $\triangle ADB$)

$d° = 180° - c° - e°$ ($\angle$ sum of $\triangle ABD$)
$= 180° - 2x° - 2x°$
$= 180° - 4x°$

For $\triangle ADE$,

$f° = 180° - x° - (180° - 4x°)$ (adj. ∠s on a str. line)

$= 180° - x° - 180° + 4x°$

$= 3x°$

$g° = 180° - 3x° - 3x°$ (∠ sum of $\triangle ADE$)

$= 180° - 6x°$

For $\triangle AEF$,

$h° = 180° - 2x° - (180° - 6x°)$ (adj. ∠s on a str. line)

$= 180° - 2x° - 180° + 6x°$

$= 4x°$

$i° = h° = 4x°$ (base ∠s of isos. $\triangle AEF$)

$i° + x° + 90° = 180°$ (∠ sum of $\triangle CEF$)

$4x° + x° + 90° = 180°$

$5x° = 180° - 90° = 90°$

$x° = 18°$

$\therefore x = 18$

(ii) Let n be the number of isosceles triangles that can be formed.

From **(i)**,

$(n + 1)x° + 90° = 180°$

When $x° = 5°$,

$(n + 1)5° + 90° = 180°$

$5(n + 1) = 90$

$n + 1 = 18$

$n = 17$

∴ There are 17 isosceles triangles that can be formed when $x = 5$.

46. **(i)** Let polygon A have a sides and polygon B have b sides.

$$\frac{360}{a} + \frac{360}{b} = 80$$

$360(a + b) = 80ab$

$9(a + b) = 2ab$

$9a + 9b = 2ab$

$9a = 2ab - 9b$

$$b = \frac{9a}{2a - 9}$$

∴ A possible solution is polygon A has 5 sides and polygon B has 45 sides.

(ii) Sum of exterior angles of any polygon = 360°.

When the exterior angle of their shared side decreases, the corresponding exterior angle of each polygon decreases.

$$\text{Number of sides} = \frac{360°}{\text{size of each exterior angle}}$$

∴ Number of sides increases as size of each interior angle in both polygons decreases.

New Trend

47. Let the first angle be $x°$.

$$x + (x - 10) + 4(x - 10) + \left(x + \frac{120}{100}x\right) = 360$$

$x + x - 10 + 4x - 40 + 2.2x = 360$

$8.2x = 410$

$x = 50$

The angles of the quadrilateral are 50°, 40°, 160° and 110°.

48. Sum of interior angles of a polygon with 6 sides

$= (n - 2) \times 180°$

$= (6 - 2) \times 180°$

$= 720°$

$d° + 125° + d° + 3d° + 70° + 110° = 720°$

$d° + d° + 3d° = 720° - 125° - 70° - 110°$

$5d° = 415°$

$d° = 83°$

$\therefore d = 83$

49. Size of each interior angle of the decagon

$$= \frac{(10 - 2) \times 180°}{10}$$

$= 144°$

Size of each interior angle of the hexagon

$$= \frac{(6 - 2) \times 180°}{6}$$

$= 120°$

$x° = 360° - 144° - 120°$ (∠s at a point)

$= 96°$

$\therefore x = 96$

50. $x° + 63° = 180°$ (int. ∠s, AD // BC)

$x° = 180° - 63°$

$= 117°$

$y° = 61°$ (alt. ∠s, DC // AB)

$AD = BC = 7.5$ cm

$z = 7.5$

$\therefore x = 117, y = 61$ and $z = 7.5$

51. **(a)** Size of each interior angle of a regular 24-sided polygon $= \dfrac{(24 - 2) \times 180°}{24}$

$= 165°$

(b) Let the number of sides of the polygon be n.

Sum of interior angles $= (n - 2) \times 180°$

$(n - 2) \times 180° = 172° + 2(158°) + (n - 3)p°$

$488 + (n - 3)p = 180n - 360$

$(n - 3)p = 180n - 848$

$$p = \frac{180n - 848}{n - 3}$$

52. **(a)** Size of each interior angle = $\frac{(n-2)\times 180°}{n}$

$$150° = \frac{(n-2)\times 180°}{n}$$

$$150n = (n-2) \times 180$$

$$150n = 180n - 360$$

$$180n - 150n = 360$$

$$30n = 360$$

$$n = 12$$

(b) Size of each exterior angle = $\frac{360°}{n}$

$$= \frac{360°}{9}$$

$$= 40°$$

53. Let the number of sides of the regular polygon be n.

Size of each interior angle = $\frac{(n-2)\times 180°}{n}$

$$\frac{(n-2)\times 180°}{n} = 165.6°$$

$$(n-2) \times 180 = 165.6n$$

$$180n - 165.6n = 2 \times 180$$

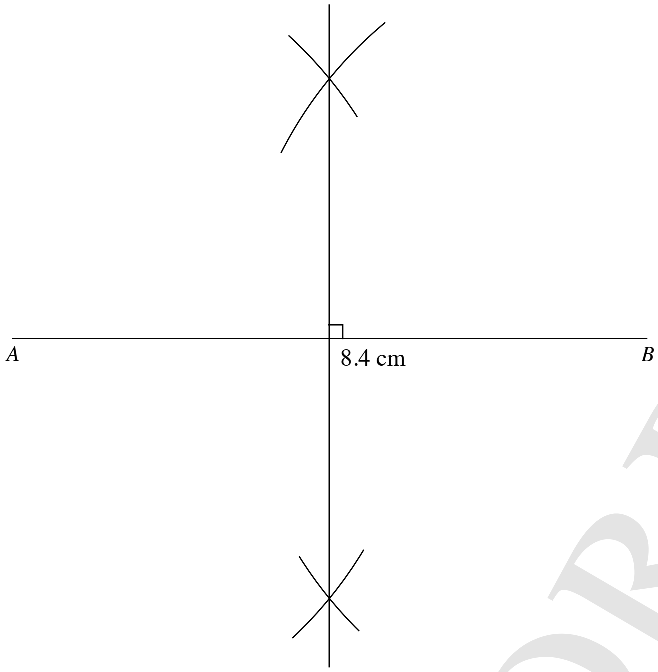
$$14.4n = 360$$

$$n = 25$$

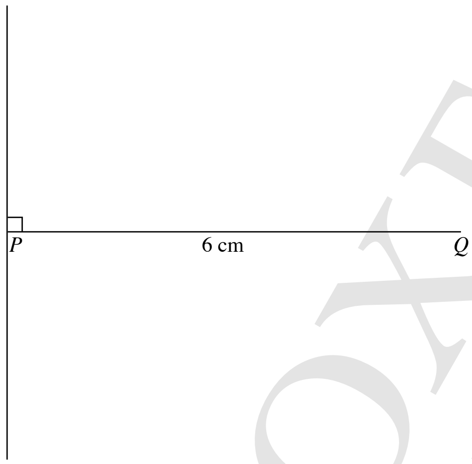
Chapter 12 Geometrical Constructions

Basic

1.



2.



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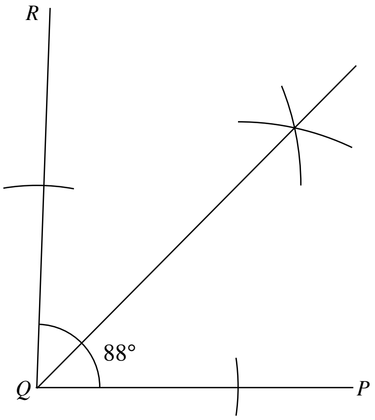

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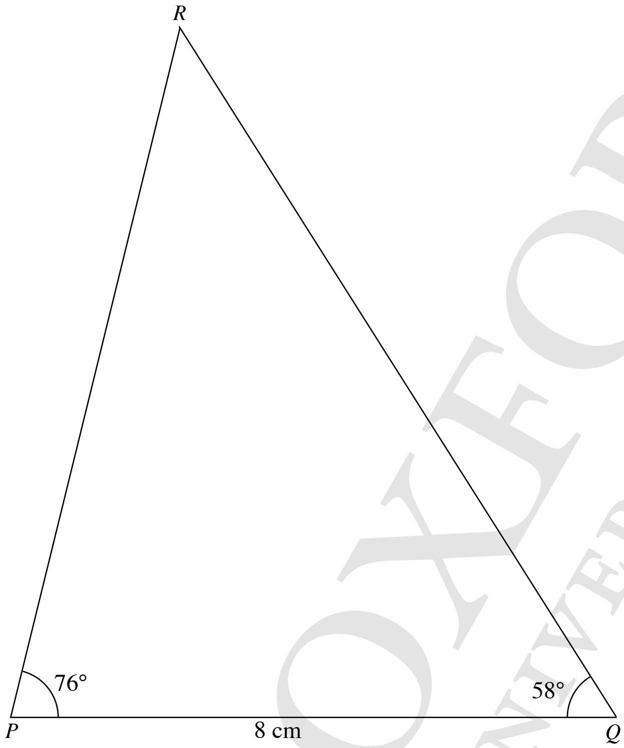
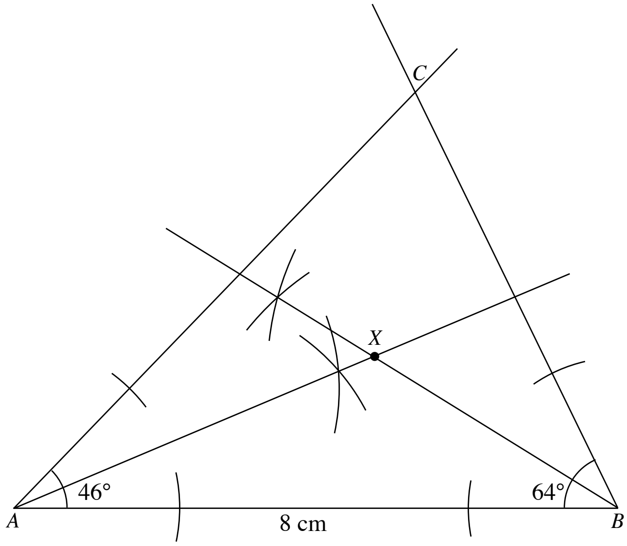


(i) $\angle PRQ = 46°$

(ii) $PR = 9.4$ cm

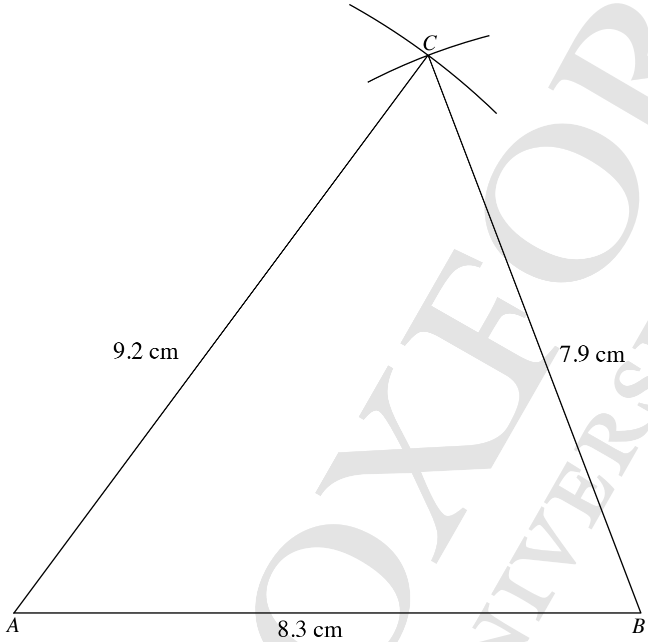
(iii) $QR = 10.8$ cm

5.



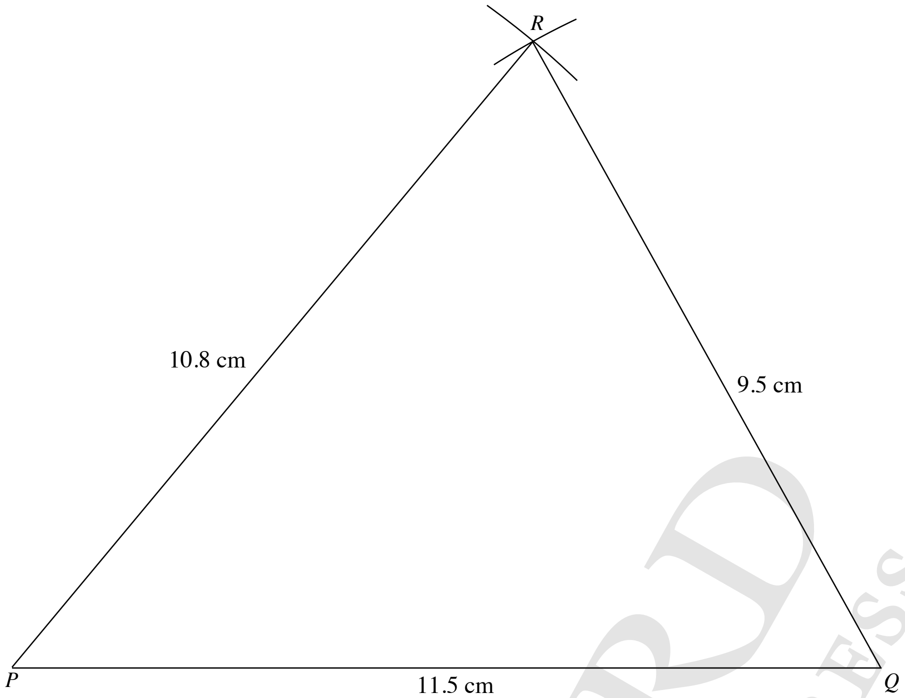
Length of AX = 5.2 cm

6.



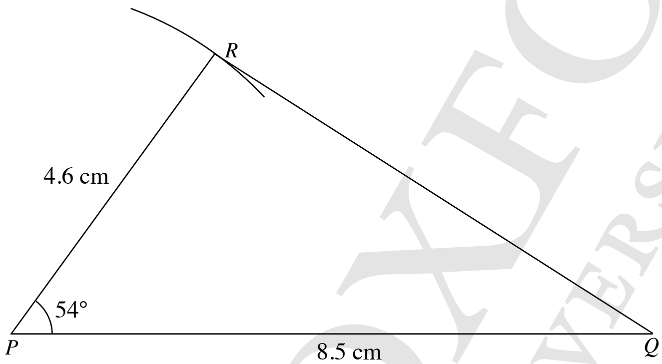
$\angle ABC = 69°$

7.



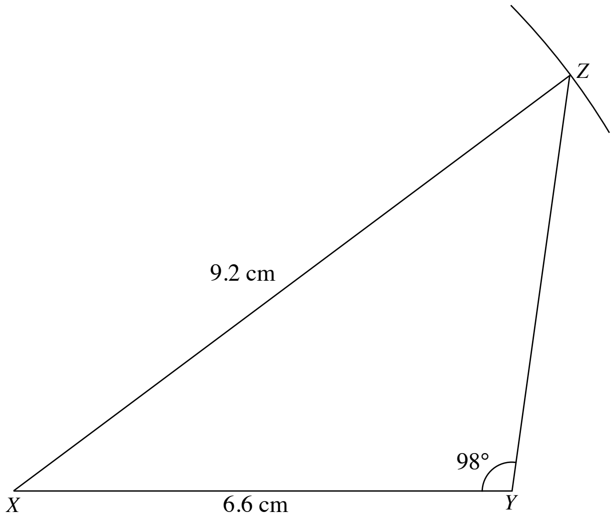
$\angle QPR = 50°$
$\angle PQR = 61°$

8.



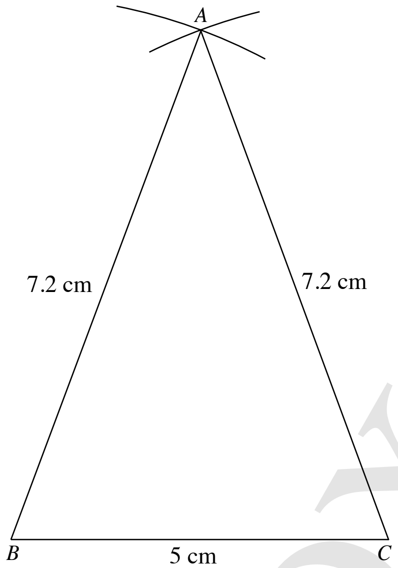
Length of QR = 6.9 cm.

9.



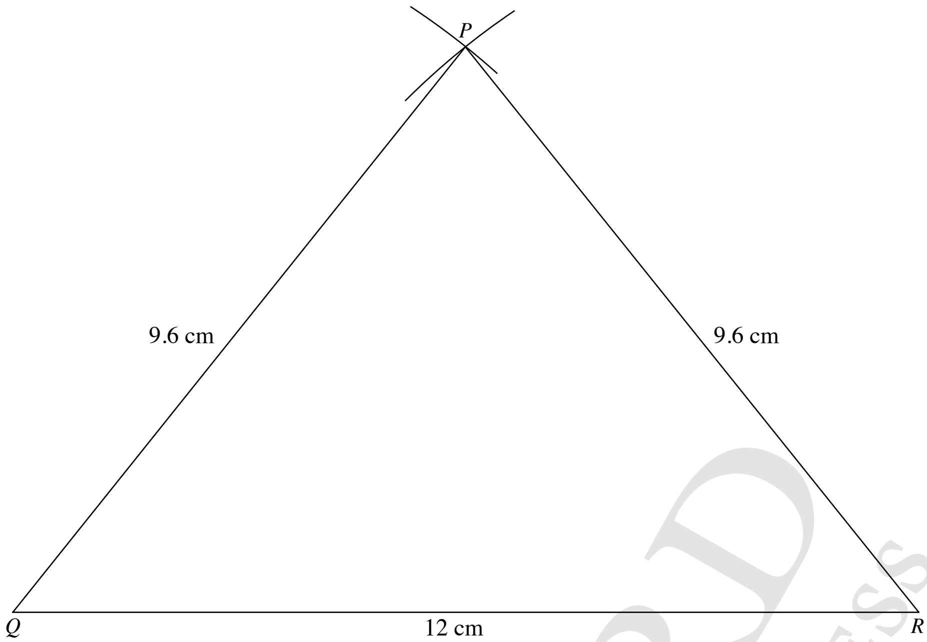
$\angle YXZ = 37°$

10.



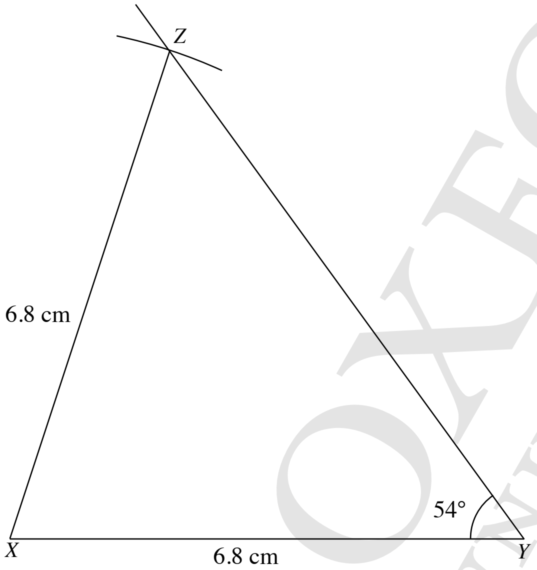
$\angle ABC = 70°$

11.



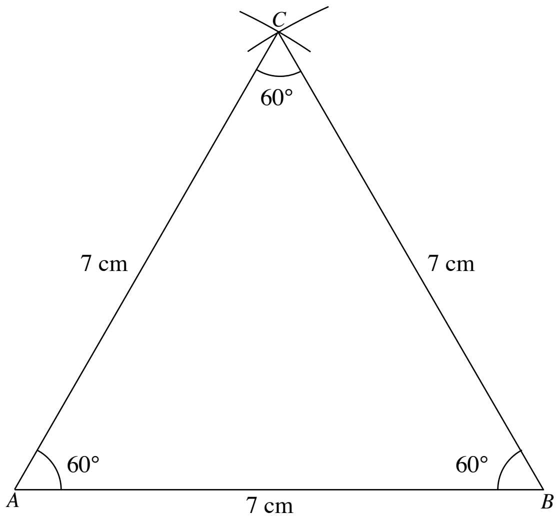
$\angle PQR = 51°$

12.

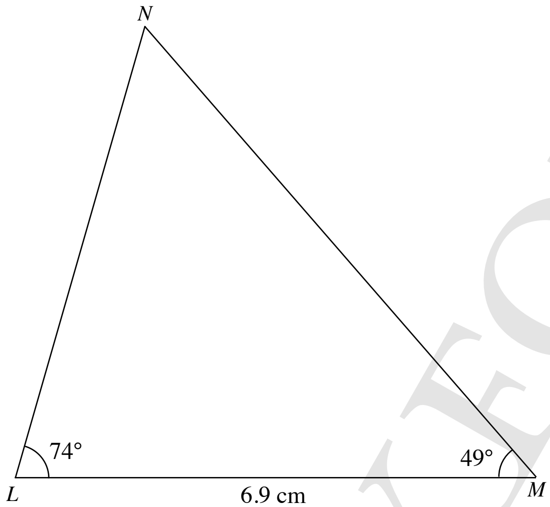


Length of YZ = 8.0 cm

13.

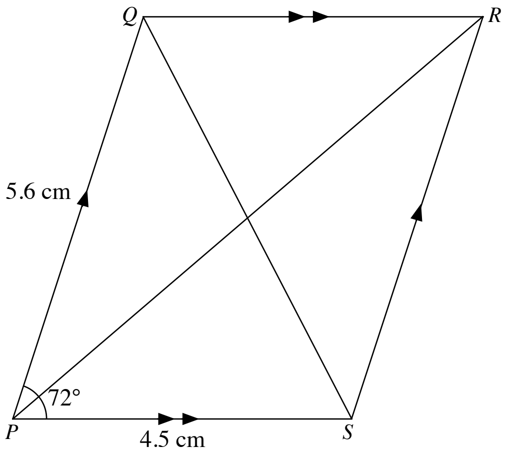


14.



Length of *LN* = 6.2 cm

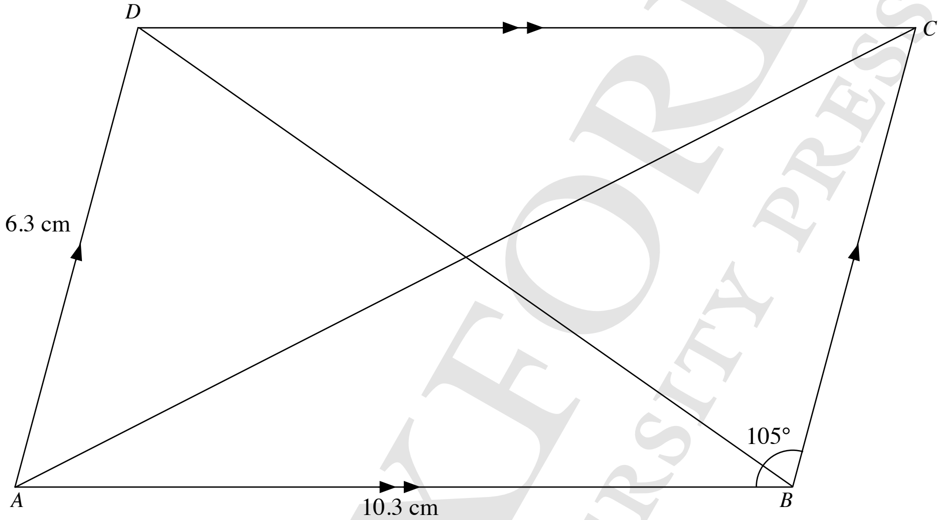
15.



Length of diagonal PR = 8.2 cm

Length of diagonal QS = 6 cm

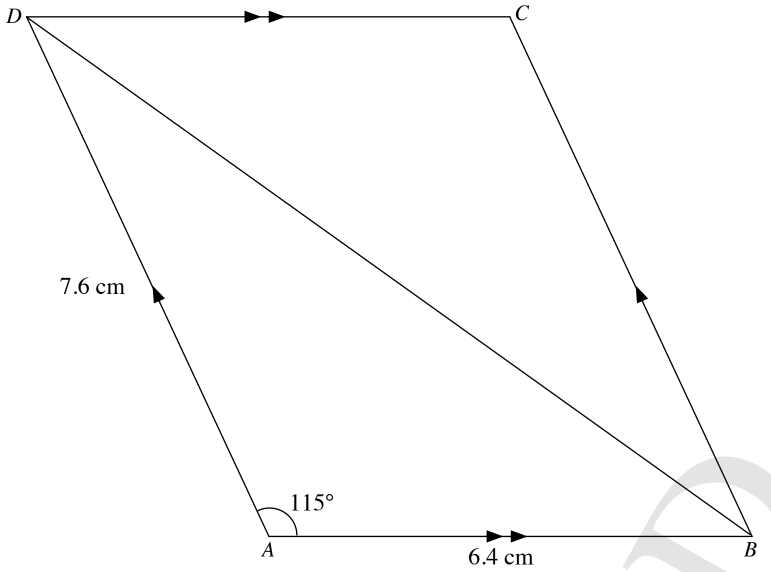
16.



Length of diagonal AC = 13.4 cm

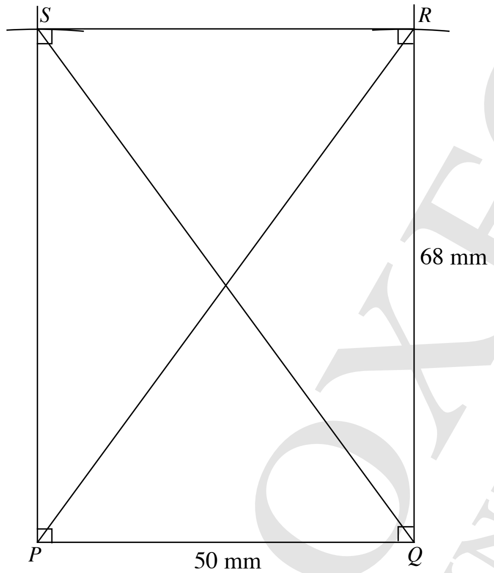
Length of diagonal BD = 10.6 cm

17.



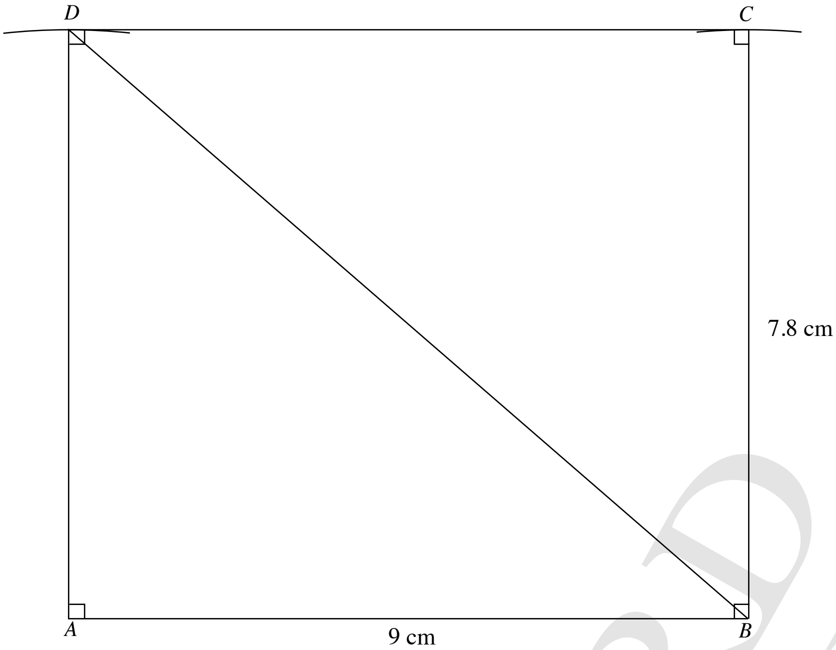
Length of BD = 11.8 cm
$\angle BDA = 29°$

18.



Length of diagonal PR = 84 mm
Length of diagonal QS = 84 mm

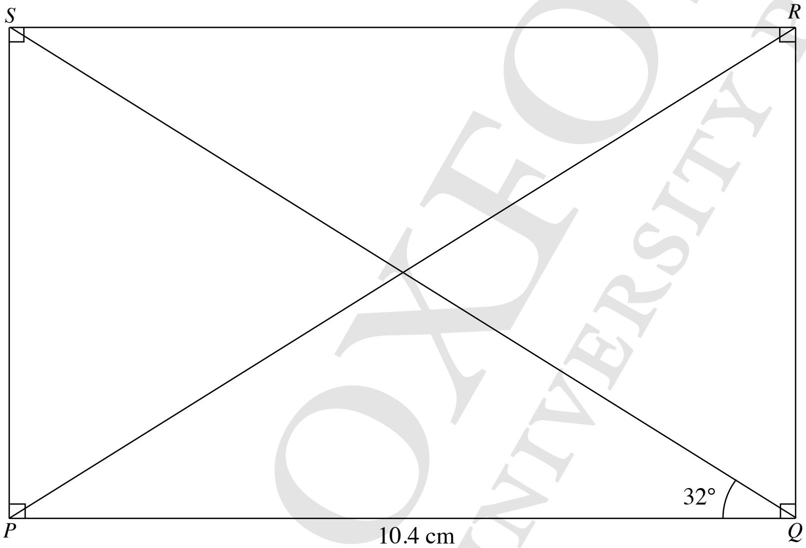
19.



Length of *BD* = 11.9 cm

∠*ABD* = 41°

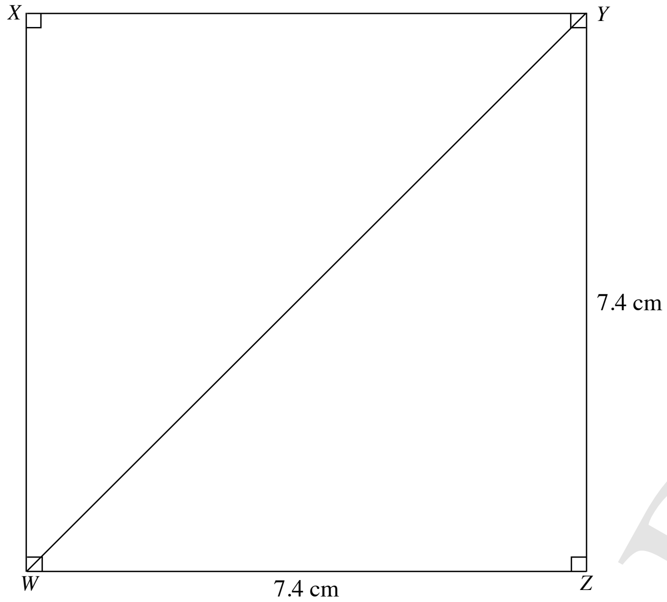
20.



Length of *PS* = 6.5 cm

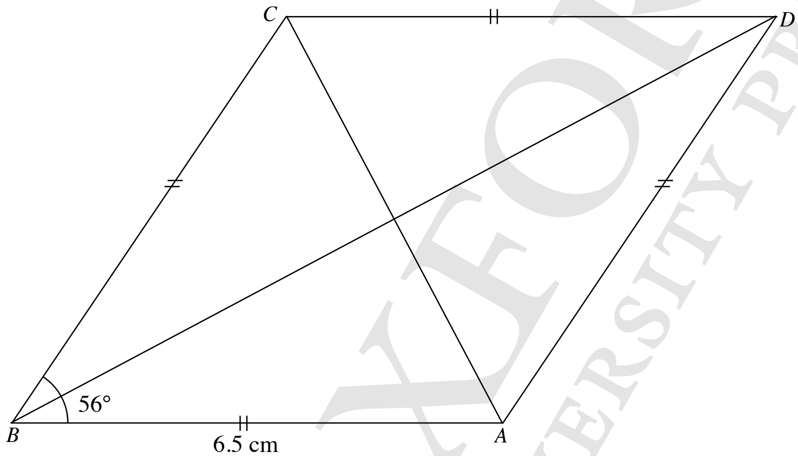
Length of *PR* = 12.3 cm

21.



Length of diagonal *WY* = 10.5 cm

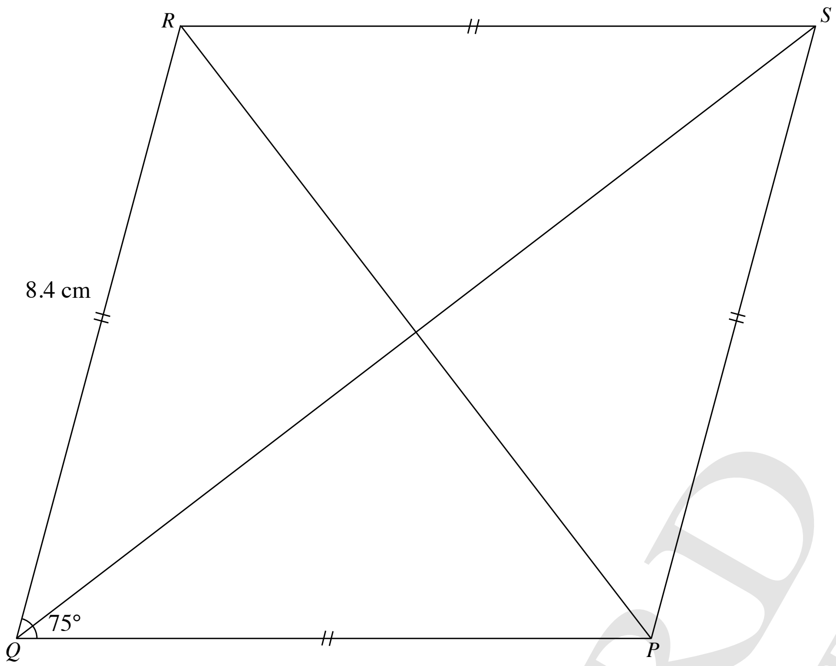
22.



Length of diagonal *BD* = 11.5 cm

Length of diagonal *AC* = 6.1 cm

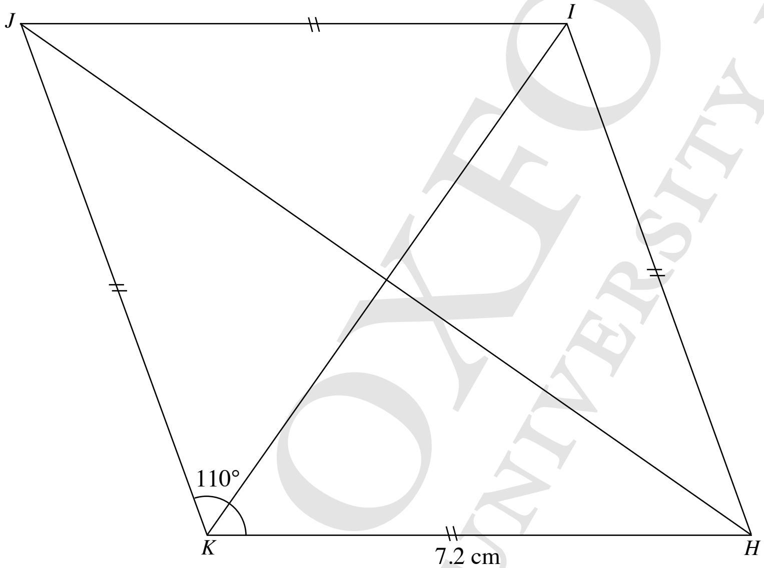
23.



Length of diagonal PR = 10.2 cm

Length of diagonal QS = 13.3 cm

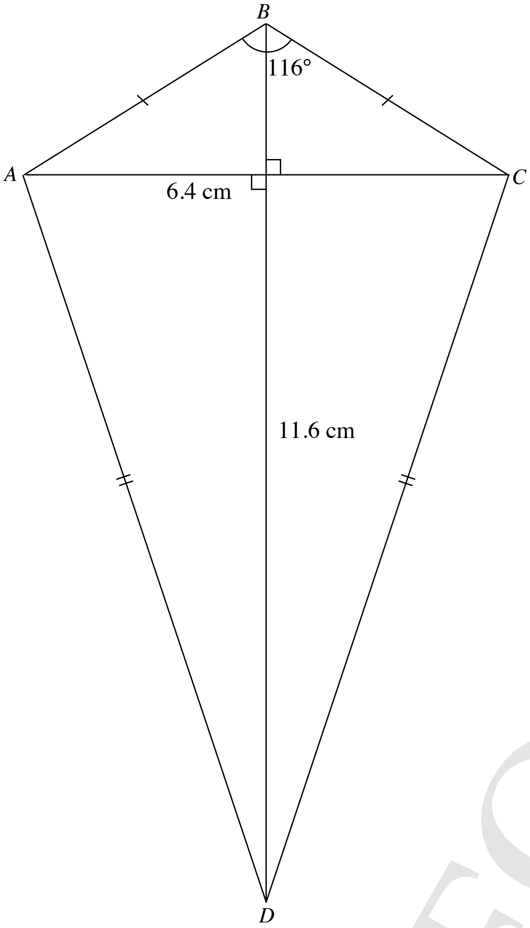
24.



Length of diagonal HJ = 11.8 cm

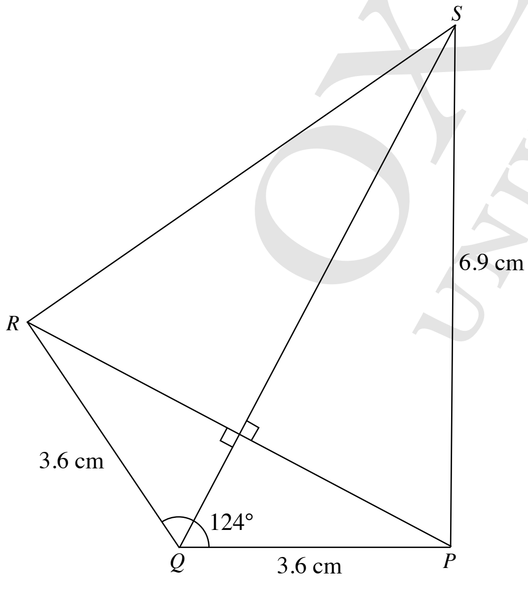
Length of diagonal KI = 8.4 cm

25.



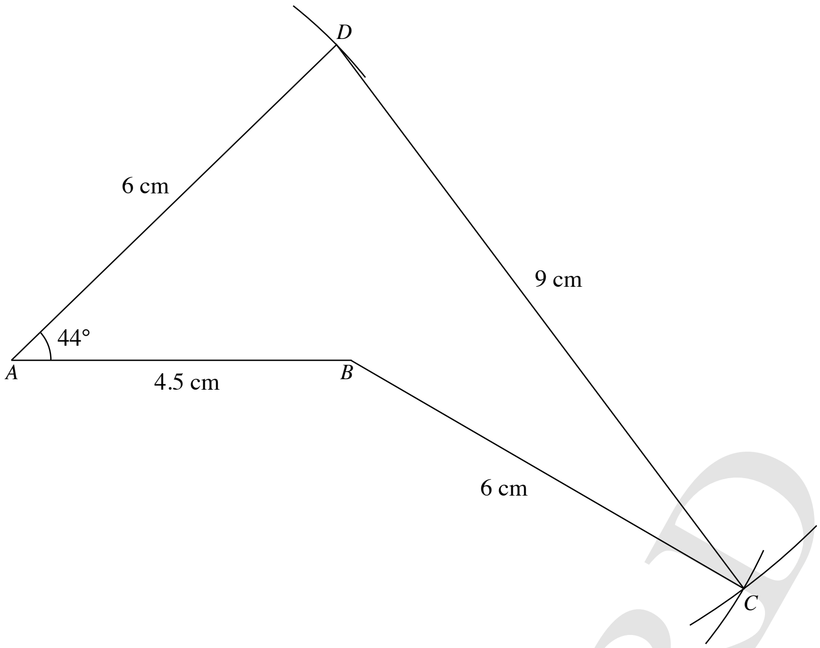
Length of AB = 3.8 cm
Length of AD = 10.1 cm

26.



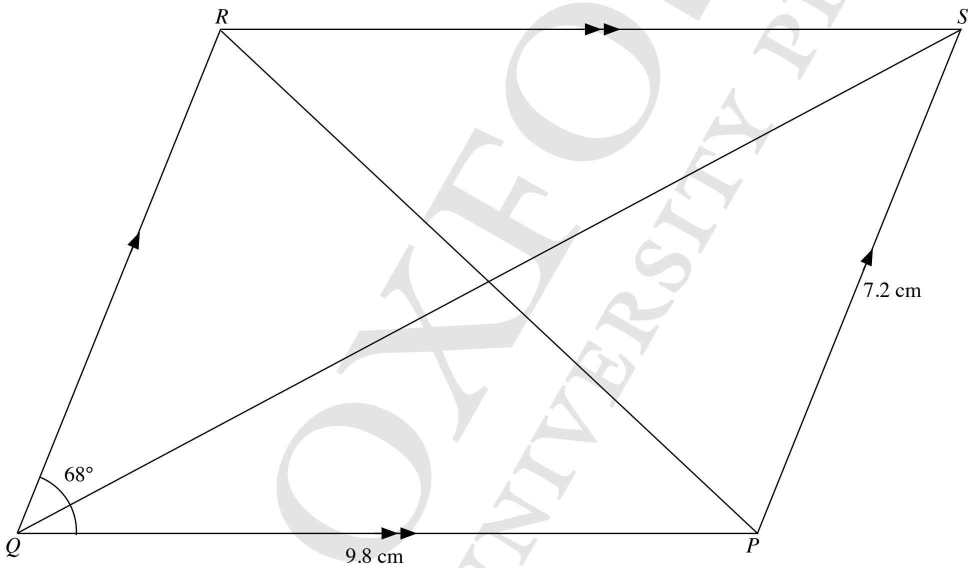
Length of diagonal PR = 6.4 cm
Length of diagonal QS = 7.8 cm

27.



$\angle ADC = 82°$

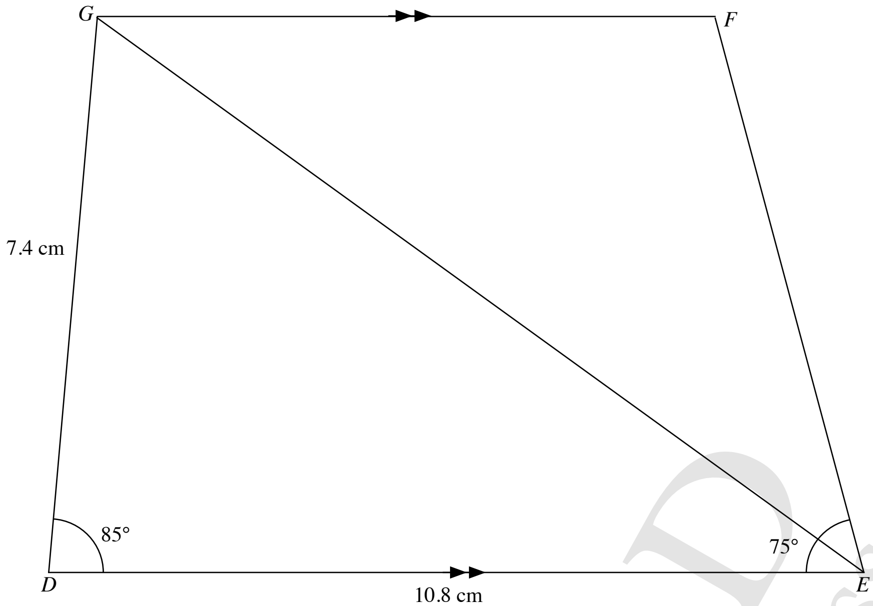
28.



Length of diagonal PR = 9.7 cm

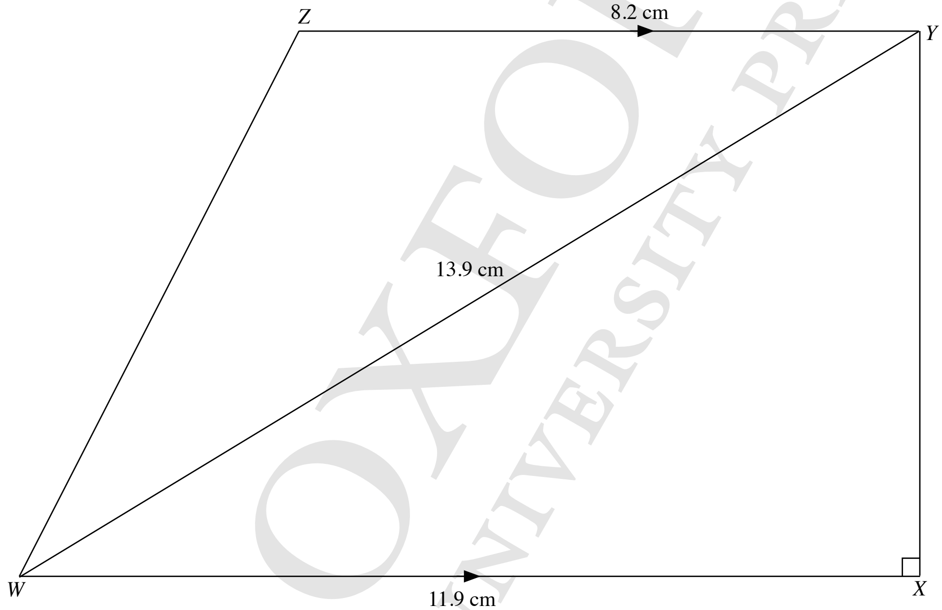
Length of diagonal QS = 14.2 cm

29.



Length of GE = 12.5 cm
Length of GF = 8.2 cm

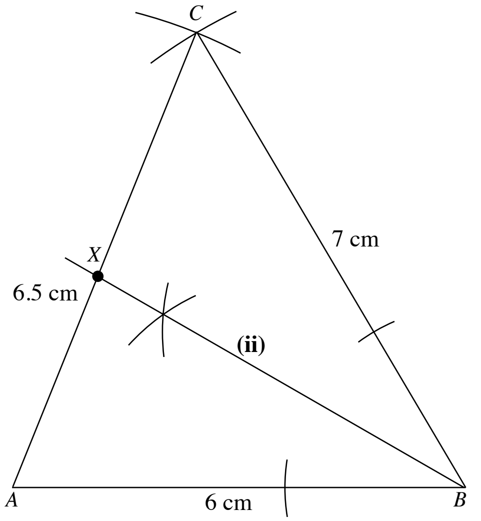
30.



Length of WZ = 8.1 cm
$\angle XWZ = 63°$

Intermediate

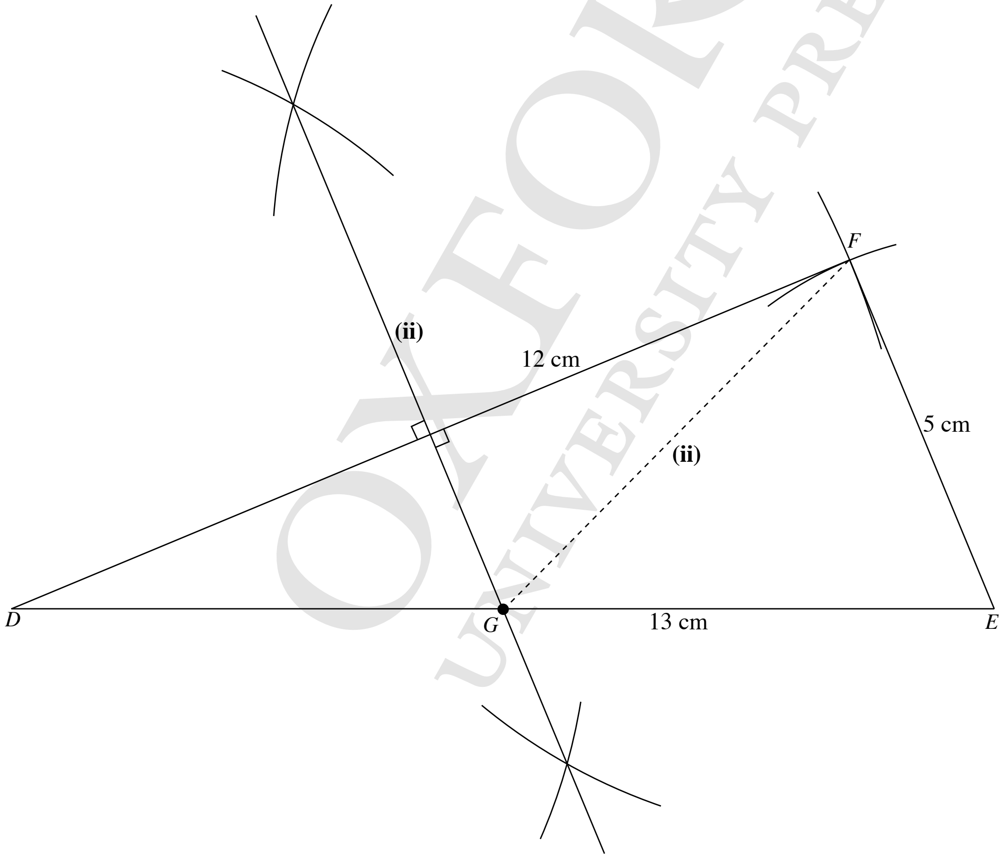
31.



(i) $\angle ABC = 58°$

(ii) Length of $BX = 5.6$ cm

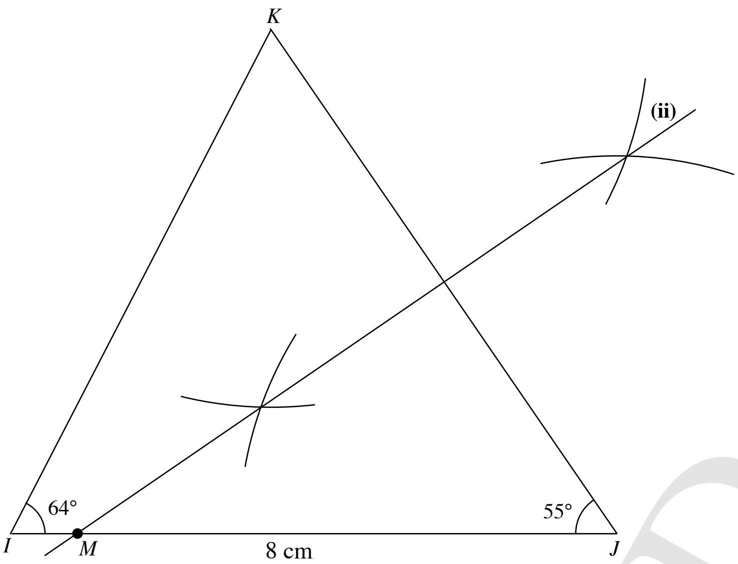
32.



(i) $\angle DEF = 67°$

(ii) Length of $GF = 6.5$ cm

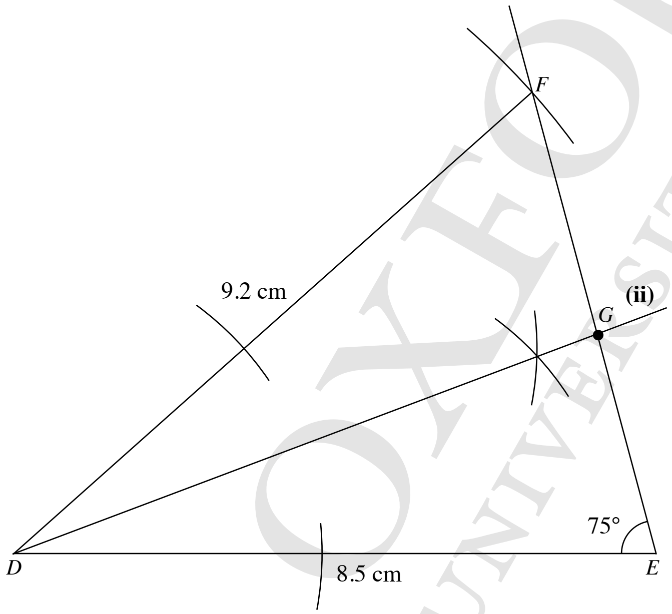
33.



(i) Length of IK = 7.5 cm
Length of JK = 8.1 cm

(ii) Length of IM = 0.9 cm

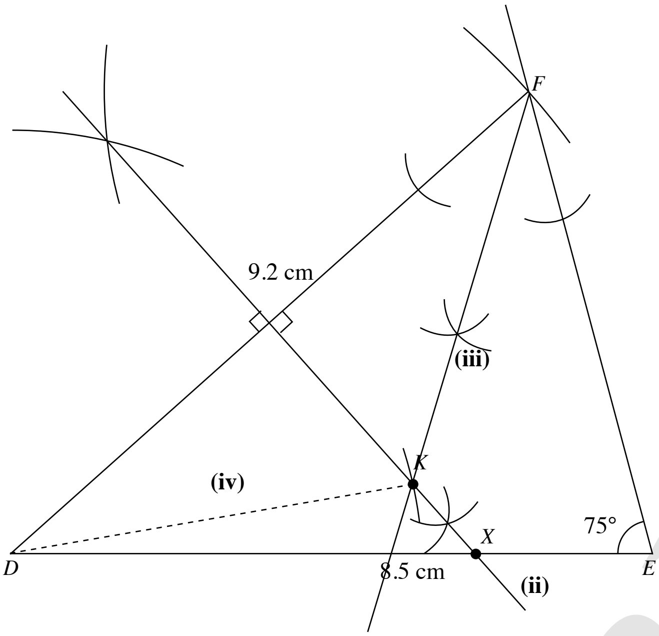
34.



(i) The angle that is facing the longest side is $\angle DEF$.
The size of $\angle DEF = 75°$.

(ii) Length of DG = 8.2 cm

35.

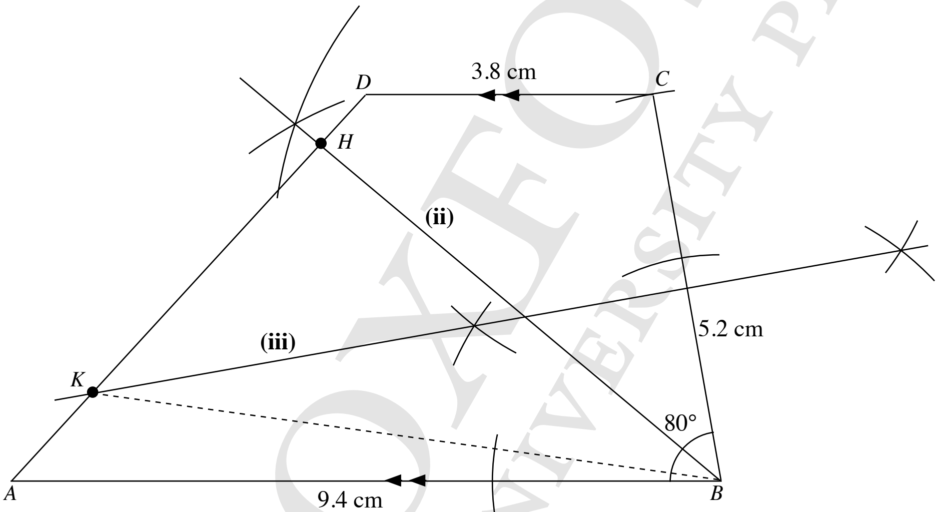


(i) Length of $EF = 6.3$ cm

(ii) Length of $DX = 6.2$ cm

(iv) Length of $DK = 5.4$ cm

36.



(i) Length of $AD = 7$ cm

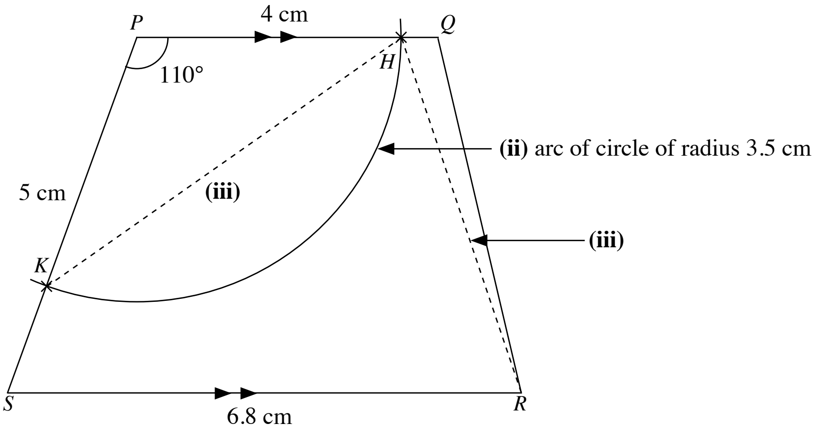
$\angle BAD = 47°$

(ii) Length of $HB = 7$ cm

(iii) Length of $KB = 8.4$ cm

(iv) Length of $HK = 4.5$ cm

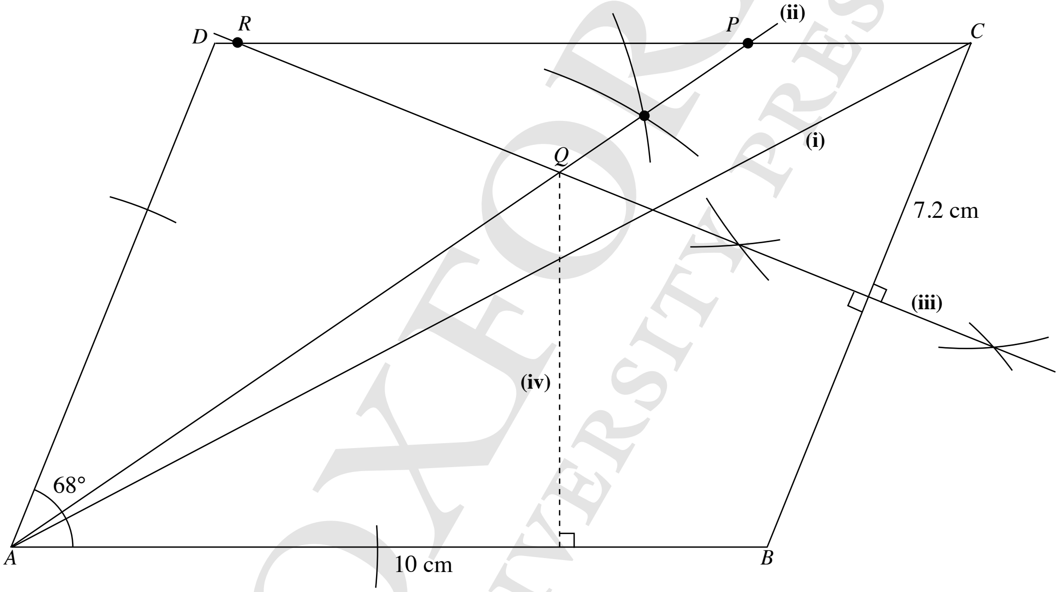
37.



(i) Length of QR = 4.8 cm
$\angle PQR$ = 104°

(iii) Length of RH = 4.9 cm
Length of HK = 5.8 cm

38.



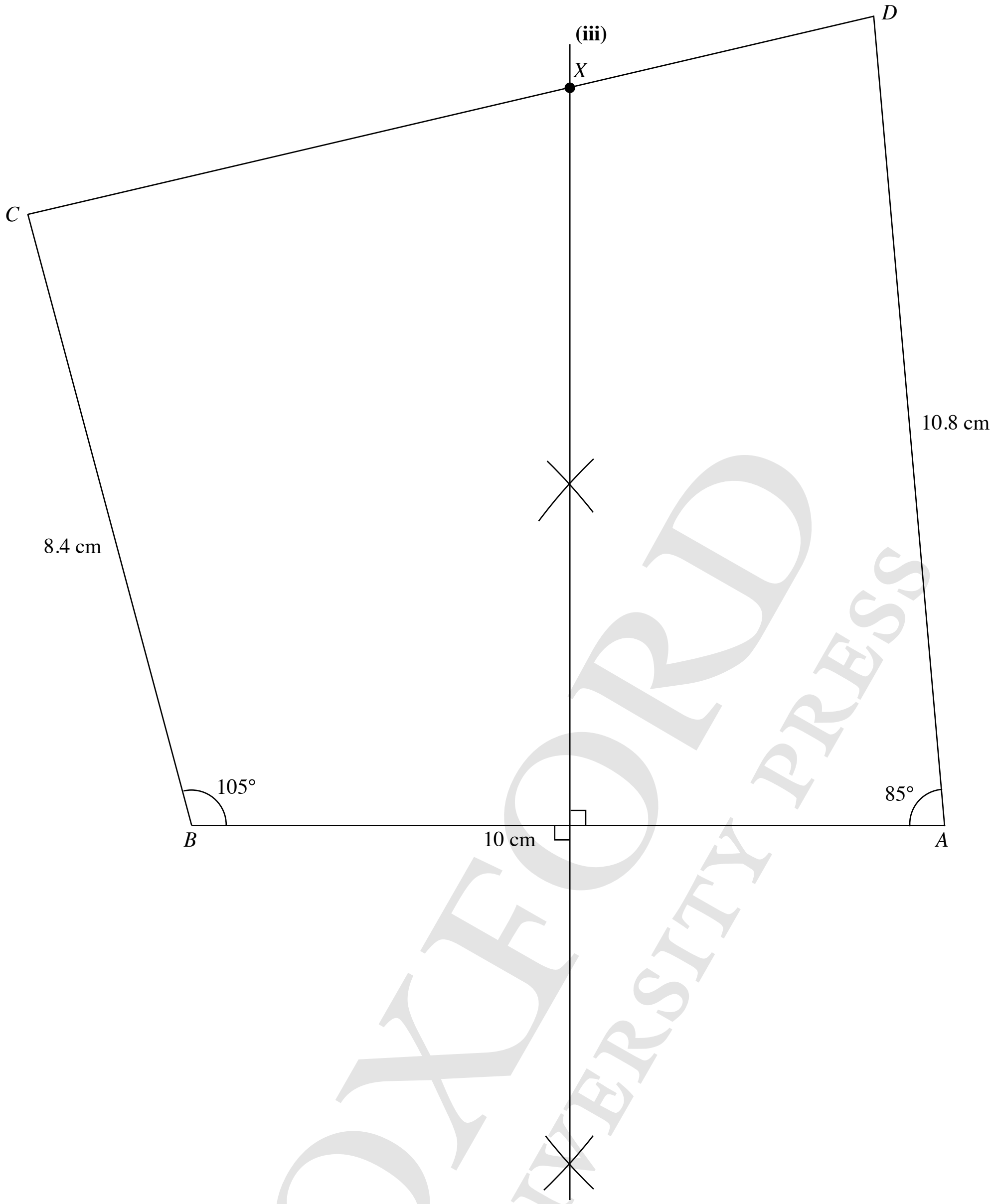
(i) Length of diagonal AC = 14.3 cm

(ii) Length of PC = 3.0 cm

(iii) Length of DR = 0.3 cm

(iv) Perpendicular height of Q to the base of the parallelogram
= 5.0 cm

39.

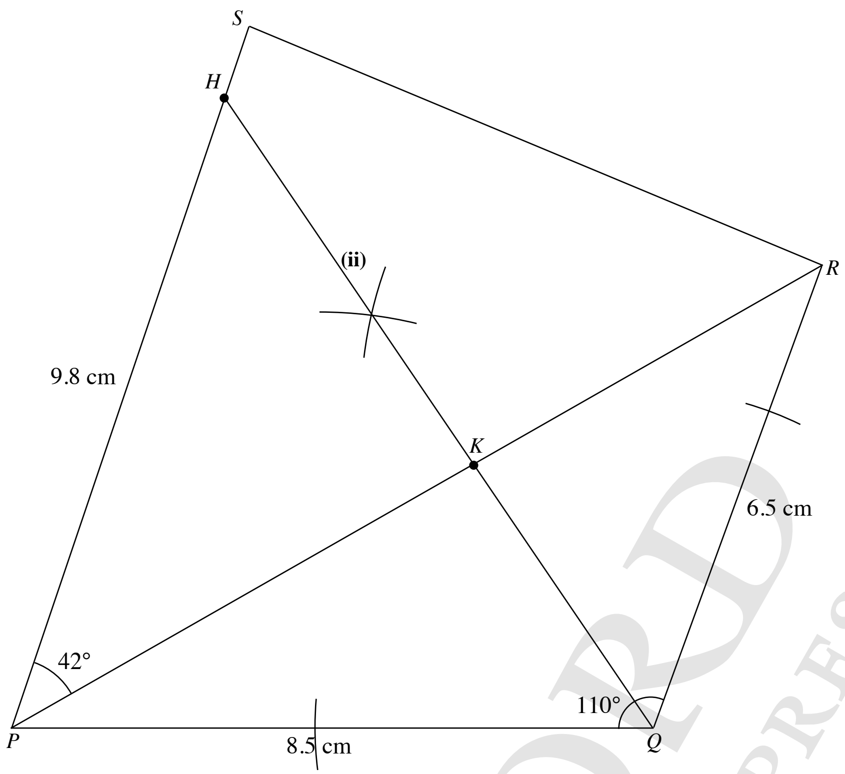


(i) Length of CD = 11.5 cm

(ii) $\angle ADC$ = 82°

(iii) Length of CX = 7.4 cm

40.



(i) Length of RS = 8.3 cm

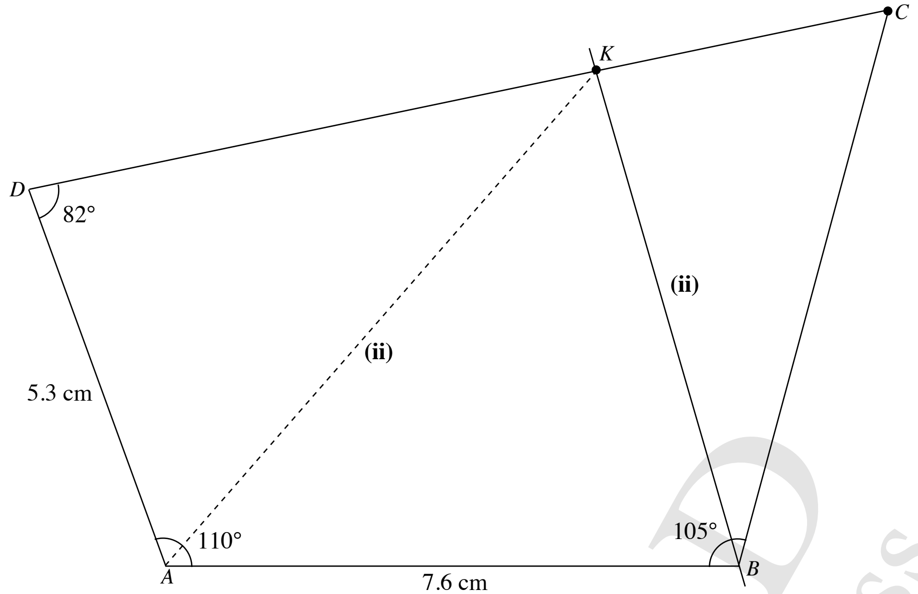
(ii) Length of QK = 4.2 cm

Length of HK = 6 cm

(iii) Ratio of QK: KH = 4.2 : 6

= 7 : 10

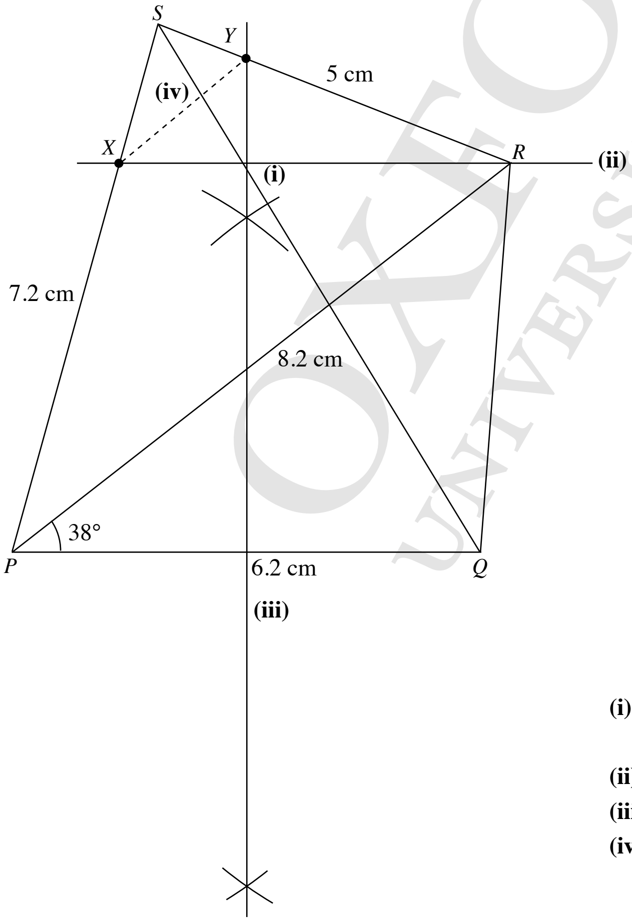
41.



(i) Length of CD = 11.6 cm
Length of BC = 7.6 cm

(ii) Length of AK = 8.7 cm
Length of BK = 6.8 cm

42.



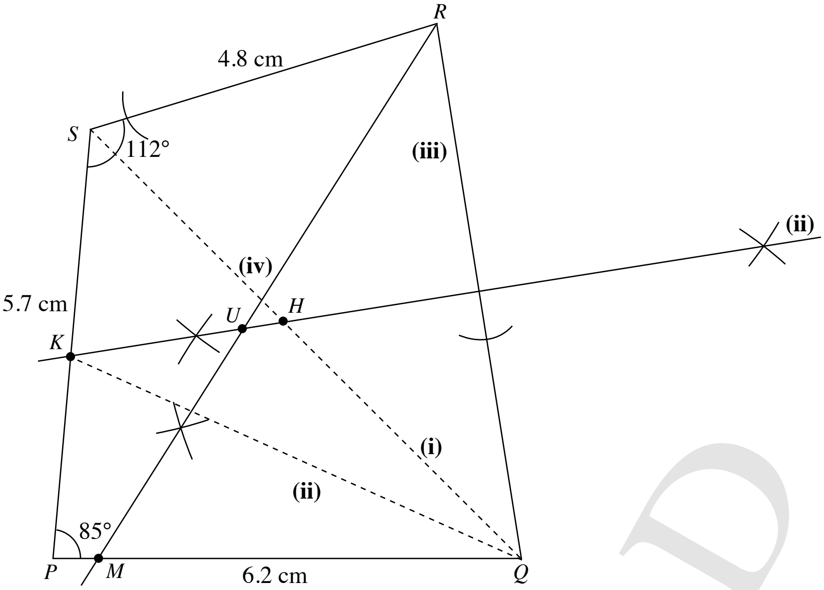
(i) Length of SQ = 8.2 cm
$\angle PRS = 60°$

(ii) Length of PX = 5.3 cm

(iii) Length of RY = 3.7 cm

(iv) Length of XY = 2.3 cm

43.



(i) Length of QR = 7.2 cm
Length of QS = 8.1 cm

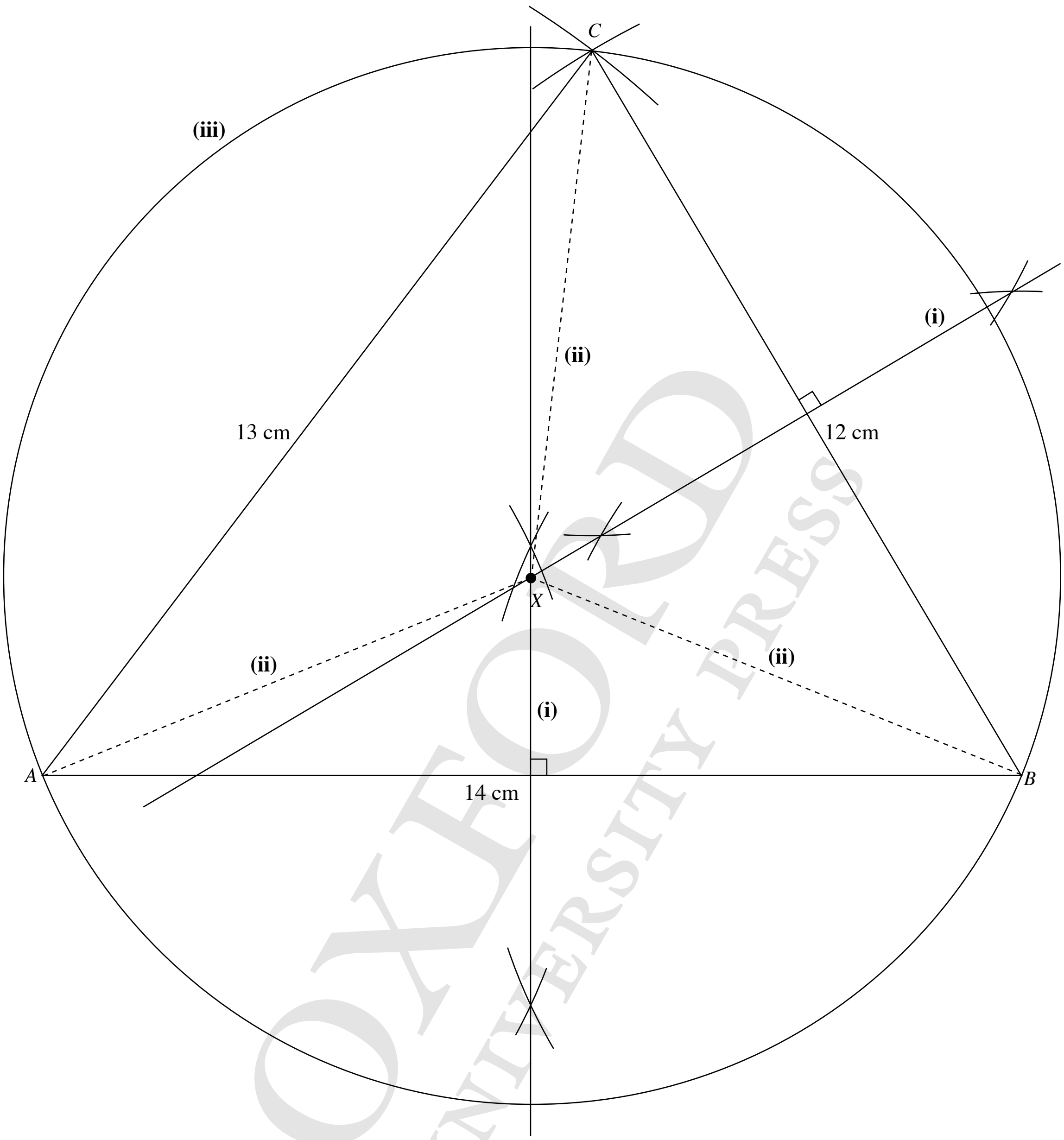
(ii) Length of SH = 3.6 cm
Length of KQ = 6.6 cm

(iii) Length of RM = 8.4 cm

(iv) Length of UM = 3.7 cm

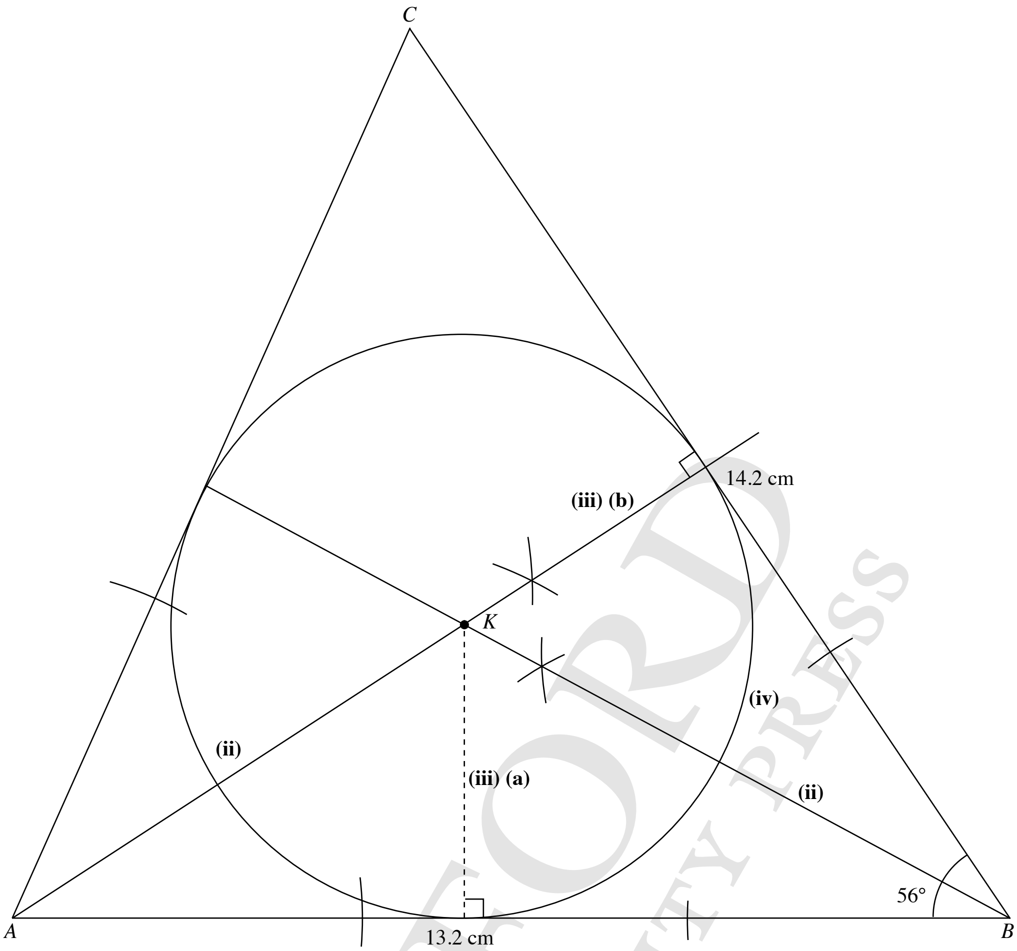
Advanced

44.



(ii) The length of *AX*, of *BX* and of *CX* = 7.6 cm

45.



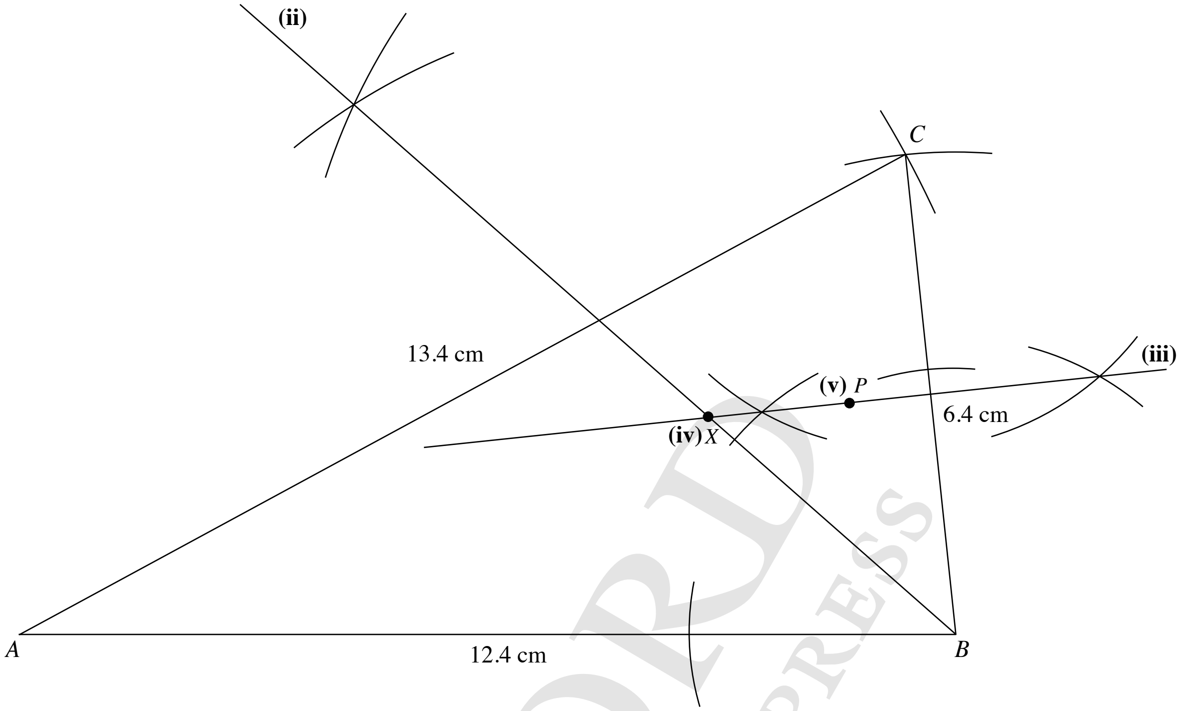
(i) Length of AC = 12.9 cm

(iii) (a) Shortest distance of K from AB = 4.0 cm

(b) Shortest distance of K from BC = 4.0 cm

New Trend

46.



(i) The angle that is facing the longest side is ∠*ABC*.
∠*ABC* = 84°

(iv) The point *X* is equidistant from the points <u>*B*</u> and <u>*C*</u>, and equidistant from the lines <u>*AB*</u> and <u>*BC*</u>.

(v) Point *P* is on the perpendicular bisector to the right of the angle bisector, closer to *BC* than *BA*.

Revision Test C1

1. Mr Lee's salary in February without allowance

$= \frac{108}{100} \times 1800$

$= \$1944$

His salary in February with allowance

$= 1944 + 20$

$= \$1964$

Increase in salary $= \$1964 - \$1800 = \$164$

Percentage increase

$= \frac{164}{1800} \times 100\%$

$= 9\frac{1}{9}\ \%$

2. **(a)** Total parts required to manufacture an article

$= 9 + 5 + 3$

$= 17$

Cost of labour

$= \frac{9}{17} \times 918$

$= \$486$

(b) **(i)** Time at which the train arrives at Station *B*

$= 1056 + 1$ hour 32 minutes

$= 1156 + 32$ minutes

$= 0000 + 28$ minutes

$= 0028$

(ii) Speed of train

$= \frac{\text{Distance}}{\text{Time taken}}$

$= \frac{161}{1\frac{32}{60}}$

$= 105$ km/h

3. **(a)** **(i)** Time taken to plant a row of lettuce

$= 5 \times 7$

$= 35$ man-days

Time taken to plant a row a cabbage

$= 2 \times 4$

$= 8$ man-days

Total time taken to complete the job

$= 35 + 8$

$= 43$ man-days

(ii) Total cost to complete the job

$= 7 \times \$40 + 4 \times \$50$

$= \$280 + \$200$

$= \$480$

(b) Cost of planting 3 rows of lettuce

$= 3 \times \$40$

$= \$120$

Cost of planting the cabbage

$= \$610 - \$120$

$= \$490$

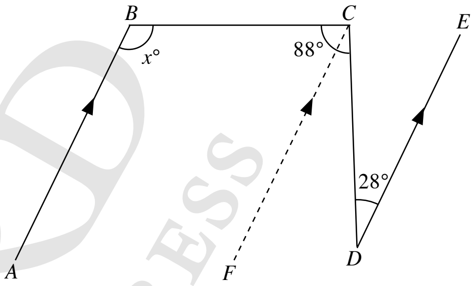
Maximum number of rows of cabbage that can be planted

$= \frac{490}{50}$

$= 9.8$

≈ 9 rows

4.



$\angle DCF = 28°$ (alt. $\angle$s, $FC \parallel DE$)

$\angle BCF = 88° - 28° = 60°$

$x° = 180° - 60°$ (int. $\angle$s, $AB \parallel FC$)

$x° = 120°$

$\therefore x = 120$

5. $112° + (27° + x°) = 180°$ (int. $\angle$s, $CD \parallel AB$)

$x° = 180° - 112° - 27°$

$= 41°$

$49° + y° + x° = 180°$ ($\angle$ sum of $\triangle$)

$y° = 180° - 49° - 41°$

$= 90°$

$\therefore x = 41$ and $y = 90$

6. **(a)** **(i)** $\angle TQR = 180° - 120°$ (adj. $\angle$s on a str. line)

$= 60°$

$3x° + 60° = 5x°$ (ext. $\angle$ of $\triangle$)

(ii) $60 + 3x = 5x$

$5x - 3x = 60$

$2x = 60$

$x = 30$

(b) **(i)** Sum of angles of a pentagon

$= (5 - 2) \times 180°$

$= 540°$

$3x + 4x + 5x + (3x - 20) + (5x - 50) = 540$

$3x + 4x + 5x + 3x + 5x - 20 - 50 = 540$

$20x - 70 = 540$

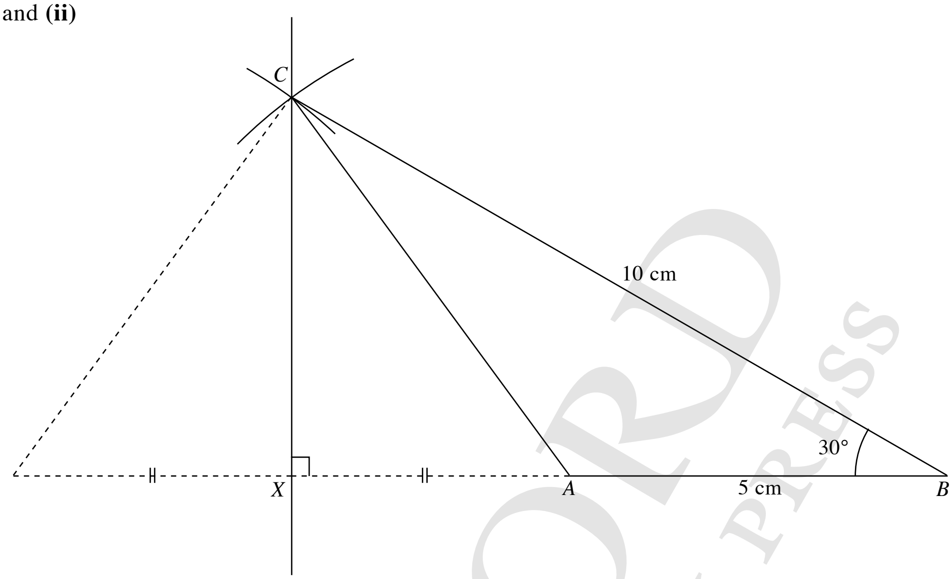
$20x = 610$

$x = 30.5$

(ii) Largest interior angle
$= (5x)°$
$= (5 \times 30.5)°$
$= 152.5°$

(iii) Smallest exterior angle
$= 180° - 152.5°$
$= 27.5°$

7. **(i)** and **(ii)**



(ii) From the diagram,
the length of CX = 5.0 cm.

Revision Test C2

1. Length of $RQ = 10 - 6$
$= 4$ cm
Length of RQ after decrease
$= 91\% \times 4$
$= \frac{91}{100} \times 4$
$= 3.64$ cm
Length of $PR = 10 - 3.64 = 6.36$ cm
Increase in the length of PR
$= 6.36 - 6$
$= 0.36$ cm
Percentage increase in the length of PR
$= \frac{0.36}{6} \times 100\%$
$= 6\%$

2. (i) Extra distance travelled
$= 560 - 280$
$= 280$ km
Extra charge
$= 280 \times 0.25$
$= \$70$
Total hire charges
$= (\$75 \times 3) + \$25 + \$70$
$= \$320$

(ii) Charges based on the total distance travelled
$= \$415 - (\$75 \times 4) - \$25$
$= \$90$
Extra distance travelled
$= \frac{90}{0.25}$
$= 360$ km
Total distance travelled
$= 280 + 360$
$= 640$ km

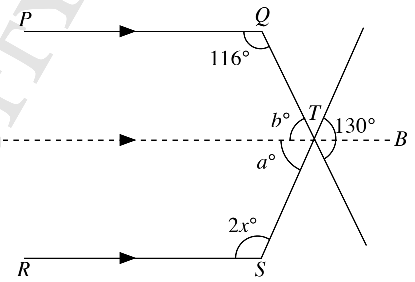
(iii) Amount that is chargeable
$= (320 - 280) \times 0.25$
$= \$10$
Hire amount
$= \$185 - \$25 - \$10$
$= \$150$
Number of days that he hired the car
$= \frac{150}{75}$
$= 2$ days

3. (a) Hourly rate of the tutor
$= \frac{124}{2\frac{1}{2}}$
$= \$49.60$
Amount charged for a lesson that lasts $3\frac{3}{4}$ hours
$= 49.6 \times 3\frac{3}{4}$
$= \$186$

(b) 6 printers can print 200 copies in $1\frac{1}{2}$ hours.
1 printer can print 200 copies in $1\frac{1}{2} \times 6 = 9$ hours.
8 printers can print 200 copies in $9 \div 8 = 1\frac{1}{8}$ hours.
$\therefore$ 8 printers can print 800 copies in
$1\frac{1}{8} \times 4 = 4\frac{1}{2}$ hours.

4. $\angle DFE = 360° - 308°$ ($\angle$s at a point)
$= 52°$
$x° = 52°$ (corr. $\angle$s, CD // EF)
$(76 + x)° + y° = 180°$ (int. $\angle$s, AB // CD)
$y° = 180° - 76° - 52°$
$= 52°$
$\therefore x = 52$ and $y = 52$

5.



Draw a line AB through T that is parallel to PQ and RS.
$116° + b° = 180°$ (int. $\angle$s, PQ // AB)
$b° = 180° - 116°$
$= 64°$
$a° + 64° = 130°$ (vert. opp. $\angle$s)
$a° = 130° - 64°$
$= 66°$
$2x° + 66° = 180°$ (int. $\angle$s, AB // RS)
$2x° = 180° - 66°$
$= 114°$
$x° = 57°$
$\therefore x = 57$

6. **(i)** If AB // CD, then

$\angle DFG = 180° - 125°$ (adj. s on a str. line)

$= 55°$

$\angle FGB = \angle EFG$ (corr. ∠s, AB // CD)

$= 125°$

$\angle GJK = 180° - 65°$ (adj. ∠s, n a str. line)v

$= 115°$

$\angle FKJ = \angle KJB$ (alt. ∠s, AB // CD)

$= 65°$

$\angle DFG = \angle FGB = 55° + 125°$

$= 180°$

$\angle GJK = \angle FKJ = 115° + 65°$

$= 180°$

By the converse of interior angle theorem, AB is parallel to CD.

(ii) $x° + \angle KJB = 180°$ (int. ∠s, Ab // Cd)

$x° = 180° - 65°$

$= 115°$

$y° = \angle FGB$ (vert. opp. ∠s)

$= 125°$

$x° + y° = 115° + 125°$

$= 240°$

$x + y = 240$

7. $88° + 99° + [(n - 2) \times 163°] = (n - 2) \times 180°$

$187° + 163n° - 326° = 180n° - 360°$

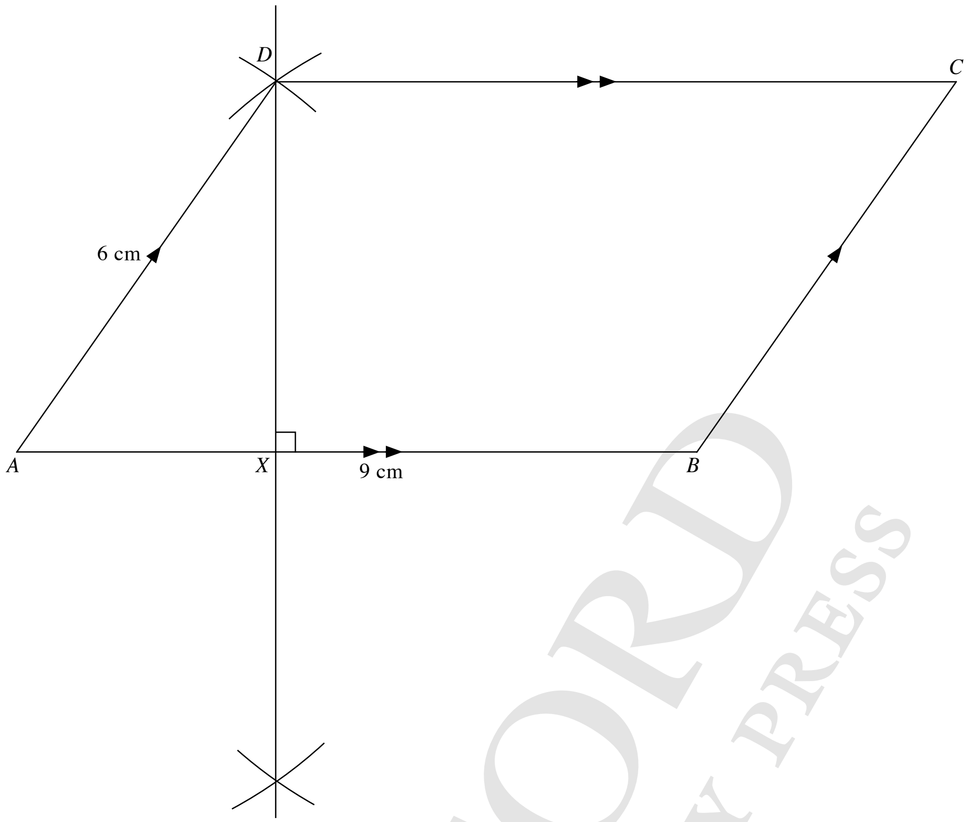
$187° - 326° + 360° = 180n° - 163n°$

$17n° = 221°$

$n° = 13°$

$\therefore n = 13$

8. **(i)** and **(ii)**



(ii) Length of the perpendicular line from D to AB (DX) = 5 cm

Chapter 13 Perimeter and Area of Plane Figures

Basic

1. **(a)** $7.3\text{ cm}^2 = 7.3 \times 10 \times 10$
$= 730\text{ mm}^2$

(b) $4.65\text{ m}^2 = 4.65 \times 10\,000$
$= 46\,500\text{ cm}^2$

(c) $3650\text{ mm}^2 = 3650 \div 100$
$= 36.5\text{ cm}^2$

(d) $200\,000\text{ cm}^2 = 200\,000 \div 10\,000$
$= 20\text{ m}^2$

(e) $50\,000\text{ mm}^2 = 50\,000 \div 100 \div 10\,000$
$= 0.05\text{ m}^2$

2. **(a)** Breadth of rectangle $= \dfrac{48}{8}$
$= 6\text{ cm}$
Perimeter of rectangle $= 2(6 + 8)$
$= 28\text{ cm}$

(b) Breadth of rectangle $= \dfrac{0.9}{1.2}$
$= 0.75\text{ m}$
Perimeter of rectangle $= 2(0.75 + 1.2)$
$= 3.9\text{ m}$

(c) Length of rectangle $= \dfrac{1.76}{0.8}$
$= 2.2\text{ cm}$
Perimeter of rectangle $= 2(0.8 + 2.2)$
$= 6\text{ cm}$

3. Perimeter of square $= 4 \times$ length of square
$48 = 4 \times$ length of square
Length of square $= \dfrac{48}{4}$
$= 12\text{ cm}$
$\therefore$ Area of square $= 12 \times 12$
$= 144\text{ cm}^2$

4. Circumference of a circle $= 2\pi r$
Area of a circle $= \pi r^2$

	Diameter	Radius	Circumference	Area
(a)	2×10 $= 20$ cm	10 cm	$2 \times 3.142 \times 10$ $= 62.8$ cm (to 3 s.f.)	3.142×10^2 $= 314\text{ cm}^2$ (to 3 s.f.)
(b)	2×0.7495 $= 0.150$ m (to 3 s.f.)	$0.471 \div (2 \times 3.142)$ $= 0.07495$ $= 0.0750$ m (to 3 s.f.)	0.471 m	3.142×0.07495^2 $= 0.0177\text{ m}^2$ (to 3 s.f.)
(c)	1.2 m	$1.2 \div 2$ $= 0.6$ m	$2 \times 3.142 \times 0.6$ $= 3.77$ m (to 3 s.f.)	3.142×0.6^2 $= 1.13\text{ m}^2$ (to 3 s.f.)
(d)	3.999×2 $= 8.00$ cm (to 3 s.f.)	$\sqrt{50.24 \div 3.142}$ $= 3.999$ cm $= 4.00$ cm (to 3 s.f.)	$2 \times 3.142 \times 3.999$ $= 25.1$ cm (to 3 s.f.)	50.24 cm^2
(e)	2×11.996 $= 24.0$ cm (to 3 s.f.)	$\sqrt{452.16 \div 3.142}$ $= 11.996$ cm $= 12.0$ cm (to 3 s.f.)	$2 \times 3.142 \times 11.996$ $= 75.4$ cm (to 3 s.f.)	452.16 cm^2
(f)	2×14 $= 28$ cm	14 cm	$2 \times 3.142 \times 14$ $= 88.0$ cm	3.142×14^2 $= 616\text{ cm}^2$ (to 3 s.f.)
(g)	2×4.2 $= 8.4$ cm	4.2 cm	$2 \times 3.142 \times 4.2$ $= 26.4$ cm (to 3 s.f.)	3.142×4.2^2 $= 55.4\text{ cm}^2$ (to 3 s.f.)
(h)	2×19.987 $= 40.0$ m (to 3 s.f.)	$125.6 \div (2 \times 3.142)$ $= 19.987$ m $= 20.0$ m (to 3 s.f.)	125.6 m	3.142×19.987^2 $= 1260\text{ m}^2$ (to 3 s.f.)
(i)	84 mm	$84 \div 2$ $= 42$ mm	$2 \times 3.142 \times 42$ $= 264$ mm (to 3 s.f.)	3.142×42^2 $= 5540\text{ mm}^2$ (to 3 s.f.)
(j)	2×21.0057 $= 42.0$ cm (to 3 s.f.)	$132 \div (2 \times 3.142)$ $= 21.0057$ cm $= 21.0$ cm (to 3 s.f.)	132 cm	3.142×21.0057^2 $= 1390\text{ cm}^2$ (to 3 s.f.)
(k)	2×12.4920 $= 25.0$ cm (to 3 s.f.)	$78.5 \div (2 \times 3.142)$ $= 12.4920$ cm $= 12.5$ cm (to 3 s.f.)	78.5 cm	3.142×12.4920^2 $= 490\text{ cm}^2$ (to 3 s.f.)
(l)	56 cm	$56 \div 2$ $= 28$ cm	$2 \times 3.142 \times 28$ $= 176$ cm (to 3 s.f.)	3.142×28^2 $= 2460\text{ cm}^2$ (to 3 s.f.)
(m)	2×38.9752 $= 78.0$ mm (to 3 s.f.)	$244.92 \div (2 \times 3.142)$ $= 38.9752$ mm $= 39.0$ mm (to 3 s.f.)	244.92 mm	3.142×38.9752^2 $= 4770\text{ mm}^2$ (to 3 s.f.)
(n)	60 cm	$60 \div 2$ $= 30$ cm	$2 \times 3.142 \times 30$ $= 189$ cm (to 3 s.f.)	3.142×30^2 $= 2830\text{ cm}^2$ (to 3 s.f.)
(o)	2×4.9984 $= 10.0$ cm (to 3 s.f.)	$\sqrt{78.5 \div 3.142}$ $= 4.9984$ cm $= 5.00$ cm	$2 \times 3.142 \times 4.9984$ $= 31.4$ cm (to 3 s.f.)	78.5 cm^2

5. **(a)** **(i)** Perimeter of figure
$= 2 + 3 + 1 + 2 + 1 + 1$
$= 10\text{ cm}$

(ii) Area of figure
$= (2 \times 1) + (2 \times 1)$
$= 2 + 2$
$= 4\text{ cm}^2$

(b) **(i)** Perimeter of figure
$= 3 + 9 + 3 + 3 + 3 + 3 + 3 + 3$
$= 30\text{ cm}$

(ii) Area of figure
$= (9 \times 3) + (3 \times 3)$
$= 27 + 9$
$= 36 \text{ cm}^2$

(c) (i) Perimeter of figure
$= 12 + 6 + 6 + 6 + 12 + 6 + 6 + 6$
$= 60$ cm

(ii) Area of figure
$= 2(12 \times 6)$
$= 2(72)$
$= 144 \text{ cm}^2$

(d) (i) Perimeter of figure
$= 14 + 7 + 7 + 7 + 14 + 7 + 7 + 7$
$= 70$ cm

(ii) Area of figure
$= 2(14 \times 7)$
$= 2(98)$
$= 196 \text{ cm}^2$

6. (a) (i) Perimeter of figure
$$= \left[\frac{1}{2} \times 2 \times 3.142 \times \left(\frac{49}{2}\right)\right] + 49$$
$= 76.979 + 49$
$= 125.979$
$= 126$ cm (to 3 s.f.)

(ii) Area of figure
$$= \frac{1}{2} \times 3.142 \times \left(\frac{49}{2}\right)^2$$
$= 942.992\ 75$
$= 943 \text{ cm}^2$ (to 3 s.f.)

(b) (i) Perimeter of figure
$$= \left[\frac{1}{2} \times 2 \times 3.142 \times \left(\frac{21}{2}\right)\right] + 20 + 21 + 20$$
$= 32.991 + 61$
$= 93.991$
$= 94.0$ cm (to 3 s.f.)

(ii) Area of figure
$$= (20 \times 21) + \left[\frac{1}{2} \times 3.142 \times \left(\frac{21}{2}\right)^2\right]$$
$= 420 + 173.202\ 75$
$= 593.202\ 75$
$= 593 \text{ cm}^2$ (to 3 s.f.)

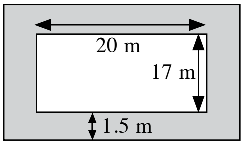
(c) (i) Perimeter of figure
$$= \left[\frac{1}{2} \times 2 \times 3.142 \times \left(\frac{21}{2}\right)\right] + \left[\frac{1}{2} \times 2 \times 3.142 \times \left(\frac{14}{2}\right)\right] + 21 + 14$$
$= 32.991 + 21.994 + 21 + 14$
$= 89.985$
$= 90.0$ cm (to 3 s.f.)

(ii) Area of figure
$$= (14 \times 21) + \left[\frac{1}{2} \times 3.142 \times \left(\frac{21}{2}\right)^2\right] + \left[\frac{1}{2} \times 3.142 \times \left(\frac{14}{2}\right)^2\right]$$
$= 294 + 173.202\ 75 + 76.979$
$= 544.181\ 75$
$= 544 \text{ cm}^2$ (to 3 s.f.)

(d) (i) Perimeter of figure
$$= \left[2 \times 3.142 \times \left(\frac{28}{2}\right)\right] + 16 + 16$$
$= 87.976 + 16 + 16$
$= 119.976$
$= 120$ cm (to 3 s.f.)

(ii) Area of figure
$= 28 \times 16$
$= 448 \text{ cm}^2$
(Note: The semicircle removed from the rectangle can be replaced by the semicircle that is placed beside the rectangle. Therefore, the area of the figure is that of a rectangle of 28 cm by 16 cm.)

7. Length of the pool with the walkway
$= 20 + 1.5 + 1.5$
$= 23$ m



Breadth of the pool with the walkway
$= 17 + 1.5 + 1.5$
$= 20$ m
Area of pool with walkway
$= 23 \times 20$
$= 460 \text{ m}^2$
Area of the swimming pool
$= 20 \times 17$
$= 340 \text{ m}^2$
Area of walkway $= 460 - 340$
$= 120 \text{ m}^2$

8. **(i)** Perimeter of the shaded region

$= 40 + 40 + \left[2 \times 3.142 \times \frac{28}{2}\right]$

$= 80 + 87.976$

$= 167.976$

$= 168$ cm (to 3 s.f.)

(ii) Area of the shaded region

$= (40 \times 28) - \left[3.142 \times \left(\frac{28}{2}\right)^2\right]$

$= 1120 - 615.832$

$= 504.168$

$= 504$ cm^2 (to 3 s.f.)

9. **(i)** Perimeter of quadrant

$= \left(\frac{1}{4} \times 2 \times 3.142 \times 10\right) + 10 + 10$

$= 15.71 + 20$

$= 35.71$

$= 35.7$ cm (to 3 s.f.)

(ii) Area of quadrant

$= \frac{1}{4} \times 3.142 \times 10^2$

$= 78.55$

$= 78.6$ cm^2 (to 3 s.f.)

10.

	Base	Height	Area
(a)	10 cm	12 cm	$10 \times 12 = 120$ cm^2
(b)	$100 \div 5$ $= 20$ m	5 m	100 m^2
(c)	5.2 mm	$50.96 \div 5.2$ $= 9.8$ mm	50.96 mm^2

11.

	Parallel side 1	Parallel side 2	Height	Area
(a)	5 cm	11 cm	4 cm	$\frac{1}{2}(5 + 11) \times 4$ $= 32$ cm^2
(b)	6 m	14 m	$65 \div \left[\frac{1}{2}(6+14)\right]$ $= 6.5$ m	65 m^2
(c)	2 mm	$(34.65 \div 8.25) \times 2 - 2$ $= 6.4$ mm	8.25 mm	34.65 mm^2

12. **(a)** The figure shown is a trapezium.

Area of the trapezium

$= \frac{1}{2}(11 + 13) \times 9$

$= 108$ cm^2

(b) The figure shown is a parallelogram.

Area of parallelogram

$= 16 \times 9$

$= 144$ cm^2

(c) If we rearrange the figure, it turns out to be a parallelogram

Area of the figure

$= 18 \times \left(\frac{1}{2} \times 16\right)$

$= 144$ cm^2

(d) The figure is a rhombus and it is a special case of parallelogram.

Area of rhombus

$= 32 \times \left(\frac{1}{2} \times 18\right)$

$= 288$ cm^2

(e) The figure is a trapezium.

Area of trapezium

$= \frac{1}{2}(8.3 + 11.7) \times 7.2$

$= 72$ cm^2

(f) The figure is a trapezium and a rectangle.

Area of figure

$= \left[\frac{1}{2}(9 + 26) \times (32 - 10)\right] + (26 \times 10)$

$= 385 + 260$

$= 645$ cm^2

(g) The figure is made up of two trapeziums.

Area of figure

$= \left[\frac{1}{2}(9 + 23) \times 10\right] + \left[\frac{1}{2}(9 + 17) \times 7\right]$

$= 160 + 91$

$= 251$ cm^2

13. **(a)** Area of the figure

$= \frac{1}{2} \times 11 \times 14$

$= 77$ cm^2

Area of figure $= \frac{1}{2} \times k \times 16$

$77 = \frac{1}{2} \times k \times 16$

$77 = 8k$

$\therefore k = \frac{77}{8} = 9\frac{5}{8}$

(b) Area of parallelogram = $16 \times x$

$$144 = 16x$$
$$x = 9$$

(c) Area of $ABCD = \frac{1}{2}(18 + 24) \times h$

$$273 = \frac{1}{2}(18 + 24) \times h$$
$$273 = 21h$$
$$h = 13$$

(d) Area of $ABCD = \frac{1}{2}(32 + y) \times 24$

$$912 = \frac{1}{2}(32 + y) \times 24$$
$$38 = \frac{1}{2}(32 + y)$$
$$76 = 32 + y$$
$$y = 76 - 32$$
$$= 44$$

(e) Area of trapezium = $\frac{1}{2}(27 + 37) \times x$

$$480 = \frac{1}{2}(27 + 37) \times x$$
$$960 = 64x$$
$$x = 15$$

14. Let the perpendicular height be h cm.

Area of parallelogram = $(4 + 3) \times h$

$$35 = 7h$$
$$h = 5$$

Area of $\triangle PQT = \frac{1}{2} \times 4 \times 5$

$$= 10 \text{ cm}^2$$

15. (i) Area of parallelogram $ABCD$

$= 28 \times 22$

$= 616 \text{ cm}^2$

(ii) Area of parallelogram $ABCD = 18 \times AB$

$616 = (18 \times AB) \text{ cm}^2$

$AB = 34\frac{2}{9}$ cm

Perimeter of parallelogram

$= 2\left(22 + 34\frac{2}{9}\right)$

$= 112\frac{4}{9}$ cm

16. Let the length of the other parallel side be y cm.

Area of trapezium = $\frac{1}{2}(6 + y) \times 5$

$$45 = \frac{1}{2}(6 + y) \times 5$$
$$90 = 5(6 + y)$$
$$18 = 6 + y$$
$$y = 18 - 6$$
$$= 12$$

The length of the other parallel side is 12 cm.

17. (a) Area of shaded region

$= \left(\frac{1}{2} \times 4.6 \times 8\right) + \left(\frac{1}{2} \times 6.5 \times 8\right)$

$= 18.4 + 26$

$= 44.4 \text{ cm}^2$

(b) Area of circle with radius 10 cm

$= 3.142 \times 10^2$

$= 314.2 \text{ cm}^2$

Area of circle with radius 6 cm

$= 3.142 \times 6^2$

$= 113.112 \text{ cm}^2$

Area of shaded region

$= 314.2 - 113.112$

$= 201.088$

$= 201 \text{ cm}^2$ (to 3 s.f.)

(c) Area of circle of radius 10 cm

$= 3.142 \times 10^2$

$= 314.2 \text{ cm}^2$

Area of square

$= 14.14 \times 14.14$

$= 199.9396 \text{ cm}^2$

Area of shaded region

$= 314.2 - 199.9396$

$= 114.2604$

$= 114 \text{ cm}^2$ (to 3 s.f.)

(d) Area of circle with diameter 32 cm

$= 3.142 \times \left(\frac{32}{2}\right)^2$

$= 804.352 \text{ cm}^2$

Area of circle with diameter 20 cm

$= 3.142 \times \left(\frac{20}{2}\right)^2$

$= 314.2 \text{ cm}^2$

Area of shaded region

$= 804.352 - 314.2$

$= 490.152$

$= 490 \text{ cm}^2$ (to 3 s.f.)

(e) Area of square = $16 \times 16 = 256$ cm^2

Area of circle of diameter 16 cm

$= 3.142 \times \left(\frac{16}{2}\right)^2$

$= 201.088$ cm^2

Area of shaded region

$= 256 - 201.088$

$= 54.912$

$= 54.9$ cm^2 (to 3 s.f.)

(f) Area of shaded region

$= (3.5 \times 4.6) + [(3.8 + 3.5 + 3.7) \times 3.4]$

$= 16.1 + 11 \times 3.4$

$= 16.1 + 37.4$

$= 53.5$ cm^2

(g) Area of rectangle

$= 13 \times 11$

$= 143$ cm^2

Area of triangle with perpendicular height of 5 cm

$= \frac{1}{2} \times 11 \times 5$

$= 27.5$ cm^2

Area of triangle with perpendicular height of 4 cm

$= \frac{1}{2} \times 11 \times 4$

$= 22$ cm^2

Area of shaded region

$= 143 - 27.5 - 22$

$= 93.5$ cm^2

(h) Area of circle of radius 80 mm (8 cm)

$= 3.142 \times 8^2$

$= 201.088$ cm^2

(Note: The centre of the rectangle is not at the centre of the circle.)

Area of rectangle

$= 5.6 \times 8.5$

$= 47.6$ cm^2

Area of shaded region

$= 201.088 - 47.6$

$= 153.488$

$= 153$ cm^2 (to 3 s.f.)

Intermediate

18. (a) Let the length of the square be n cm.

$\therefore n^2 = 900$

Thus $n = \sqrt{900} = 30$ cm

Perimeter of square $= 4 \times 30$

$= 120$ cm

(b) Let the length of the square be x cm.

$12.8 = 4x$

$x = 3.2$

Area of the square $= (3.2)^2$

$= 10.24$ cm^2

19. (a) (i) Let the breadth of the rectangle be y cm.

$2[y + (y + 8)] = 80$

$y + y + 8 = 40$

$2y = 40 - 8$

$2y = 32$

$y = 16$

The length of the rectangle is $(16 + 8)$ $= 24$ cm.

(ii) Area of the rectangle

$= 16 \times 24$

$= 384$ cm^2

(b) Let the length of the rectangle be x m.

$0.464 \times x = 11.6$

$x = 25$ m

Perimeter of rectangle

$= 2(25 + 0.464)$

$= 50.928$ m

(c) Let the breadth of the rectangle be y cm.

Then the length of the rectangle is $(3y)$ cm.

Perimeter of rectangle $= 2(3y + y)$ cm

$1960 = 2(3y + y)$

$980 = 4y$

$\therefore y = 245$

The breadth is 245 cm and the length is 735 cm.

Area of the rectangle

$= 735 \times 245$

$= 180\ 075$ cm^2

$= 180\ 075 \div 10\ 000$

$= 18.0075$ m^2

(d) Let the breadth of the rectangle be x cm.
Then the length of the rectangle is $2x$ cm.
Circumference of the wire
$= 2 \times 3.142 \times \frac{35}{2}$
$= 3.142 \times 35$
$= 109.97$ cm
Circumference of the wire is the perimeter of the rectangle.
$109.97 = 2(x + 2x)$
$54.985 = x + 2x$
$3x = 54.985$
$x = 18.328\ 33$ (to 5 d.p.)
Area of the rectangle
$= 18.328\ 33 \times 2(18.328\ 33)$
$= 672$ cm^2 (to 3 s.f.)

20. (a) Area of $\triangle ACD = \frac{1}{2} \times DC \times AB$
$8.4 = \frac{1}{2} \times 4 \times AB$
$2AB = 8.4$
$AB = 4.2$ cm

(b) Area of $\triangle ABC$
$= \frac{1}{2} \times BC \times AB$
$= \frac{1}{2} \times 6 \times 4.2$
$= 12.6$ cm^2

21. (a) The height from X to the length $PQ = \frac{10}{2}$
$= 5$ cm
Area of $\triangle PQX = \frac{1}{2} \times 16 \times 5$
$= 40$ cm^2

(b) Area of $\triangle PQR = \frac{1}{2} \times (16 \times 10)$
$= 80$ cm^2
Area of $\triangle QRX = 80 - 40$
$= 40$ cm^2

22. (a) Perimeter of quadrant $= r + r +$ arc length PQ
$50 = r + r +$ arc length PQ
Arc length $PQ = 50 - r - r$
$= (50 - 2r)$ cm
$\frac{1}{4} \times 2 \times \frac{22}{7} \times r = 50 - 2r$
$\frac{11}{7} \times r = 50 - 2r$
$\frac{11}{7} \times r + 2r = 50$
$3\frac{4}{7} r = 50$
$r = 50 \div 3\frac{4}{7}$
$= 14$ cm
Area of quadrant $= \frac{1}{4} \times \frac{22}{7} \times 14^2 = 154$ cm^2

(b) Circumference of wheel
$= 2 \times 3.142 \times \left(\frac{25}{2}\right)$
$= 78.55$ cm
Number of complete revolutions
$= \frac{200}{78.55 \div 100}$
≈ 254.61
$= 254$ revolutions (to 3 s.f.)
Note: The answer cannot be 255 as the wheel has made 254 revolutions but has not yet completed the 255$^{\text{th}}$ revolution.

(c) Distance moved by the tip of the hand for 26 minutes
$= \frac{26}{60} \times 2 \times 3.142 \times 8$
$= 21.8$ cm (to 1 d.p.)

(d) Distance travelled in 5 minutes
$= 90 \times \frac{5}{60}$
$= 7.5$ km
Circumference of car wheel
$= 2 \times 3.142 \times 0.000\ 35$
$= 0.002\ 199\ 4$ km
Number of revolutions made
$= 7.5 \div 0.002\ 199\ 4$
$= 3410$ (to 3 s.f.)

(e) Distance covered when the athlete runs round the track once
$= \frac{4}{8}$
$= 0.5$ km
$= 500$ m
Let the radius of the track be r m.
Circumference of the track $= 2 \times 3.142 \times r$
$500 = 2 \times 3.142 \times r$
$\therefore r = \frac{500}{2 \times 3.142}$
$= 79.57$ m (to 2 d.p.)

23. (a) Area of the rhombus
= length of the diagonal
× perpendicular height to the diagonal
90 = 18 × perpendicular height to the diagonal
Perpendicular height to the diagonal = $\frac{90}{18}$
= 5 cm
Length of the other diagonal = 5 × 2
= 10 cm

(b) Perpendicular height of the rhombus to the diagonal
= $\frac{24}{2}$
= 12 cm
Area of rhombus
= 28 × 12
= 336 cm^2

(c) **(i)** Area of trapezium
= $\frac{1}{2}$ (sum of its parallel sides) × 12
210 = $\frac{1}{2}$ (sum of its parallel sides) × 12
Sum of its parallel sides = 210 × 2 ÷ 12
= 35 cm

(ii) Let the length of the shorter side be n cm.
$35 = 2\frac{1}{2}n + n$
$3\frac{1}{2}n = 35$
$n = 35 \div 3\frac{1}{2}$
= 10
Length of the longer side = $2\frac{1}{2} \times 10$
= 25 cm

24. (a) Length of arc PR
= $\frac{1}{4} \times 2 \times 3.142 \times 5$
= 7.855 cm
Perimeter of the shaded region
= 5.66 + 7.855 + 3 + (4 + 5) + 4
= 29.515
= 29.5 cm (to 3 s.f.)

(b) Area of rectangle
= 9 × 8
= 72 cm^2
Area of $\triangle APQ$
= $\frac{1}{2} \times 4 \times 4$
= 8 cm^2
Area of quadrant BPR
= $\frac{1}{4} \times 3.142 \times 5^2$
= 19.6375 cm^2
Area of shaded region
= 72 – 8 – 19.6375
= 44.3625
= 44.4 cm^2 (to 3 s.f.)

25. Area of rectangle $ABCD$
= 60 × 28
= 1680 cm^2
Area of semicircle BXC
= $\frac{1}{2} \times 3.142 \times \left(\frac{28}{2}\right)^2$
= 307.916 cm^2
Area of $\triangle ADX$
= $\frac{1}{2} \times 28 \times (60 - 14)$
= 644 cm^2
Area of the shaded region
= 1680 – 307.916 – 644
= 728.084
= 728 cm^2 (to 3 s.f.)

26. (a) Circumference of the pond
= 2 × 3.142 × 3.2
= 20.1088 m
Circumference of the pond with concrete path
= 2 × 3.142 × (3.2 + 1.4)
= 2 × 3.142 × 4.6
= 28.9064 m
Perimeter of the shaded region
= 28.9064 + 20.1088
= 49.0152
= 49.0 m (to 3 s.f.)

(b) Area of the pond

$= 3.142 \times 3.2^2$

$= 32.174\ 08\ \text{m}^2$

Area of the pond with concrete path

$= 3.142 \times 4.6^2$

$= 66.484\ 72\ \text{m}^2$

Area of the shaded region

$= 66.484\ 72 - 32.174\ 08$

$= 34.310\ 64$

$= 34.3\ \text{m}^2$ (to 3 s.f.)

$\therefore$ The area of the concrete path is $34.3\ \text{m}^2$.

27. (a) Area of shaded region A

= area of circle with radius 5 cm

$= 3.142 \times 5^2$

$= 78.55$

$= 78.6\ \text{cm}^2$ (to 3 s.f.)

(b) Area of circle with radius 10 cm

$= 3.142 \times 10^2$

$= 314.2\ \text{cm}^2$

Area of circle with radius 8 cm

$= 3.142 \times 8^2$

$= 201.088\ \text{cm}^2$

Area of shaded region B

$= 314.2 - 201.088$

$= 113.112$

$= 113\ \text{cm}^2$ (to 3 s.f.)

28. (a) $1.25\ \text{m} = 1.25 \times 100$

$= 125\ \text{cm}$

The largest possible radius is 125.4 cm or 1.254 m.

(b) Smallest possible radius = 124.5 cm = 1.245 m

Smallest possible area

$= 3.142 \times (1.245)^2$

$= 4.870\ 178\ 55$

$= 4.870\ \text{m}^2$ (to 4 s.f.)

29. Area of quadrant

$= \frac{1}{4} \times 3.142 \times 21^2$

$= 346.4055\ \text{cm}^2$

Area of $\triangle OCA$

$= \frac{1}{2} \times 13 \times 21$

$= 136\frac{1}{2}\ \text{cm}^2$

Area of shaded region

$= 346.4055 - 136\frac{1}{2}$

$= 209.9055$

$= 210\ \text{cm}^2$ (to 3 s.f.)

30. (a) Area of quadrant

$= \frac{1}{4} \times 3.142 \times 40^2$

$= 1256.8\ \text{cm}^2$

Area of triangle

$= \frac{1}{2} \times 40 \times 40$

$= 800\ \text{cm}^2$

Area of shaded region

$= 1256.8 - 800$

$= 456.8$

$= 457\ \text{cm}^2$ (to 3 s.f.)

(b) Area of square

$= 24 \times 24$

$= 576\ \text{cm}^2$

Area of a circle with radius 12 cm

$= 3.142 \times 12^2$

$= 452.448\ \text{cm}^2$

Area of shaded region

$= 576 - 452.448$

$= 123.552$

$= 124\ \text{cm}^2$ (to 3 s.f.)

(c) Area of semicircle with radius 5 cm

$= \frac{1}{2} \times 3.142 \times 5^2$

$= 39.275\ \text{cm}^2$

Area of triangle

$= \frac{1}{2} \times 6 \times 8$

$= 24\ \text{cm}^2$

Area of shaded region

$= 39.275 - 24$

$= 15.275$

$= 15.3\ \text{cm}^2$ (to 3 s.f.)

(d) Area of trapezium

$= \frac{1}{2}(23 + 33) \times 19$

$= 532\ \text{cm}^2$

Area of triangle

$= \frac{1}{2} \times 33 \times 19$

$= 313.5\ \text{cm}^2$

Area of shaded region

$= 532 - 313.5$

$= 218.5\ \text{cm}^2$

(e) Area of semicircle with diameter 5 cm

$= \frac{1}{2} \times 3.142 \times \left(\frac{5}{2}\right)^2$

$= 9.818\ 75\ cm^2$

Area of semicircle with diameter 2 cm

$= \frac{1}{2} \times 3.142 \times \left(\frac{2}{2}\right)^2$

$= 1.571\ cm^2$

Area of semicircle with diameter 3 cm

$= \frac{1}{2} \times 3.142 \times \left(\frac{3}{2}\right)^2$

$= 3.534\ 75\ cm^2$

Area of shaded region

$= 9.818\ 75 - 1.571 + 3.534\ 75$

$= 11.7825$

$= 11.8\ cm^2$ (to 3 s.f.)

(f) Area of trapezium

$= \frac{1}{2} \times (19 + 29) \times 21$

$= 504\ cm^2$

Area of circle with diameter 21 cm

$= 3.142 \times \left(\frac{21}{2}\right)^2$

$= 346.4055\ cm^2$

Area of shaded region

$= 504 - 346.4055$

$= 157.5945$

$= 158\ cm^2$ (to 3 s.f.)

(g) Total shaded area

= area of semicircle with radius 12 cm
+ area of rectangle 23 cm by 12 cm
+ area of rectangle 17 cm by 12 cm
+ area of triangle

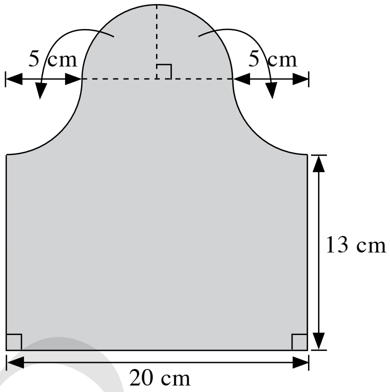
$= \left(\frac{1}{2} \times 3.142 \times 12^2\right) + (12 \times 23) + (17 \times 12) + \left(\frac{1}{2} \times 6 \times 4\right)$

$= 226.224 + 276 + 204 + 12$

$= 718.224$

$= 718\ cm^2$ (to 3 s.f.)

(h) Place the quadrants and fill the gap. The figure is then changed into a rectangle with dimensions 20 cm by 18 cm.



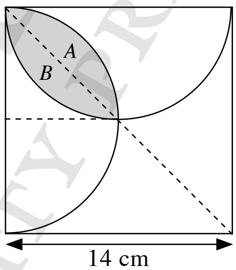
Area of shaded region

= area of rectangle with dimension 20 cm by 18 cm

$= 20 \times 18$

$= 360\ cm^2$

(i)



Area of region *A*

$= \left(\frac{1}{4} \times 3.142 \times 7^2\right) - \left(\frac{1}{2} \times 7 \times 7\right)$

$= 38.4895 - 24.5$

$= 13.9895\ cm^2$

Area of region *B* = area of region *A*

$= 13.9895\ cm^2$

$\therefore$ Area of shaded region

$= 2 \times 13.9895$

$= 27.979$

$= 28.0\ cm^2$ (to 3 s.f.)

31. (a) Since *ABCD* is a square, then

$3x = 22$

$x = 7\frac{1}{3}$

(b) Area of shaded region

= area of square *ABCD* – area of *PQRC*

$403 = (22 \times 22) - y^2$

$y^2 = (22 \times 22) - 403$

$= 484 - 403$

$= 81$

$\therefore y = 9$

32. (a) (i) Perimeter of rectangle

$= 2[(3x + 4) + (4x - 13)]$

$94 = 2[(3x + 4) + (4x - 13)]$

$94 = 2[3x + 4x + 4 - 13]$

$94 = 2[7x - 9]$

$94 = 14x - 18$

$14x = 94 + 18$

$14x = 112$

$\therefore x = 8$

(ii) Length of rectangle $= 3 \times 8 + 4$

$= 28$ cm

Breadth of rectangle $= 4 \times 8 - 13$

$= 19$ cm

Area of rectangle $= 28 \times 19$

$= 532 \text{ cm}^2$

(b) Area of trapezium

$= \frac{1}{2} \times [(x + 5) + (3x + 1)] \times 6$

$= 3[(x + 5) + (3x + 1)]$

$66 = 3[(x + 5) + (3x + 1)]$

$66 = 3[x + 3x + 5 + 1]$

$66 = 3[4x + 6]$

$66 = 12x + 18$

$12x = 66 - 18$

$12x = 48$

$\therefore x = 4$

33. (a) Perimeter of semicircle

$= \left(\frac{1}{2} \times 2 \times 3.142 \times \frac{2x}{2}\right) + 2x$

$= 3.142x + 2x$

$= 5.142x$ cm

Perimeter of rectangle

$= 2[(x + 11) + (x - 3)]$

$= 2(x + x + 11 - 3)$

$= 2(2x + 8)$ cm

$5.142x = 2(2x + 8)$

$5.142x = 4x + 16$

$5.142x - 4x = 16$

$1.142x = 16$

$x = 14.0105$

$= 14.0$ (to 3 s.f.)

(b) Area of semicircle

$= \frac{1}{2} \times 3.142 \times \left(\frac{2 \times 14.01}{2}\right)^2$

$= 308.356\ 037\ 1 \text{ cm}^2$

Length of rectangle $= 14.01 + 11$

$= 25.01$ cm

Breadth of rectangle $= 14.01 - 3$

$= 11.01$ cm

Area of rectangle $= 25.01 \times 11.01$

$= 275.3601 \text{ cm}^2$

Difference in area $= 308.356\ 037\ 1 - 275.3601$

$= 32.9959$

$= 33.0 \text{ cm}^2$ (to 3 s.f.)

34. (a) Number of slabs needed along its length

$= \frac{25 \times 100}{25}$

$= 100$

(b) Number of slabs needed along its row

$= \frac{12 \times 100}{25}$

$= 48$

(c) Area of rectangular courtyard

$= (25 \times 100) \times (12 \times 100)$

$= 3\ 000\ 000 \text{ cm}^2$

Area of each slab

$= 25 \times 25$

$= 625 \text{ cm}^2$

Number of slabs needed to pave the whole courtyard

$= \frac{3\ 000\ 000}{625}$

$= 4800$

(d) Total cost of paving the courtyard

$= \$0.74 \times 4800$

$= \$3552$

35. (a) Let the radius of the semicircle be r cm.

Area of semicircle

$= \frac{1}{2} \times 3.142 \times r^2$

$= 1.571r^2 \text{ cm}^2$

Area of triangle AFE

$= \frac{1}{2} \times 2r \times r$

$= r^2 \text{ cm}^2$

Area of shaded region $= 1.571r^2 - r^2$

$73 = 1.571r^2 - r^2$

$0.571r^2 = 73$

$r^2 = 127.845\ 884\ 4$

$r = 11.3$ (to 3 s.f.)

Length of $AE = 2 \times 11.306\ 895$

$= 22.6138$

$= 22.6$ cm (to 3 s.f.)

(b) Area of trapezium $ABDE$

$= \frac{1}{2} \times (48 + 22.6138) \times 20$

$= 706.138$

$= 706 \text{ cm}^2$ (to 3 s.f.)

36. (a) Let the length AB be h cm.

Area of quadrilateral $ABCD$

$= 8 \times h$

$= 8h$ cm^2

Area of quadrilateral $EFGH$

$= 10 \times h$

$= 10h$ cm^2

Area of $\triangle IJK$

$= \frac{1}{2} \times 14 \times h$

$= 7h$ cm^2

Ratio of area of $ABCD$ to area of $EFGH$ to area of $\triangle IJK$

$= 8h : 10h : 7h$

$= 8 : 10 : 7$

(b) Area of $\triangle IJK = 56$

$\frac{1}{2} \times 14 \times h = 56$

$7h = 56$

$h = 8$

The quadrilateral $LMNO$ is a trapezium.

Area of quadrilateral $LMNO$

$= \frac{1}{2} \times (3 + 17) \times 8$

$= 80$ cm^2

Advanced

37. (a) Perimeter of triangle ABC

$= 2x + (x + 5) + (4x - 2)$

$= 2x + x + 4x + 5 - 2$

$= (7x + 3)$ cm

Perimeter of rectangle $PQRS$

$= 2[(7x - 10) + (2x + 1)]$

$= 2(7x + 2x - 10 + 1)$

$= 2(9x - 9)$ cm

$= 18(x - 1)$ cm

The equation is $1\frac{1}{2}(7x + 3) = 18(x - 1)$.

(b) $1\frac{1}{2}(7x + 3) = 18(x - 1)$

$3(7x + 3) = 36(x - 1)$

$21x + 9 = 36x - 36$

$36x - 21x = 9 + 36$

$15x = 45$

$x = 3$

Perimeter of triangle $ABC = 7 \times 3 + 3$

$= 24$ cm

Area of triangle ABC

$= \frac{1}{2} \times (2 \times 3) \times (3 + 5)$

$= \frac{1}{2} \times 6 \times 8$

$= 24$ cm^2

(c) Area of rectangle $PQRS$

$= (2 \times 3 + 1) \times (7 \times 3 - 10)$

$= 7 \times 11$

$= 77$ cm^2

Difference between the area of triangle ABC and the area of rectangle $PQRS$

$= 77 - 24$

$= 53$ cm^2

38. Let the radius of each circle be r cm.

Area of each circle $= \pi r^2$

$36\pi = \pi r^2$

$r^2 = 36$

$\therefore r = \sqrt{36} = 6$

Length of $CD = 6 + 6$

$= 12$ cm

39. $AM : MJ = 1 : 1$

$AM = \frac{1}{2} \times 10$

$= 5$ cm

Area of $AMIB$

$= \frac{1}{2} \times (5 + 10) \times 10$

$= 75$ cm^2

$LG = 2$ cm (based on L divides DG in the ratio of 1 : 2)

Area of $BIGL$

$= \frac{1}{2} \times (2 + 10) \times 17$

$= 102$ cm^2

Area of $\triangle LGF$

$= \frac{1}{2} \times 3 \times 2$

$= 3$ cm^2

Total area of the shaded region

$= 75 + 102 + 3$

$= 180$ cm^2

New Trend

40. (a) $A = \pi\left(\frac{5r+5kr}{2}\right)^2$

$= \pi\left[\frac{5r(1+k)}{2}\right]^2$

$= \frac{25}{4}\pi r^2(1+k)^2$

(b) When $k = 3$,

Area of large circle $= \frac{25}{4}\pi r^2(4)^2$

$= 100\pi r^2 \text{ cm}^2$

Area of shaded region

$= \frac{100\pi r^2}{2} + \frac{1}{2}\pi\left(\frac{5kr}{2}\right)^2 - \frac{1}{2}\pi\left(\frac{5r}{2}\right)^2$

$= 50\pi r^2 + \frac{1}{2}\pi\left(\frac{225r^2}{4}\right) - \frac{1}{2}\pi\left(\frac{25r^2}{4}\right)$

$= 50\pi r^2 + \frac{225\pi r^2}{8} - \frac{25\pi r^2}{8}$

$= 75\pi r^2 \text{ cm}^2$

Area of unshaded region

$= \frac{100\pi r^2}{2} + \frac{1}{2}\pi\left(\frac{5r}{2}\right)^2 - \frac{1}{2}\pi\left(\frac{5kr}{2}\right)^2$

$= 50\pi r^2 + \frac{25\pi r^2}{8} - \frac{225\pi r^2}{8}$

$= 50\pi r^2 - 25\pi r^2$

$= 25\pi r^2 \text{ cm}^2$

Difference in area $= 75\pi r^2 - 25\pi r^2$

$= 50\pi r^2 \text{ cm}^2$

Chapter 14 Volume and Surface Area of Prisms and Cylinders

Basic

1. (a) $6.2 \text{ m}^3 = 6.2 \times 100 \times 100 \times 100$
$= 6\ 200\ 000 \text{ cm}^3$

(b) $2.9 \text{ m}^3 = 2.9 \times 100 \times 100 \times 100$
$= 2\ 900\ 000 \text{ cm}^3$

(c) $35\ 000 \text{ cm}^3 = 35\ 000 \div 100 \div 100 \div 100$
$= 0.035 \text{ m}^3$

(d) $75 \text{ cm}^3 = 75 \div 100 \div 100 \div 100$
$= 0.000\ 075 \text{ m}^3$

(e) $97.8\ l = 97.8 \times 1000$
$= 97\ 800 \text{ cm}^3$

(f) $1 \text{ cm}^3 = 1 \text{ m}l$
$0.07 \text{ cm}^3 = 0.07 \text{ m}l$

2. (a) (i) Volume of cube $= 5^3$
$= 125 \text{ cm}^3$

(ii) Total surface area
$= 6l^2$
$= 6 \times 5^2 = 150 \text{ cm}^2$

(b) (i) Volume of cube $= 2.4^3$
$= 13.824 \text{ cm}^3$

(ii) Total surface area
$= 6 \times 2.4^2$
$= 34.56 \text{ cm}^2$

(c) (i) Volume of rectangular cuboid
$= 30 \times 25 \times 12$
$= 9000 \text{ cm}^3$

(ii) Total surface area of cuboid
$= 2[(30 \times 25) + (30 \times 12) + (25 \times 12)]$
$= 2[750 + 360 + 300]$
$= 2820 \text{ cm}^2$

(d) (i) Volume of rectangular cuboid
$= 1.2 \times 0.8 \times 0.45$
$= 0.432 \text{ m}^3$

(ii) Total surface area of cuboid
$= 2[(1.2 \times 0.8) + (1.2 \times 0.45) + (0.8 \times 0.45)]$
$= 2[0.96 + 0.54 + 0.36]$
$= 3.72 \text{ m}^2$

3. (a) The base is a triangle with height 12 cm and base length 16 cm.
Base area = area of triangle
$= \frac{1}{2} \times 12 \times 16$
$= 96 \text{ cm}^2$
Volume of prism = base area × height
$= 96 \times 14$
$= 1344 \text{ cm}^3$
Total surface area of prism
$= 96 + 96 + (16 \times 14) + (12 \times 14) + (14 \times 20)$
$= 864 \text{ cm}^2$

(b) The shape of the base is a cross.
Base area
$= (14 \times 14) - 4(5 \times 5)$
$= 96 \text{ cm}^2$
Volume of prism = base area × height
$= 96 \times 3$
$= 288 \text{ cm}^3$
Total surface area of solid
$= (2 \times 96) + 8(5 \times 3) + 4(3 \times 4)$
$= 192 + 120 + 48$
$= 360 \text{ cm}^2$

(c) The base is a triangle with height 8 cm and base length 6 cm.
Base area $= \frac{1}{2} \times 8 \times 6$
$= 24 \text{ cm}^2$
Volume of prism = base area × height
$= 24 \times 14$
$= 336 \text{ cm}^3$
Total surface area of the solid
$= 24 + 24 + (10 \times 14) + (14 \times 8) + (6 \times 14)$
$= 48 + 140 + 112 + 84$
$= 384 \text{ cm}^2$

(d) The base is a U-shape.
Base area $= (21 \times 15) - (9 \times 7)$
$= 315 - 63$
$= 252 \text{ cm}^2$
Volume of prism = base area × height
$= 252 \times 10$
$= 2520 \text{ cm}^3$
Total surface area of the solid
$= (252 \times 2) + 2(6 \times 10) + 2(7 \times 10) + (9 \times 10) + (21 \times 10) + 2(15 \times 10)$
$= 504 + 120 + 140 + 90 + 210 + 300$
$= 1364 \text{ cm}^2$

4. Volume of a closed cylinder = $\pi r^2 h$

Total surface area of closed cylinder = $2\pi r^2 + 2\pi rh$

	Diameter	Radius	Height	Volume	Total Surface Area
(a)	24×2 $= 48$ cm	24 cm	21 cm	$3.142 \times (24)^2 \times 21$ $= 38\ 000$ cm^3 (to 3 s.f.)	$(2 \times 3.142 \times (24)^2)$ $+ (2 \times 3.142 \times 24 \times 21)$ $= 3619.584 + 3167.136$ $= 6790$ cm^2 (to 3 s.f.)
(b)	1.45×2 $= 2.9$ cm	1.45 cm	1.4 cm	$3.142 \times (1.45)^2 \times 1.4$ $= 9.25$ cm^3 (to 3 s.f.)	$(2 \times 3.142 \times (1.45)^2)$ $+ (2 \times 3.142 \times 1.45 \times 1.4)$ $= 13.212\ 11 + 12.756\ 52$ $= 26.0$ cm^2 (to 3 s.f.)
(c)	28×2 $= 56$ cm	0.28 m = 28 cm	45 cm	$3.142 \times (28)^2 \times 45$ $= 111\ 000$ cm^3 (to 3 s.f.)	$(2 \times 3.142 \times (28)^2)$ $+ (2 \times 3.142 \times 28 \times 45)$ $= 4926.656 + 7917.84$ $= 12\ 800$ cm^2 (to 3 s.f.)
(d)	18.2×2 $= 36.4$ cm	182 mm = 18.2 cm	7.5 cm	$3.142 \times (18.2)^2 \times 7.5$ $= 7810$ cm^3 (to 3 s.f.)	$(2 \times 3.142 \times (18.2)^2)$ $+ (2 \times 3.142 \times 18.2 \times 7.5)$ $= 2081.512\ 16 + 857.766$ $= 2940$ cm^2 (to 3 s.f.)
(e)	4.998×2 $= 10.0$ cm (to 3 s.f.)	$\sqrt{(2826) \div (3.142 \times 36)}$ $= 4.998$ $= 5.00$ cm (to 3 s.f.)	36 cm	2826 cm^3	$(2 \times 3.142 \times (4.998)^2)$ $+ (2 \times 3.142 \times 4.998 \times 36)$ $= 156.97 + 1130.67$ $= 1290$ cm^2 (to 3 s.f.)
(f)	1.118×2 $= 2.236$ cm	$\sqrt{(30.615) \div (3.142 \times 7.8)}$ $= 1.118$ cm $= 1.12$ cm (to 3 s.f.)	7.8 cm	30.615 cm^3	$(2 \times 3.142 \times (1.118)^2)$ $+ (2 \times 3.142 \times 1.118 \times 7.8)$ $= 7.854\ 52 + 54.799$ $= 62.7$ cm^2 (to 3 s.f.)
(g)	19.994×2 $= 40.0$ cm (to 3 s.f.)	$\sqrt{(8164) \div (3.142 \times 6.5)}$ $= 19.994$ cm $= 20.0$ cm (to 3 s.f.)	65 mm = 6.5 cm	8164 cm^3	$(2 \times 3.142 \times (19.994)^2)$ $+ (2 \times 3.142 \times 19.994 \times 6.5)$ $= 2512.092 + 816.6749$ $= 3330$ cm^2 (to 3 s.f.)
(h)	5.6×2 $= 11.2$ cm	5.6 cm	$532 \div (3.142 \times 5.6^2)$ $= 5.3992$ cm $= 5.40$ cm (to 3 s.f.)	532 cm^3	$(2 \times 3.142 \times (5.6)^2)$ $+ (2 \times 3.142 \times 5.6 \times 5.3992)$ $= 197.066\ 24 + 190.00$ $= 387$ cm^2 (to 3 s.f.)
(i)	2.65×2 $= 5.3$ cm	2.65 cm	$20.74 \div (3.142 \times 2.65^2)$ $= 0.940$ cm (to 3 s.f.)	20.74 cm^3	$(2 \times 3.142 \times (2.65)^2)$ $+ (2 \times 3.142 \times 2.65 \times 0.940)$ $= 44.129\ 39 + 15.6534$ $= 59.8$ cm^2 (to 3 s.f.)
(j)	15×2 $= 30$ cm	15 cm	$5400 \div (3.142 \times 15^2)$ $= 7.6384$ cm $= 7.64$ cm (to 3 s.f.)	0.0054 m^3	$(2 \times 3.142 \times (15)^2)$ $+ (2 \times 3.142 \times 15 \times 7.6384)$ $= 1413.9 + 719.996$ $= 2130$ cm^2 (to 3 s.f.)

5. **(a)** Let the height of the room be h m.

Volume of room = $(12 \times 9 \times h)$ m^3

$540 = 12 \times 9 \times h$

$h = 5$

$\therefore$ The height of the room is 5 m.

(b) Let the length of the box be n cm.

$60 = n \times 4 \times 2$

$\therefore n = 7.5$

The length of the box is 7.5 cm.

6. **(a)** Number of cubes that can be obtained along the length

$= 20 \div 4$

$= 5$

Number of cubes that can be obtained along the breadth

$= 16 \div 4$

$= 4$

Number of cubes that can be obtained along the height

$= 8 \div 4$

$= 2$

Therefore, the number of cubes that can be obtained

$= 5 \times 4 \times 2$

$= 40$

(b) Number of cubes that can be obtained along the length

$= 80 \div 4$

$= 20$

Number of cubes that can be obtained along the breadth

$= 25 \div 4$

≈ 6

Number of cubes that can be obtained along the height

$= 35 \div 4$

≈ 8

Therefore, the number of cubes that can be obtained

$= 20 \times 6 \times 8$

$= 960$

(c) Number of cubes that can be obtained along the length

$= 120 \div 4$

$= 30$

Number of cubes that can be obtained along the breadth

$= 85 \div 4$

≈ 21

Number of cubes that can be obtained along the height

$= 50 \div 4$

≈ 12

Therefore, the number of cubes that can be obtained

$= 30 \times 21 \times 12$

$= 7560$

7. Number of cubes that can be cut along the length

$= 420 \div 20$

$= 21$

Number of cubes that can be cut along the breadth

$= 140 \div 20$

$= 7$

Number of cubes that can be cut along the height

$= 120 \div 20$

$= 6$

Therefore, the number of cubes that can be cut

$= 21 \times 7 \times 6$

$= 882$

(Note: For questions 6 and 7, understand the difference between "cut" and "melt" and "recast".)

8. Total volume of water

$= 37 + 20$

$= 57 \text{ m}^3$

Let the depth of water in the trough be h m.

Volume of water $= 8 \times 3 \times h$

$= 24h \text{ m}^3$

$57 = 24h$

$\therefore h = 2.375$

The depth of the water, after 20 m^3 of water is added, is 2.375 m.

9. **(i)** Volume of air = volume of cuboid

$= 12 \times 7 \times 3$

$= 252 \text{ m}^3$

(ii) Number of students allowed staying in the dormitory

$= 252 \div 14$

$= 18$

10. **(i)** Volume of hall

= volume of cuboid + volume of half-cylindrical ceiling

$= (30 \times 80 \times 10) + \left(\frac{1}{2} \times 3.142 \times \left(\frac{30}{2}\right)^2 \times 80\right)$

$= 24\,000 + 28\,278$

$= 52\,278$

$= 52\,300 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of hall

$= [2(30 \times 10) + 2(80 \times 10) + (80 \times 30)] + \frac{1}{2}[(2 \times 3.142 \times 15^2) + (2 \times 3.142 \times 15 \times 80)]$

$= [600 + 1600 + 2400] + \frac{1}{2}[1413.9 + 7540.8]$

$= 4600 + 4477.35$

$= 9077.35$

$= 9080 \text{ m}^2$ (to 3 s.f.)

Intermediate

11. **(a)** Density of solid

$= \frac{\text{mass}}{\text{volume}}$

$= \frac{45}{8}$

$= 5.625 \text{ g/cm}^3$

(b) Density of solid

$= \frac{\text{mass}}{\text{volume}}$

$= \frac{1.35 \times 1000}{250}$

$= 5.4 \text{ g/cm}^3$

(c) Density of solid

$= \frac{\text{mass}}{\text{volume}}$

$= \frac{0.46 \times 1000}{78000 \div 10 \div 10 \div 10}$

$= 5.90 \text{ g/cm}^3$ (to 3 s.f.)

(d) Density of solid

$= \frac{\text{mass}}{\text{volume}}$

$= \frac{0.325 \times 1000}{85}$

$= 3.82 \text{ g/cm}^3$ (to 3 s.f.)

12. Volume of block = volume of cube

$= (28)^3$

$= 21\ 952 \text{ cm}^3$

Let one unit of the length of the block be y cm.

Then $(5y) \times (4y) \times (3y) = 21\ 952$

$60y^3 = 21\ 952$

$y^3 = 365\frac{13}{15}$

$y = 7.152$

Longest side of the cuboid

$= 5 \times 7.152$

$= 35.761$

$= 35.8$ cm (to 1 d.p.)

13. **(a)** Length of the square base

$= \sqrt{1225}$

$= 35$ cm

Length of the square base

= diameter of the cylindrical pillar

Base area of cylinder with diameter 35 cm

$= 3.142 \times \left(\frac{35}{2}\right)^2$

$= 962.2375 \text{ cm}^2$

Volume of the pillar

= base area × height

$= 962.2375 \times (3.5 \times 100)$

$= 336\ 783.125$

$= 337\ 000 \text{ cm}^3$ (to 3 s.f.)

(b) Volume of block of wood

= base area × length

$= 1225 \times (3.5 \times 100)$

$= 428\ 750 \text{ cm}^3$

Volume of block left after making the pillar

$= 428\ 750 - 336\ 783.125$

$= 91\ 966.875$

$= 92\ 000 \text{ cm}^3$ (to 3 s.f.)

14. **(a)** **(i)** Convert 12 litres to cm^3.

$12\ l = 12 \times 1000 = 12\ 000 \text{ cm}^3$

Height of water

= volume of water ÷ base area of tank

$= 12\ 000 \div (40 \times 28)$

$= 10.714$

$= 10.7$ cm (to 3 s.f.)

(ii) Surface area in contact with the water

$= (40 \times 28) + 2[(40 \times 10.714)$
$+ (28 \times 10.714)]$

$= 1120 + 2[428.56 + 299.992]$

$= 2577.104$

$= 2580 \text{ cm}^2$ (to 3 s.f.)

(b) **(i)** Volume of tank

$= 65 \times 42 \times 38$

$= 103\ 740 \text{ cm}^3$

Volume of each cylindrical cup

$= 3.142 \times (3.5)^2 \times 12$

$= 461.874 \text{ cm}^3$

Number of cups that can fill the tank

$= \frac{103740}{461.874}$

≈ 224.61

= 224 complete cups

(Note: The answer is not 225 as the question requires the number of **complete** cups.)

(ii) Volume of cup = 224×461.874
$= 103\ 460\ \text{cm}^3$
Volume of sugarcane left in the tank
$= 103\ 740 - 103\ 460$
$= 280\ \text{cm}^3$

15. (i) Let the length of the cube be l cm.
Total surface area of cube = $6l^2$
$294 = 6l^2$
$6l^2 = 294$
$l = 7$
Volume of cube = 7^3
$= 343\ \text{cm}^3$

(ii) Convert 343 cm^3 to m^3.
$343\ \text{cm}^3 = 343 \div 100 \div 100 \div 100$
$= 3.43 \times 10^{-4}\ \text{m}^3$
Density of solid cube
$= \frac{\text{mass}}{\text{volume}}$
$= \frac{1.47}{3.43 \times 10^{-4}}$
$= 4285.714$
$= 4290\ \text{kg/m}^3$ (to 3 s.f.)

16. (a) (i) Base area = $(8 \times 3) + \frac{1}{2}(3 + 6) \times 4$
$= 24 + 18$
$= 42\ \text{cm}^2$
Volume of prism
= base area × height
$= 42 \times 6$
$= 252\ \text{cm}^3$

(ii) Total surface area
= area of all the surfaces
$= 42 + 42 + 2(6 \times 3) + (5 \times 6) + (6 \times 2)$
$+ (6 \times 7) + (8 \times 6)$
$= 84 + 36 + 30 + 12 + 42 + 48$
$= 252\ \text{cm}^2$

(iii) Mass of solid = density × volume
$= 2.8 \times 252$
$= 705.6$ g

(b) (i) Base area = $\frac{1}{2}(9 + 6) \times 4$
$= 30\ \text{cm}^2$
Volume of prism = 30×8
$= 240\ \text{cm}^3$

(ii) Total surface area
$= 30 + 30 + (8 \times 9) + (6 \times 8) + (8 \times 5)$
$+ (4 \times 8)$
$= 60 + 72 + 48 + 40 + 32$
$= 252\ \text{cm}^2$

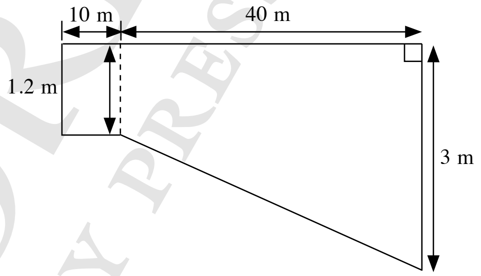
(iii) Mass of solid = density × volume
$= 2.8 \times 240$
$= 672$ g

17. (i) Area of face $ABQP$
$= \frac{1}{2}(7 + 13) \times 8$
$= 80\ \text{cm}^2$

(ii) Base area = area of face $ABQP$ = 80 cm^2
Volume of solid
= base area × height
$= 80 \times 40$
$= 3200\ \text{cm}^3$

(iii) Total surface area
= area of all the faces
$= 2(80) + (13 \times 40) + (7 \times 40) + (8 \times 40)$
$+ (10 \times 40)$
$= 160 + 520 + 280 + 320 + 400$
$= 1680\ \text{cm}^2$

18. A drawing of the cross-section of the swimming pool is helpful in solving the problem.



Area of cross-section
$= \frac{1}{2} \times (1.2 + 3) \times 40 + 10 \times 1.2$
$= 96\ \text{m}^2$
Volume of water in the pool when it is full
= area of cross-section × width
$= 96 \times 32$
$= 3072\ \text{cm}^3$

19. (i) Let the radius of the base of the cylinder be r cm.
Circumference of base of cylinder = $2\pi r$
$88 = 2 \times 3.142 \times r$
$r = 14.004$
Total surface area
$= 2\pi r^2$ + (circumference × height)
$= (2 \times 3.142 \times 14.004^2) + (88 \times 10)$
$= 1232.367\ 909 + 880$
$= 2112.367\ 909$
$= 2110\ \text{cm}^2$ (to 3 s.f.)

(ii) Volume of cylinder
$= \pi r^2 h$
$= 3.142 \times 14.004^2 \times 10$
$= 6161.840$
$= 6160\ \text{cm}^3$ (to 3 s.f.)

20. Volume of water in container P

$= 3.142 \times \left(\frac{3}{2}\right)^2 \times 24$

$= 169.668 \text{ cm}^3$

Let the height of water in container Q be h cm.

Volume of water in container $Q = 169.668 \text{ cm}^3$

Base area of container Q

$= 3.142 \times \left(\frac{8}{2}\right)^2$

$= 50.272 \text{ cm}^2$

$50.272 \times h = 169.668$

$h = 3\frac{3}{8}$

The height of water in container Q is $3\frac{3}{8}$ cm.

21. Volume of water in the cylinder when it is filled to the brim

$= 3.142 \times \left(\frac{10}{2}\right)^2 \times 30$

$= 2356.5 \text{ cm}^3$

Volume of water in the tank before the ball bearings are added

$= \frac{3}{8} \times 2356.5$

$= 883.6875 \text{ cm}^3$

Volume of water and ball bearings

$= \frac{1}{2} \times 2356.5$

$= 1178.25 \text{ cm}^3$

Volume of 8 ball bearings

$= 1178.25 - 883.6875$

$= 294.5625 \text{ cm}^3$

Volume of each ball bearing

$= 294.5625 \div 8$

$= 36.820$

$= 36.8 \text{ cm}^3$ (to 3 s.f.)

22. **(i)** Total surface area of an open cylinder

$= \pi r^2 + 2\pi rh$

$= (3.142 \times 14^2) + (2 \times 3.142 \times 14 \times 30)$

$= 615.832 + 2639.28$

$= 3255.112$

$= 3260 \text{ cm}^2$ (to 3 s.f.)

(ii) $1 \text{ m}^2 = 1 \times 100 \times 100 = 10\ 000 \text{ cm}^2$

$10\ 000 \text{ cm}^2$ costs 750 cents

3255.112 cm^2 costs 244.1334 cents

= 244 cents (to the nearest cent)

23. **(a)** Volume of metal cube $= (46)^3$

$= 97\ 336 \text{ cm}^3$

Volume of each cylindrical rod

$= 3.142 \times 2^2 \times 3.2$

$= 40.2176 \text{ cm}^3$

Maximum number of rods that can be obtained

$= \frac{97\ 336}{40.2176}$

≈ 2420

(b) Volume of the metal disc

$= 3.142 \times 8^2 \times 3$

$= 603.264 \text{ cm}^3$

Volume of each bar

$= 3.142 \times 1^2 \times 4.2$

$= 13.1964 \text{ cm}^3$

Maximum number of bars that can be obtained

$= \frac{603.264}{13.1964}$

$= 45.7143$

≈ 45

(c) Volume of butter

$= 3.142 \times 3^2 \times 10$

$= 282.78 \text{ cm}^3$

Volume of each circular disc

$= 3.142 \times (1.5)^2 \times 0.8$

$= 5.6556 \text{ cm}^3$

Maximum number of discs formed

$= \frac{282.78}{5.6556}$

$= 50$

24. Volume of the metal

$= 12 \times 18 \times 10$

$= 2160 \text{ cm}^3$

Volume of each cylindrical plate

$= 2160 \div 45$

$= 48 \text{ cm}^3$

Let the thickness of each plate be t cm.

$48 = 3.142 \times 1.2^2 \times t$

$t = 10.61$ cm (to 2 d.p.)

$\therefore$ The thickness of each plate is 10.61 cm.

25. **(i)** Internal curved surface area

$= 2\pi rh$

$= 2 \times 3.142 \times 9 \times (12 \times 100)$

$= 67\ 867.2 \text{ cm}^2$

$= 6.79 \text{ m}^2$ (to 3 s.f.)

(ii) External radius of the pipe $= 9 + 0.5$

$= 9.5$ cm

Volume of metal

$= [3.142 \times (9.5)^2 \times 1200] - (3.142 \times 9^2 \times 1200)$

$= [3.142 \times 1200](9.5^2 - 9^2)$

$= 34\ 876.2$

$= 34\ 900 \text{ cm}^3$ (to 3 s.f.)

26. (i) Convert 385 litres to cm^3.

385 litres = $385 \times 1000 = 385\ 000\ cm^3$

(ii) Base area = $3.142 \times (70)^2$

$= 15\ 395.8\ cm^2$

Volume of water in tank = base area × height h

$385\ 000 = 15\ 395.8 \times h$

$h = 25.007$

$= 25.0$ cm (to 3 s.f.)

(iii) Total surface area of the liquid in contact with the cylindrical tank

$= (3.142 \times 70^2) + (2 \times 3.142 \times 70 \times 25.007)$

$= 15\ 395.8 + 11\ 000.08$

$= 26\ 395.88$

$= 26\ 400\ cm^2$ (to 3 s.f.)

27. (a) Since water is discharged through the pipe at a rate of 28 m/min, the volume of water discharged in 1 minute is the volume of water that fills the pipe to a length of 28 m.

In 1 minute, volume of water discharged

= volume of pipe of length 28 m

$= \pi r^2 h$

$= 3.142 \times (4.2 \div 100)^2 \times 28$

$= 0.155\ 189\ 664\ cm^3$

Volume of water in rectangular tank

$= 4 \times 2.5 \times 2.4$

$= 24\ m^3$

Amount of time needed to fill the tank completely

$= \dfrac{24}{0.155\ 189\ 664}$

$= 154.6495$ minutes

= 2 hours and 35 minutes (to the nearest minute)

(b) Since water is discharged through the pipe at a rate of 3.4 m/s, the volume of water discharged in 1 second is the volume of water that fills the pipe to a length of 3.4 m.

In 1 second, volume of water discharged

= volume of pipe of length 3.4 m

$= \pi r^2 h$

$= 3.142 \times [(5.2 \div 2) \div 100]^2 \times 3.4$

$= 0.007\ 221\ 572\ 8\ m^3$

Volume of cylindrical tank

$= 3.142 \times (2.3)^2 \times 1.6$

$= 26.593\ 888\ 8\ m^3$

Amount of time needed to fill the tank

$= 26.593\ 888\ 8 \div 0.007\ 221\ 572\ 8$

$= 3682.561\ 893$ seconds

= 61 minutes (to the nearest minute)

(c) Base area of trapezium

$= \dfrac{1}{2} \times (7 + 5) \times 2.5$

$= 15\ m^2$

In 1 hour, volume of water discharged

$= 15 \times (12 \times 1000)$

$= 180\ 000\ m^3$

In 1 second,

volume of water discharged

$= (180\ 000 \div 3600)$

$= 50\ m^3$

In 5 seconds, the volume of water discharged

$= 5 \times 50 = 250\ m^3$

(d) Since water is discharged through the pipe at a rate of 18 km/h, the volume of water discharged in 1 hour is the volume of water that fills the pipe to a length of 18 km = 18 000 m.

In 1 hour, volume of water discharged

= volume of pipe of length 18 km

$= 3.142 \times (4 \div 100)^2 \times 18\ 000$

$= 90.4896\ m^3$

In $1\dfrac{2}{3}$ hours, the volume of water discharged

$= 1\dfrac{2}{3} \times 90.4896$

$= 150.816\ m^3$

Volume of swimming pool = $50 \times 25 \times$ height h

$150.816 = 1250 \times h$

$\therefore h = 0.120\ 652\ 8$ m

$= 12.1$ cm (to 3 s.f.)

28. (a) Volume of rectangular cuboid

$= 0.40 \times 0.25 \times 0.08$

$= 0.008\ m^3$

Density of solid

$= \dfrac{\text{mass}}{\text{volume}}$

$= \dfrac{33.6}{0.008}$

$= 4200\ kg/m^3$

(b) Convert 10.5 kg to g.

$10.5\ kg = 10.5 \times 1000 = 10\ 500$ g

Volume of metal

$= \dfrac{10\ 500}{3.5}$

$= 3000\ cm^3$

$3000 = 4 \times 3 \times x$

$x = 250$

(c) Convert 22.44 kg to g.
22.44 kg = 22.44 × 1000 = 22 440 g
Volume of metal
$= \frac{22\,440}{13.6}$
= 1650 cm^3
Let the radius of the glass cylinder be x cm.
$1650 = 3.142 \times x^2 \times 21$
$x^2 = 25.0068$
$x = 5.000\,68$
Diameter of the glass cylinder
= 2 × 5.000 68
= 10.0 cm (to 3 s.f.)

(d) Convert 14.5 kg to g.
14.5 kg = 14.5 × 1000 = 14 500 g
Volume of metal
$= \frac{14\,500}{3.8}$
= 3815.789 474 cm^3 (to 6 d.p.)
Let the length of the rod be l cm.
$3815.789\,474 = 3.142 \times 6^2 \times l$
$\therefore l = 33.7346$
= 34 cm (to the nearest cm)

29. (i) Base area $= \frac{1}{2} \times (12 + 8) \times 7$
= 70 cm^2
Volume of block = base area × length
= 70 × 28
= 1960 cm^3

(ii) Total surface area of block
= 70 + 70 + (12 × 28) + (7 × 28) + (28 × 8.06)
+ (8 × 28)
= 70 + 70 + 336 + 196 + 225.68 + 224
= 1121.68 cm^2

(iii) Mass of the block = density × volume
= 1.12 × 1960
= 2195.2 g

30. (i) Total surface area of the solid block
$= (2 \times 3.142 \times 14^2)$
$+ (2 \times 3.142 \times 14 \times [1.2 \times 100])$
= 1231.664 + 10 557.12
= 11 788.784
= 11 800 cm^2 (to 3 s.f.)

(ii) Volume of block
$= 3.142 \times (14)^2 \times (1.2 \times 100)$
= 73 899.84
= 73 900 cm^3 (to 3 s.f.)

(iii) Convert 92.4 kg to g.
92.4 kg = 92.4 × 1000 = 92 400 g
Density of block
$= \frac{92\,400}{79\,899.84}$
= 1.250 34…
= 1.25 g/cm^3 (to 3 s.f.)

31. (i) Volume of box
= 48 × 36 × 15
= 25 920 cm^3
Number of items in box
= (48 ÷ 7) × (36 ÷ 7) × (15 ÷ 7)
≈ 6 × 5 × 2
= 60
Total volume of items = 60×7^3
= 20 580 cm^3
Volume of sawdust
= 25 920 – 20 580
= 5340 cm^3

(ii) Mass of sawdust = density × volume
= 0.75 × 5340
= 4005 g

32. (i) Total surface area of the cuboid
= 2[(30 × 25) + (30 × 15) + (25 × 15)]
= 2[750 + 450 + 375]
= 3150 cm^2

(ii) Volume of cuboid
= 30 × 25 × 15
= 11 250 cm^3
Volume of each coin
$= 3.142 \times 1.5^2 \times (2.4 \div 10)$
= 1.696 68 cm^3
Number of coins that can be made
= 11 250 ÷ 1.696 68
≈ 6630
(Note: The answer is not 6631 as the number of coins is 6630.6, which is less than 6631.)

(iii) Total volume of coins
= 6630 × 1.696 68
= 11 248.9884 cm^3
Volume of molten metal left behind
= 11 250 – 11 248.9884
= 1.0116
= 1.01 cm^3 (to 3 s.f.)

(iv) Mass of each coin
= density × volume
= 6.5 × 1.696 68
= 11.028 42 g
= 11.0 g (to 3 s.f.)

33. (i) Convert 3780 litres to m^3.

$3780\ l = (3780 \times 1000) \div 100 \div 100 \div 100$

$= 3.78\ m^3$

Let the depth of the liquid in the tank be d m.

$3.78 = 4.2 \times 1.8 \times d$

$\therefore d = 0.5$

The depth of the liquid in the tank is 0.5 m or 50 cm.

(ii) Volume of increase in liquid level

$= 420 \times 180 \times (1.6)$

$= 120\ 960\ cm^3$

Volume of one solid brick

$= \dfrac{120\,960}{380}$

$= 318.315\ 789\ 5$

$= 318\ cm^3$ (to 3 s.f.)

(iii) Mass of bricks

$= 1.8 \times 318.315\ 789\ 5 \times 380$

$= 217\ 728$ g

Mass of liquid

$= 1.2 \times 3\ 780\ 000$

$= 4\ 536\ 000$ g

Total mass

$= 4\ 536\ 000 + 217\ 728$

$= 4\ 753\ 728$ g

$= 4753.728$ kg

$= 4753.73$ kg (to 2 d.p.)

34. (i) Volume of open rectangular tank

$= 110 \times 60 \times 40$

$= 264\ 000\ cm^3$

Amount of liquid required to fill up the tank

$= \dfrac{3}{8} \times 264\ 000$

$= 99\ 000\ cm^3$

$= 99$ litres

(ii) Amount of time needed, in minutes, to fill up the tank

$= \dfrac{99}{5.5}$

$= 18$ minutes

(iii) Volume of liquid in tank, in m^3

$= 264\ 400 \div 100 \div 100 \div 100$

$= 0.264\ m^3$

Mass of liquid in the whole tank

= density × volume of liquid in tank

$= 800 \times 0.264$

$= 211.2$ kg

35. (i) Volume of closed container

= volume of cuboid + volume of half – cylinder

$= (28 \times 60 \times 40) + \left(\dfrac{1}{2} \times 3.142 \times \left(\dfrac{28}{2}\right)^2 \times 60\right)$

$= 67\ 200 + 18\ 474.96$

$= 85\ 674.96\ cm^3$

$= (85\ 674.96 \div 1000)$ litres

$= 85.7$ litres (to 3 s.f.)

(ii) Total surface area of the container

= surface area of the cuboid (without the top surface)

+ surface area of the half cylinder

$= 2 \times \left[28 \times 40 + \dfrac{1}{2} \times 3.142 \times 14^2\right]$

$+ 2(60 \times 40) + (28 \times 60)$

$+ \left(\dfrac{1}{2} \times 2 \times 3.142 \times 14 \times 60\right)$

$= 2[1120 + 307.916] + 4800 + 1680 + 2639.28$

$= 2855.832 + 4800 + 1680 + 2639.28$

$= 11\ 975.112\ cm^2$

$= 1.197\ 511\ 2\ m^2$

$= 1.20\ m^2$ (to 3 s.f.)

36. Volume of two cylindrical discs

$= 2\left[\pi \times \left(\dfrac{120}{2}\right)^2 \times 12\right]$

$= 86\ 400\pi\ cm^3$

Volume of the connecting cylinder of diameter 40 cm

$= \pi \times \left(\dfrac{40}{2}\right)^2 \times (94 - 12 - 12)$

$= 28\ 000\pi\ cm^3$

Total volume of drum (before the cylinder of diameter of 16 cm is removed)

$= (86\ 400 + 28\ 000)\pi$

$= 114\ 400\pi\ cm^3$

Volume of cylinder, of diameter 16 cm, removed from the drum

$= \pi \times \left(\dfrac{16}{2}\right)^2 \times 94$

$= 6016\pi\ cm^3$

Volume of wood used to make the drum

$= (114\ 400 - 6016)\pi$

$= 108\ 384\pi\ cm^3$

$= 108\ 000\pi\ cm^3$ (to 3 s.f.)

37. (i) Volume of cylindrical container

$= 3.142 \times (14)^2 \times 40$

$= 24\ 633.28$

$= 24\ 600\ cm^3$ (to 3 s.f.)

(ii) Surface area of one cylindrical container
$= (2 \times 3.142 \times 14 \times 40) + (2 \times 3.142 \times 14^2)$
$= 4750.704 \text{ cm}^2$
Surface area of 450 cylinders
$= 450 \times 4750.704$
$= 2\ 137\ 816.8 \text{ cm}^2$
4200 cm^2 surface requires 0.24 litres of paint.
$2\ 137\ 816.8 \text{ cm}^2$ requires 122.160 96 litres of paint.
123 litres of paint must be purchased to paint all 450 containers.
Cost to paint the containers
$= \$8.70 \times 123$
$= \$1070.10$

38. (i) Volume of the rectangular block
$= 12 \times 18 \times 10$
$= 2160 \text{ cm}^3$
Volume of cylinder removed from the block
$= 3.142 \times \left(\frac{7}{2}\right)^2 \times 18$
$= 692.811 \text{ cm}^3$
Volume of remaining solid
$= 2160 - 692.811$
$= 1467.189$
$= 1470 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of the remaining solid
$= 2\left[12 \times 10 - \left(3.142 \times \left(\frac{7}{2}\right)^2\right)\right]$
$\quad + 2[(18 \times 12) + (18 \times 10)]$
$= 2[120 - 38.4895] + 2[396]$
$= 955.021$
$= 955 \text{ cm}^2$ (to 3 s.f.)

Advanced

39. (i) Let the radius of cylinder A be r cm and the height be h cm.
Volume of cylinder A
$= \pi r^2 h$
$= 343 \text{ cm}^3$
Then the radius of cylinder B will be $\frac{1}{3}r$ cm and the height will be h cm.
Volume of cylinder B
$= \pi \times \left(\frac{1}{3}r\right)^2 \times h$
$= \pi \times \frac{1}{9} \times r^2 \times h$
$= \frac{1}{9} \times \pi r^2 h$
$= \frac{1}{9} \times 343$
$= 38.111$
$= 38.1 \text{ cm}^3$ (to 3 s.f.)

(ii) Number of cubes formed
$= \frac{38.111}{2^3}$
$= 4.763\ldots$
The maximum number of cubes formed is 4.

40. Total thickness of the paper towel after it is being rolled
$= (1 \div 10) \times 90$
$= 9 \text{ cm}$
Total radius of the paper towel and roll
$= 9 + 2.5$
$= 11.5 \text{ cm}$
Base area of the paper towel, in the form of a cylinder
$= \frac{22}{7} \times [(11.5)^2 - (2.5)^2]$
$= 396 \text{ cm}^2$
Volume of paper towel
= base area × width of towel
$= 396 \times 14$
$= 5544$
$= 5540 \text{ cm}^3$ (to 3 s.f.)

New Trend

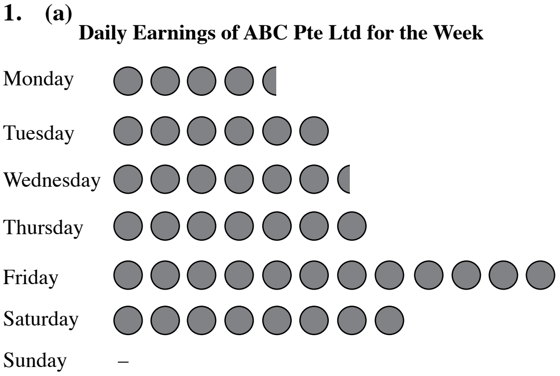
41. Surface area of cross-section
$= \pi\left(\frac{1.3}{2}\right)^2 + \frac{1}{2} \times 2.6 \times 2.25 - \pi\left(\frac{0.5}{2}\right)^2$
$= 0.4225\pi + 2.925 - 0.0625\pi$
$= (0.36\pi + 2.925) \text{ cm}^2$
Volume of platinum $= 0.2(0.36\pi + 2.925)$
$= (0.072\pi + 0.585) \text{ cm}^3$
Price of platinum used $= 21.5(0.072\pi + 0.585) \times \$43.48$
$= \$758.3210$ (to 4 d.p.)
Total value of pendant $= \$4200 + \$758.3210$
$= \$4958.32$ (to the nearest cent)

42. Convert 100 litres to cm^3.
$100\ l = 100 \times 1000 = 100\ 000 \text{ cm}^3$
Volume of tank = cross-sectional area × height
Height = volume of tank ÷ cross-sectional area
$= 100\ 000 \div \pi(30)^2$
$= 35.4 \text{ cm}$ (to 3 s.f.)

Chapter 15 Statistical Data Handling

Basic

1. **(a)**



Each circle represents $1000.

(b) Total earnings for the week

= 4500 + 6000 + 6500 + 7000 + 12 000 + 8000

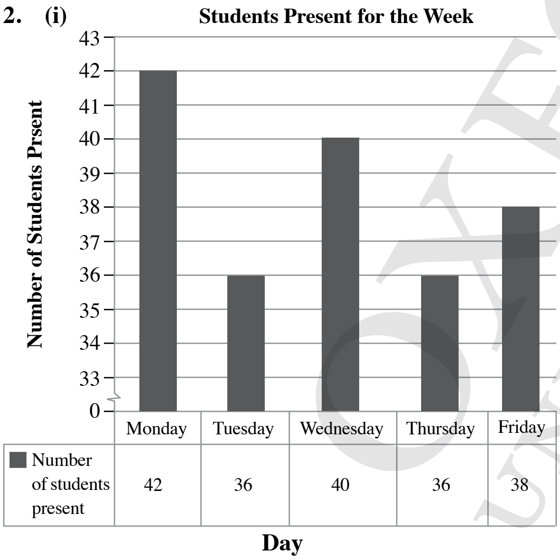
= $44 000

Percentage of Friday's earning to the total earnings for the week

$= \frac{12\ 000}{44\ 000} \times 100\%$

$= 27\frac{3}{11}\ \%$

2. **(i)**



(ii) All students were present on Monday.

(iii) Number of absentees on Friday = 42 – 38

= 4

Percentage of absentees on Friday

$= \frac{4}{42} \times 100\%$

= 9.52% (to 3 s.f.)

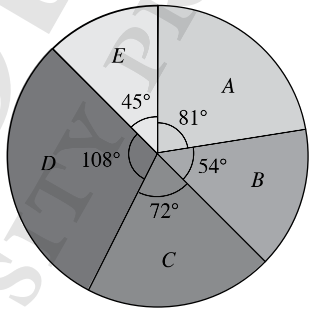
(iv) Ethan is right to say that because on Monday, everyone is present. So, if student *A* is absent from Tuesday to Friday, he is still present at least once in that week and not absent for the whole week.

3. **(a)** Total number of foreign countries

= 9 + 6 + 8 + 12 + 5 = 40

Number of foreign countries	Angle of sector
A	$\frac{9}{40} \times 360° = 81°$
B	$\frac{6}{40} \times 360° = 54°$
C	$\frac{8}{40} \times 360° = 72°$
D	$\frac{12}{40} \times 360° = 108°$
E	$\frac{5}{40} \times 360° = 45°$

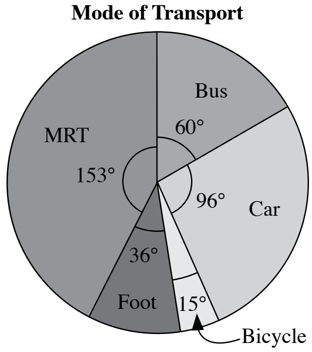
Number of Foreign Countries



(b) Total number of students surveyed

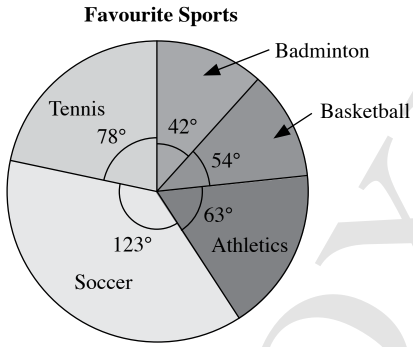
= 40 + 64 + 10 + 24 + 102 = 240

Mode of Transport	Angle of sector
Bus	$\frac{40}{240} \times 360° = 60°$
Car	$\frac{64}{240} \times 360° = 96°$
Bicycle	$\frac{10}{240} \times 360° = 15°$
Foot	$\frac{24}{240} \times 360° = 36°$
MRT	$\frac{102}{240} \times 360° = 153°$



(c)

Sports	Angle of sector
Badminton	$\frac{70}{600} \times 360° = 42°$
Basketball	$\frac{90}{600} \times 360° = 54°$
Athletics	$\frac{105}{600} \times 360° = 63°$
Soccer	$\frac{205}{600} \times 360° = 123°$
Tennis	$\frac{130}{600} \times 360° = 78°$



4. Total number of students in the school

$= 30 + 20 + 10 + 20$

$= 80$

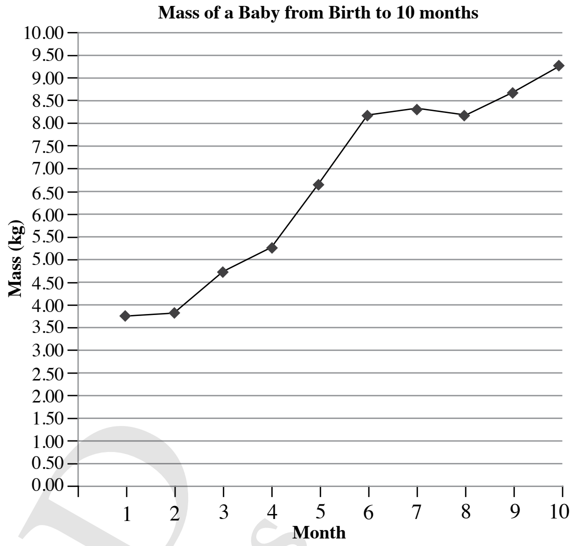
Angle of the smallest sector

$= \frac{10}{80} \times 360°$

$= 45°$

It represents the number of Secondary 3 students in a school for the year 2013.

5. (i)



(ii) From the line graph, the increase in the mass of the baby is the largest between the 5th and 6th months.

(iii) From the line graph, the first decrease in the mass is on the 8th month.

(iv) Total mass of the baby from birth to 10 months

$= 3.7 + 3.8 + 4.7 + 5.4 + 6.6 + 8.2 + 8.3 + 8.2 + 8.7 + 9.4$

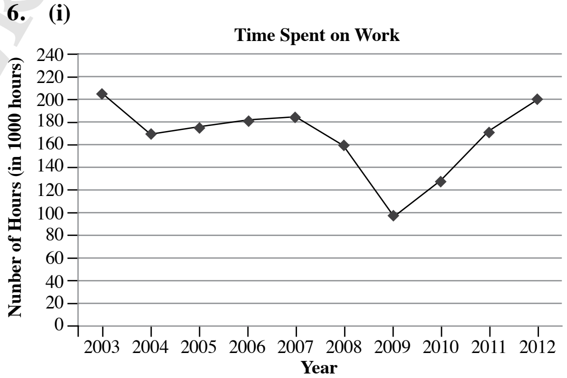
$= 67$ kg

Average mass of the baby

$= \frac{67}{10}$

$= 6.7$ kg

6. (i)



(ii) The years in which there was a decrease in the number of hours the workers spent in work are 2004, 2008 and 2009.

(iii) The years in which there was an increase in the number of hours the workers spent in work are 2005, 2006, 2007, 2010, 2011 and 2012.

(iv) From the line graph, the year in which the increase is the largest is 2011 and the year in which the increase is the least is 2007.

(v) The possible years in which the workers spent more than 172 000 hours in work are 2003, 2005, 2006, 2007 and 2012.

Intermediate

7. **(i)** **(a)** Number of cars produced on Tuesday
$= 6.5 \times 20$
$= 130$

(b) Number of cars produced on Thursday
$= 5 \times 20$
$= 100$

(c) Number of cars produced on Saturday
$= 0 \times 20 = 0$

(ii) The greatest number of cars produced was on Tuesday.

(iii) Production line has stopped for half a day on Wednesday. One possible indication is that the number of cars produced is low as compared to the other days. The number of cars produced on Wednesday is approximately half the number of cars produced on Monday and on Thursday.

(iv) Increase in production of cars from Monday to Tuesday
$= 130 - 80$
$= 50$
Percentage increase
$= \frac{50}{80} \times 100\%$
$= 62.5\%$

(v) One possible explanation may be the workers are resting on weekends. The other reason may be there may not be orders on weekends and the number of cars produced on Friday may be sufficient to meet the demands for the coming week.

(vi) Total number of cars produced
$= 80 + 130 + 50 + 100 + 120$
$= 480$

8. **(a)** **(i)** Number of students in the class
$= 6 + 7 + 10 + 8 + 4 + 3 + 3$
$= 41$

(ii) Most students have 2 coins.

(iii) Total number of coins
$= 7 \times 1 + 10 \times 2 + 8 \times 3 + 4 \times 4 + 3 \times 5 + 3 \times 6$
$= 7 + 20 + 24 + 16 + 15 + 18$
$= 100$
Average number of coins
$= \frac{100}{41}$
$= 2.44$ (to 3 s.f.)

(iv) Number of students having 4 or more coins
$= 4 + 3 + 3$
$= 10$
Percentage of students having 4 or more coins
$= \frac{10}{41} \times 100\%$
$= 24.4\%$ (to 3 s.f.)

(b) Angle representing students having 0 coins
$= \frac{6}{41} \times 360°$
$= 52.7°$
Angle representing students having 1 coin
$= \frac{7}{41} \times 360°$
$= 61.5°$
Angle representing students having 2 coins
$= \frac{10}{41} \times 360°$
$= 87.8°$
Angle representing students having 3 coins
$= \frac{8}{41} \times 360°$
$= 70.2°$
Angle representing students having 4 or more coins
$= \frac{10}{41} \times 360°$
$= 87.8°$

9. **(a)** **(i)** February
(ii) June
(iii) August

(b) The month in which he is the heaviest is in the month of November. His weight is about 54 kg.

(c) The months in which his weights were the same are May, October and December.

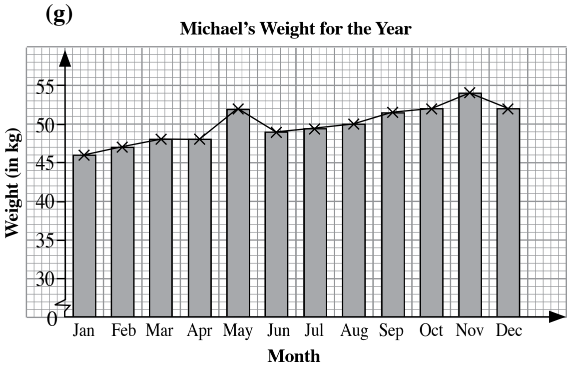
(d) His largest weight = 54 kg
His smallest weight = 46 kg
Range of weight = 54 – 46
= 8 kg

(e) **(i)** On 1st June, he lost weight greatly after his weight increased for the past 5 months. Therefore, he was sick in May.

(ii) On 1st December, he lost weight slightly after his weight increased for the past 5 months. Therefore, he was controlling his diet in November.

(f) November

(g)



(h) Line graph is more suitable to represent and interpret the above data as we can observe the trends of his weight over the months easily.

We can observe the increase or decrease of his weight easily from the line graph.

10. (a) (i) The value of sales in 2007 is $64 \times \$10\,000$
$= \$640\,000$.

(ii) The value of sales in 2009 is $110 \times \$10\,000$
$= \$1\,100\,000$.

(iii) The value of sales in 2011 is $140 \times \$10\,000$
$= \$1\,400\,000$.

(b) The value of sales is \$1 000 000 in 2008.

(c) Between 2009 and 2010, the increase in the value of sales is the greatest.

The maximum value of sales (from 2009 to 2010)
$= (160 \times \$10\,000) - \$1\,100\,000$
$= \$500\,000$

(d) Amount exceeded the sales target
$= \$1\,600\,000 - \$1\,300\,000$
$= \$300\,000$

Percentage of amount exceeded the target

$$= \frac{300\,000}{1\,300\,000} \times 100\%$$

$$= 23\frac{1}{13}\%$$

(e) Amount below the sales target
$= \$1\,650\,000 - \$1\,400\,000$
$= \$250\,000$

Percentage of amount below the target

$$= \frac{250\,000}{1\,650\,000} \times 100\%$$

$$= 15\frac{5}{33}\%$$

(f) Total value of sales over the past 6 years
$= (64 + 100 + 110 + 160 + 140 + 50) \times \$10\,000$
$= \$6\,240\,000$

(g) The sudden increase may be due to the increase in the popularity of the product. Another reason may be the population in the country has increased over the past year and the demand for the product increases as it is a necessity.

11. (i)

$$5x° + 2x° + 52° = 360°$$
$$7x° + 52° = 360°$$
$$7x° = 308°$$
$$x° = 44°$$
$$\therefore x = 44$$

(ii) $2 \times 44° = 88°$ represents 66 vehicles
1° represents 0.75 vehicles
360° represents $0.75 \times 360 = 270$ vehicles
The total number of vehicles included in the survey is 270.

12. (i) When it rained the whole day, the average temperature should be the lowest among the 10 days. In this case, the day in which it rained the whole day is Monday during the 1st week and its temperature is 24°C.

(ii) Friday, the 1st week; the temperature in the classroom on that day is 31°C.

(iii) The days when the temperature is below 29°C are 1st week on Monday, Tuesday and Wednesday and 2nd week on Friday.

(iv) Number of days in which the temperature is above 28°C
$= 6$

Percentage of days in which the temperature is above 28°C

$$= \frac{6}{10} \times 100\%$$

$$= 60\%$$

(v) The sudden increase in temperature may be due to a change in weather. Another reason may be the monsoon season has ended and the temperature has resumed to its initial temperature before the monsoon season.

13. **(i)** The sale first exceeds the 50 000 mark in year 2010.

(ii) In year 2012, the sale was exactly 100 000.

(iii) Between 2011 and 2012, the sales in the soap powder were the greatest.

(iv)

Year	2008	2009	2010	2011	2012
Number of Packets (in thousands)	40	40	60	70	100

(v) Increase in sales from 2010 to 2012

= 100 000 – 60 000

= 40 000

Percentage increase in sales from 2010 to 2012

$= \frac{40\ 000}{60\ 000} \times 100\%$

$= 66\frac{2}{3}\%$

Advanced

14. Yes.

Suggested answer:

The increase in the size of the diagram does not represent accurately that the sales have increased by 300%. What the advertisement is trying to show is that there is an increase in the sales but it is unable to represent the increase as 300%.

15. No.

Suggested answer:

The charts did show an increase in the radius of the circle by two times. However, the actual figures of the sales are not given. Therefore, it is not conclusive that the sales have doubled from the year 2010 to 2012.

Suggestion: A better representation is a bar graph which compares the sales in 2010 and 2012 using bars.

16. No. I do not agree with Amirah. The person who collected the data did not mention whether taking more projects of the same nature contributes to people involved in more community work.

Reason 1: More people may have increased their involvement from May to June by taking part in more projects within the same organisation. Therefore, the nature of the projects may not have changed but the number of projects involved has increased.

Reason 2: There may be a higher chance of people involving in community work due to demand for more volunteers as part of the school's holiday programmes.

New Trend

17. **(a)** Number of females who use public transport

= 55 – 20 – 2 – 9

= 24

(b) Angle representing students walking to school

$= \frac{16}{120} \times 360°$

= 48°

(c) For males,

percentage who travel using other modes of transport

$= \frac{12}{65} \times 100\%$

= 18.462% (to 5 s.f.)

For females,

percentage who travel using other modes of transport

$= \frac{9}{55} \times 100\%$

= 16.364% (to 5 s.f.)

Difference in percentage = 18.462% – 16.364%

= 2.10%

A greater percentage travel using other modes of transport in males as compared to females. The percentage in males is 2.10% higher.

18. **(a)** Ratio of manufactured goods and the minerals

$= \frac{85}{115}$

$= \frac{17}{23}$

= 17 : 23

(b) Angle representing agricultural produce

= 360° – 10° – 85° – 115°

= 150°

Ratio of agricultural produce and the manufactured goods

$= \frac{150}{85}$

$= \frac{30}{17}$

= 30 : 17

(c) 115° represent 23 million

1° represents 0.2 million

360° represent 72 million

The total value of exports of the country is 2012 is 72 million.

Revision Test D1

1. **(i)** Length of $OA = 28 - 11.2 = 16.8$ cm

Circumference of semicircle with diameter 28 cm

$= \frac{1}{2} \times 2 \times 3.142 \times \left(\frac{28}{2}\right)$

$= 43.988$ cm

Circumference of semicircle with diameter 11.2 cm

$= \frac{1}{2} \times 2 \times 3.142 \times \left(\frac{11.2}{2}\right)$

$= 17.5952$ cm

Perimeter of figure

$= 43.988 + 17.5952 + 16.8$

$= 78.3832$

$= 78.4$ cm (to 3 s.f.)

(ii) Area of semicircle with diameter 28 cm

$= \frac{1}{2} \times 3.142 \times \left(\frac{28}{2}\right)^2$

$= 307.916$ cm^2

Area of semicircle with diameter 11.2 cm

$= \frac{1}{2} \times 3.142 \times \left(\frac{11.2}{2}\right)^2$

$= 49.266\ 56$ cm^2

Area of shaded region

$= 307.916 - 49.266\ 56$

$= 258.649\ 44$

$= 259$ cm^2 (to 3 s.f.)

2. Length of $DC = 4.7 - 2.3 = 2.4$ cm

Area of $\triangle BDC = \frac{1}{2} \times DC \times BD$

$2.64 = \frac{1}{2} \times 2.4 \times BD$

$2.64 = 1.2 \times BD$

$BD = 2.2$ cm

Area of trapezium $= \frac{1}{2} \times (2.3 + 4.7) \times 2.2$

$= 7.7$ cm^2

3. Base area $= (140 \times 90) - [(140 - 30 - 30) \times 60]$

$= 12\ 600 - 4800$

$= 7800$ cm^2

Volume of solid

= base area × width

$= 7800 \times 50$

$= 390\ 000$ cm^3

Total surface area

$= 2(7800) + 2(50 \times 90) + 2(30 \times 50) + (140 \times 50) + 2(60 \times 50) + (80 \times 50)$

$= 15\ 600 + 9000 + 3000 + 7000 + 6000 + 4000$

$= 44\ 600$ cm^2

4. **(i)** Volume of water in the cylindrical tank

$= 3.142 \times \left(\frac{140}{2}\right)^2 \times 60$

$= 923\ 748$

$= 924\ 000$ cm^3 (to 3 s.f.)

(ii) Let the height of water in the trough be h cm.

Volume of water in the cylindrical tank

= volume of water in trough

$923\ 748 = 120 \times 80 \times h$

$h = 96.223\ 75$

$= 96.2$ cm (to 3 s.f.)

5. **(a)** Volume of cylindrical tank

$= 3.142 \times \left(\frac{350}{2}\right)^2 \times 260$

$= 25\ 018\ 175$ cm^3

$= 25\ 018.175\ l$

Time taken for the pipe to fill the tank

$= \frac{25\ 018.175}{25}$

$= 1000.727$

$= 1001$ seconds

(to the nearest second)

(b) Volume of pipe

$= \left(3.142 \times \left(\frac{6}{2}\right)^2 \times 120\right) - \left(3.142 \times \left(\frac{4.8}{2}\right)^2 \times 120\right)$

$= 3393.36 - 2171.7504$

$= 1221.6096$ cm^3

Mass of pipe

= density × volume

$= 7.6 \times 1221.6096$

$= 9284.232\ 96$ g

$= 9.28$ kg (to 3 s.f.)

6. **(a)** $5x° + 2x° + 255° = 360°$

$7x° = 360° - 255°$

$7x° = 105°$

$x° = 15°$

$\therefore x = 15$

(b) 255° represent 153 cars.

1° represents 0.6 cars.

$5 \times 15° = 75°$ represent $0.6 \times 75° = 45$.

There are 45 motorcycles in the car park.

Revision Test D2

1. (a) Area of $\triangle AKC$

$= \frac{1}{2} \times 6.8 \times 5.6$

$= 19.04 \text{ cm}^2$

Area of $\triangle AKB$

$= \frac{1}{2} \times 6.8 \times 6.4$

$= 21.76 \text{ cm}^2$

Area of shaded region

$= 19.04 + 21.76$

$= 40.8 \text{ cm}^2$

(b) Area of semicircle with diameter 24 cm

$= \frac{1}{2} \times 3.142 \times \left(\frac{24}{2}\right)^2$

$= 226.224 \text{ cm}^2$

Area of $\triangle ABC$

$= \frac{1}{2} \times 24 \times 8$

$= 96 \text{ cm}^2$

Area of shaded region

$= 226.224 - 96$

$= 130.224$

$= 130 \text{ cm}^2$ (to 3 s.f.)

2. Let the length of the cube be l cm.

Volume of solid cube $= l^3$

$\therefore l^3 = 125$

$l = 5$

Total surface area of cube

$= 6l^2$

$= 6 \times 5^2$

$= 150 \text{ cm}^2$

3. (i) Volume of the cake before it is being cut

$= 3.142 \times (14)^2 \times 8$

$= 4926.656 \text{ cm}^3$

Volume of remaining cake

$= \frac{3}{4} \times 4926.656$

$= 3694.992$

$= 3690 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of the cake after it is being cut

$= (2 \times \frac{3}{4} \times 3.142 \times (14)^2)$

$+ (2 \times \frac{3}{4} \times 3.142 \times 14 \times 8) + 2(14 \times 8)$

$= 923.748 + 527.856 + 224$

$= 1675.604$

$= 1680 \text{ cm}^2$ (to 3 s.f.)

4. Volume of water in the cylindrical container

$= \frac{11}{14} \times \left[3.142 \times \left(\frac{28}{2}\right)^2 \times 35\right]$

$= 16\,935.38 \text{ cm}^3$

Number of glasses of water

$= \frac{16\,935.38}{245}$

$= 69.124$

≈ 69

Volume of water $= 69 \times 245$

$= 16\,905 \text{ cm}^3$

Volume of water left in the container

$= 16\,935.38 - 16\,905$

$= 30.38$

$= 30.4 \text{ cm}^3$ (to 3 s.f.)

5. Since water is discharged through the pipe at a rate of 8 km/h, the volume of water discharged is the volume of water that fills the pipe to a length of 8 km or 8000 m.

In 1 hour, volume of water discharged

= volume of pipe of length 8000 m

$= \pi r^2 h$

$= 3.142 \times [2.4 \div 100]^2 \times 8000$

$= 14.478\,336 \text{ m}^3$

Volume of rectangular tank

$= 4 \times 3 \times 2.8$

$= 33.6 \text{ m}^3$

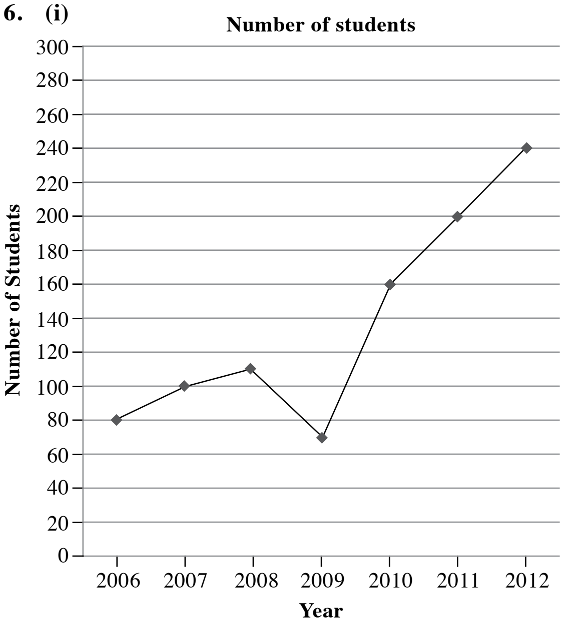
Amount of time needed to fill the tank

$= 33.6 \div 14.478\,336$

$= 2.3208$ hours

$= 139$ minutes (to the nearest minute)

6. (i)



(ii) Between 2009 and 2010, the school has the greatest increase in the number of students taking additional mathematics.

(iii) Total number of students

= 80 + 100 + 110 + 70 + 160 + 200 + 240

= 960

Angle of sector that represents the number of students taking additional mathematics in 2009

$= \frac{70}{960} \times 360°$

= 26.25

= 26.3° (to 3 s.f.)

(iv) One possible reason for the sudden increase may be the school has increased the number of classes taking additional mathematics from 2.5 classes to about 4 classes. This may be due to a change in the expectation by the Ministry of Education or an increase in demand requested by parents.

End-of-Year Examination Specimen Paper A

Part I

1. **(a)** 37 850 = 38 000 (to 2 s.f.)

(b) 1.3249 = 1.32 (to 2 d.p.)

2. **(a)** Convert 47.56 cm to mm.

47.56 cm = 47.56 × 10 = 475.6 mm

475.6 mm = 476 mm (to the nearest mm)

(b) 75 489 cm^2 = 75 500 cm^2 (to the nearest 100 cm^2)

3. **(a)** $\{[(56 + 34) \div 5 - 7] \times 3 - 17\} \div 4$

$= \{[90 \div 5 - 7] \times 3 - 17\} \div 4$

$= \{[18 - 7] \times 3 - 17\} \div 4$

$= \{11 \times 3 - 17\} \div 4$

$= \{33 - 17\} \div 4$

$= 16 \div 4$

$= 4$

(b) $(-2)^2 - (7 - 8)^2 - (11 - 15)^3$

$= 4 - (-1)^2 - (-4)^3$

$= 4 - 1 - (-64)$

$= 4 - 1 + 64$

$= 67$

4. **(a)** $3\frac{1}{4} + 1\frac{1}{4} \div \frac{3}{8}$

$= 3\frac{1}{4} + \frac{5}{4} \div \frac{3}{8}$

$= 3\frac{1}{4} + \frac{5}{4} \times \frac{8}{3}$

$= 3\frac{1}{4} + 3\frac{1}{3}$

$= 6\frac{7}{12}$

(b) $5\frac{5}{8} - \left(\frac{2}{3}\right)^3 \div \sqrt{\frac{9}{16}}$

$= 5\frac{5}{8} - \frac{8}{27} \div \frac{3}{4}$

$= 5\frac{5}{8} - \frac{8}{27} \times \frac{4}{3}$

$= 5\frac{5}{8} - \frac{32}{81} = 5\frac{149}{648}$

5.

$12 = \boxed{2^2} \times 3$

$28 = \boxed{2^2} \times 7$

$64 = \boxed{2^6}$

$\downarrow$

2^2

HCF of 12, 28 and 64 = $2^2 = 4$

$12 = \boxed{2^2} \times 3$

$28 = \boxed{2^2} \times 7$

$64 = \boxed{2^6}$

$\downarrow \quad \downarrow \quad \downarrow$

$2^6 \quad 3 \quad 7$

LCM of 12, 28 and 64 = $2^6 \times 3 \times 7 = 1344$

The HCF and LCM of 12, 28 and 64 are 4 and 1344 respectively.

6. **(a)** $2x - 2[3(1 - x) - 5(x - 2y)]$

$= 2x - 2[3 - 3x - 5x + 10y]$

$= 2x - 2[3 - 8x + 10y]$

$= 2x - 6 + 16x - 20y$

$= 2x + 16x - 20y - 6$

$= 18x - 20y - 6$

(b) **(i)** $15hx + 10hy - 5h$

$= 5h(3x + 2y - 1)$

(ii) $3p(x + h) + p(2h - 1)$

$= p[3(x + h) + (2h - 1)]$

$= p(3x + 5h - 1)$

7. **(a)** When $a = 2$, $b = 3$ and $c = -1$,

$\frac{5(2) - (-1)}{3 - (-1)^2} + \frac{3 + (-1)^3}{2^3 - 2(3)}$

$= \frac{10 + 1}{2} + \frac{3 - 1}{8 - 6}$

$= 5\frac{1}{2} + 1$

$= 6\frac{1}{2}$

(b)

$$\frac{3x - 2}{4} = \frac{x + 2}{5} - \frac{2x - 1}{10}$$

$$20 \times \frac{3x - 2}{4} = \left(\frac{x + 2}{5} - \frac{2x - 1}{10}\right) \times 20$$

$$5(3x - 2) = 4(x + 2) - 2(2x - 1)$$

$$15x - 10 = 4x + 8 - 4x + 2$$

$$15x - 10 = 4x - 4x + 8 + 2$$

$$15x = 8 + 2 + 10$$

$$15x = 20$$

$$x = 1\frac{1}{3}$$

8. **(a)** $\dfrac{63.2 \times 2.8}{5.53} + \dfrac{2.826}{0.9 \times 1.57}$

$\approx \dfrac{63 \times 3}{6} + \dfrac{3}{1 \times 2}$

$= 31.5 + 1.5$

$= 33$

$= 30$ (to 1 s.f.)

(b) Decrease $= \$38.50 - \$30.80$

$= \$7.70$

Percentage decrease $= \dfrac{7.70}{38.50} \times 100\%$

$= 20\%$

9. **(a)** Sum of interior angles in a n-sided polygon

$= (n - 2) \times 180°$

$153° + 144° + 100° + [(n - 3) \times 149°]$

$= (n - 2) \times 180°$

$397° + 149n° - 447° = 180n° - 360°$

$180n° - 149n° = 397° - 447° + 360°$

$31n° = 310°$

$n° = 10°$

$\therefore n = 10$

(b) Area of circle $= \pi r^2$

$154 = 3.142 \times r^2$

$r = 7.00$ cm (to 3 s.f.)

(c) Let the length of the cube be l cm.

Surface area of cube $= 6l^2$

$150 = 6l^2$

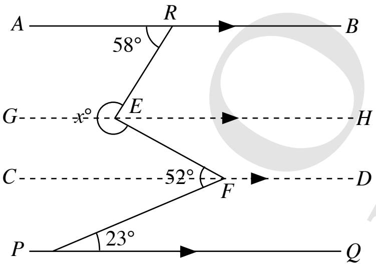
$\therefore l = 5$

Volume of a cube $= l^3$

$= 5^3$

$= 125 \text{ cm}^3$

10. **(a)** Draw a line CD through F that is parallel to AB and PQ.



$\angle PFC = 23°$ (alt. $\angle$s, CD // PQ)

$\angle CFE = 52 - 23 = 29°$

Draw a line GH through E that is parallel to AB and PQ.

$\angle GEF = 180° - 29° = 151°$ (int. $\angle$s, GH // CD)

$\angle GER = 180° - 58° = 122°$ (int. $\angle$s, GH // AB)

$x° = 151° + 122°$

$= 273°$

$\therefore x = 273$

(b) One semicircle and two quarters of diameter 14 cm are removed means a circle of diameter 14 cm is removed.

Area of square $= 14 \times 14$

$= 196 \text{ cm}^2$

Area of circle with diameter 14 cm

$= 3.142 \times \left(\dfrac{14}{2}\right)^2$

$= 153.958 \text{ cm}^2$

Area of shaded region

$= 196 - 153.958$

$= 42.042$

$= 42.0 \text{ cm}^2$ (to 3 s.f.)

11. **(i)** $5x° + 81° + 2x° + 55° = 360°$

$5x + 2x = 360 - 81 - 55$

$7x = 224$

$x = 32$ (shown)

(ii) Percentage of students who chose Science

$= \dfrac{81}{360} \times 100\%$

$= 22.5\%$

(iii) $5 \times 32 = 160°$ represent 48 students.

1° represents 0.3.

360° represent $0.3 \times 360 = 108$.

The total number of students in the group is 108.

Part II

Section A

1. **(a)** $5x > 16$

$x > \frac{16}{5}$

$x > 3\frac{1}{5}$

(b) 1 km requires $\frac{1}{12.4} = \frac{5}{62}$ litres.

300 km require $\frac{5}{62} \times 300 = 24\frac{6}{31}$ litres.

The minimum number of petrol required is 25 litres (to the nearest whole number).

(Note: The answer is not 24 litres as it requires slightly more than 24 litres of petrol to run 300 km.)

2. **(a)** $7x - \{3x - 4(2x - y) - [(5x - 3y) - (2y - 3x)]\}$

$= 7x - \{3x - 4(2x - y) - [5x - 3y - 2y + 3x]\}$

$= 7x - \{3x - 4(2x - y) - [5x + 3x - 3y - 2y]\}$

$= 7x - \{3x - 4(2x - y) - [8x - 5y]\}$

$= 7x - \{3x - 8x + 4y - 8x + 5y\}$

$= 7x - \{3x - 8x - 8x + 4y + 5y\}$

$= 7x - \{-13x + 9y\}$

$= 7x + 13x - 9y$

$= 20x - 9y$

(b)

$$\frac{1}{4 - x} + 3 = \frac{3}{x - 4}$$

$$\left(-\frac{1}{x - 4}\right) + 3 = \frac{3}{x - 4}$$

$$\frac{3}{x - 4} + \frac{1}{x - 4} = 3$$

$$\frac{4}{x - 4} = \frac{3}{1}$$

$$3(x - 4) = 4$$

$$3x - 12 = 4$$

$$3x = 4 + 12$$

$$3x = 16$$

$$x = 5\frac{1}{3}$$

(c) Let the breadth of the rectangle be x cm.

Then the length of the rectangle will be $2x$ cm.

$2(2x + x) = 24$

$2(3x) = 24$

$6x = 24$

$x = 4$

Then the length of the rectangle is 4 cm and its breadth is 8 cm.

Area of rectangle $= 4 \times 8$

$= 32\ \text{cm}^2$

3. **(a)** $0.086 = 0.086 \times 100\%$

$= 8.6\%$

(b) $42\frac{1}{8}\ \% = \frac{42\frac{1}{8}}{100}$

$= 0.42\ 125$

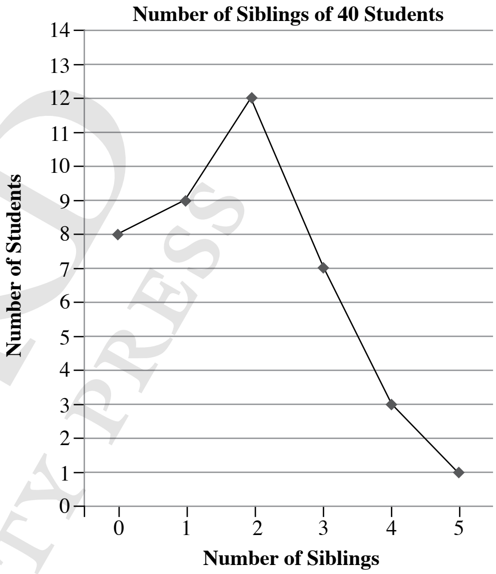
(c) $\frac{60}{100} \times 6.8\ \text{m} = 4.08\ \text{m}$

4. **(i)** $8 + 9 + x + 7 + 3 + 1 = 40$

$\therefore x = 40 - 8 - 9 - 7 - 3 - 1$

$= 12$

(ii)



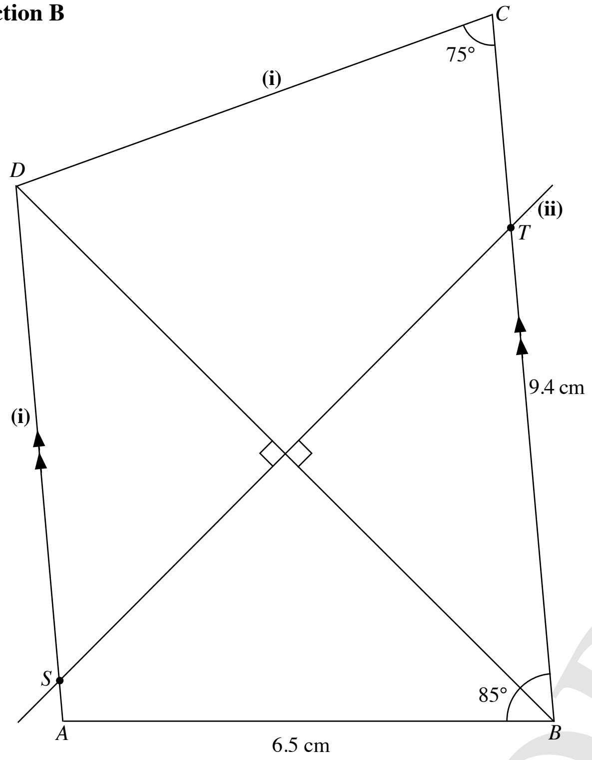
(iii) Angle representing students with 1 sibling

$= \frac{9}{40} \times 360°$

$= 81°$

Section B

5.

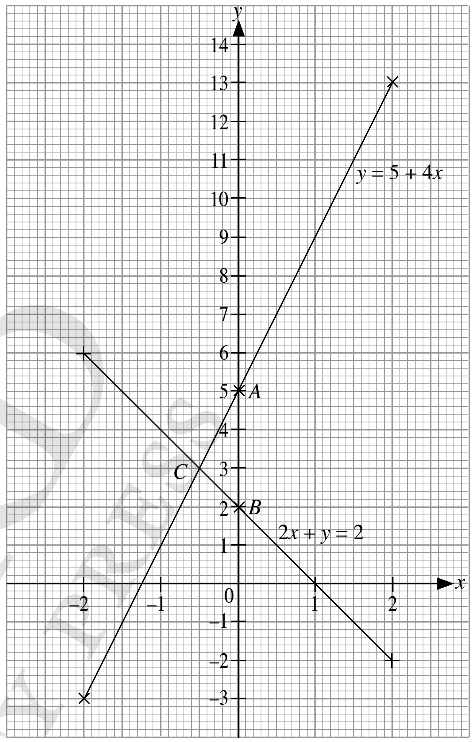


(i) Length of AD = 7.1 cm
Length of CD = 6.7 cm

(ii) Length of ST = 8.5 cm

6. **(i)**

x	–2	0	2
$y = 5 + 4x$	–3	5	13
$y = -2x + 2$	6	2	–2



(ii) From the graph, the coordinates of A, B and C are $(0, 5)$, $(0, 2)$ and $\left(-\frac{1}{2}, 3\right)$.

Area of $\triangle ABC$

$= \frac{1}{2} \times (5 - 2) \times \frac{1}{2}$

$= \frac{3}{4}$ square units

7. **(a)** Time taken for the first part of the journey

$= \frac{240}{60}$

$= 4$ hours

Time taken for the second part of the journey

$= 8\frac{4}{5} - 4$

$= 4\frac{4}{5}$ hours

Distance for the second part of the journey

$= 600 - 240$

$= 360$ km

Average speed for the second part of the journey

$= \frac{360}{4\frac{4}{5}}$

$= 75$ km/h

(b) **(i)** $\frac{1}{5} - \frac{1}{6} = \frac{1}{30}$

i.e. $30 = 5 \times 6$

(ii) $182 = x(x + 1)$

Since $13 \times 13 = 169$, we can try 13×14.

$13 \times 14 = 182$

$\therefore x = 13$

8. **(a)** **(i)** Convert 1.2 mm to cm.

1.2 mm $= 1.2 \div 10 = 0.12$ cm

Convert 200 m to cm.

200 m $= 200 \times 100 = 20\ 000$ cm

Volume of copper wire

$= \pi r^2 h$

$= 3.142 \times (0.12 \div 2)^2 \times 20\ 000$

$= 226.224$

$= 226$ cm^3 (to 3 s.f.)

(ii) Mass of wire

= density × volume of wire

$= 8.9 \times 226.224$

$= 2013.3936$

$= 2010$ g (to 3 s.f.)

(b) Area of trapezium $= \frac{1}{2}[(2x - 3) + (3x + 4)] \times 12$

$216 = \frac{1}{2}[2x - 3 + 3x + 4] \times 12$

$216 = \frac{1}{2}[2x + 3x - 3 + 4] \times 12$

$216 = 6[5x + 1]$

$30x + 6 = 216$

$30x = 216 - 6$

$30x = 210$

$x = 7$

(c) Volume of water in cylinder B

$= \pi \times [(3x) \div 2]^2 \times 20$

$= 45\pi x^2$ cm^3

Let the height of water in cylinder A be h cm.

Volume of water in cylinder A

$= \pi \times [(5x) \div 2]^2 \times h$

$45\pi x^2 = 6.25\pi x^2 h$

$h = \frac{45\pi x^2}{6.25\pi x^2}$

$= 7.2$

$\therefore$ Height of water in cylinder A is 7.2 cm.

End-of-Year Examination Specimen Paper B

Part I

1. **(a)** $4.002\ 56 = 4.00$ (to 2 d.p.)

(b) $0.002\ 045\ 6 = 0.00\ 205$ (to 3 s.f.)

(c) $10.0245 \text{ cm}^2 = 10.025 \text{ cm}^2$

$\left(\text{to the nearest } \frac{1}{1000} \text{ cm}^2\right)$

2. **(a)** $(11 - 7)^2 - 7^2 - (28 - 33)^3$

$= (4)^2 - 49 - (-5)^3$

$= 16 - 49 - (-125)$

$= 16 - 49 + 125$

$= 92$

(b) $21 + (-65) \div 5 \times \{3 + [42 \div (-7)]\}$

$= 21 + (-65) \div 5 \times \{3 + [-6]\}$

$= 21 + (-65) \div 5 \times \{-3\}$

$= 21 + 39$

$= 60$

3. **(a)** $7x - 5(4x - 5) = 2(3x - 2) - 9$

$7x - 20x + 25 = 6x - 4 - 9$

$6x - 7x + 20x = 25 + 4 + 9$

$19x = 38$

$x = 2$

(b) $\frac{x - 2}{4x + 1} = 0.5$

$\frac{x - 2}{4x + 1} = \frac{1}{2}$

$2(x - 2) = 4x + 1$

$2x - 4 = 4x + 1$

$4x - 2x = -5$

$2x = -5$

$x = -2\frac{1}{2}$

4. Let the first odd number be n.

Then the next consecutive odd number will be $n + 2$.

$4(n + 2) + \frac{1}{3}n = 73$

$4n + 8 + \frac{1}{3}n = 73$

$4\frac{1}{3}n = 73 - 8$

$4\frac{1}{3}n = 65$

$n = 15$

$\therefore$ The two consecutive odd numbers are 15 and $15 + 2 = 17$.

5. **(i)**

2	4900
2	2450
5	1225
5	245
7	49
7	7
	1

$4900 = 2^2 \times 5^2 \times 7^2$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$9261 = 3^3 \times 7^3$

(ii) $4900 = 2^2 \times 5^2 \times 7^2 = (2 \times 5 \times 7)^2$

$9261 = (3 \times 7)^3$

$\sqrt{4900} = 2 \times 5 \times 7$

$\sqrt[3]{9261} = 3 \times 7$

HCF of $\sqrt{4900}$ and $\sqrt[3]{9261} = 7$

LCM of $\sqrt{4900}$ and $\sqrt[3]{9261} = 2 \times 3 \times 5 \times 7$

$= 210$

6. **(i)** $3(x - 2) - 12 + 5(3 - x)$

$= 3x - 6 - 12 + 15 - 5x$

$= 3x - 5x - 6 - 12 + 15$

$= -2x - 3$

(ii) $3(x - 2) - 12 + 5(3 - x) \leqslant 5$

$3x - 6 - 12 + 15 - 5x \leqslant 5$

$-2x - 3 \leqslant 5$

$2x \leqslant -3 - 5$

$2x \geqslant -8$

$x \geqslant -4$

7. **(a)** Time taken for the first 120 km

$= \frac{120}{40}$

= 3 hours

Distance for the remaining journey

= 200 – 120

= 80 km

Time taken for the remaining 80 km

$= \frac{80}{60}$

$= 1\frac{1}{3}$ hours

Total time taken for the whole journey

$= 3 + 1\frac{1}{3}$

$= 4\frac{1}{3}$ hours or 4 hours 20 minutes

(b) Let the distance of *AB* be *d* km.

$\frac{d}{16} + \frac{d}{24} = 5$

$\frac{3d + 2d}{48} = 5$

$\frac{5d}{48} = 5$

$5d = 5 \times 48$

$5d = 240$

$d = 48$

∴ The distance of *AB* is 48 km.

8. The ratio is 8 : 7 : 5.

8 + 7 = 15 parts represent \$270.

1 part represents \$18.

8 + 7 + 5 = 20 parts represent \$18 × 20 = \$360.

The sum of money to be divided among the three boys is \$360.

9. $\angle AGB = 49°$ (vert. opp. ∠s)

$55° + 49° + y° = 180°$ (∠ sum of △)

$y° = 180° - 55° - 49°$

$= 76°$

$\angle GBD = 49°$ (alt. ∠s, *AE* // *BD*)

$x° + 76° + 49° + 128° = 360°$

$x° = 360° - 76° - 49° - 128°$

$= 107°$

∴ $x = 107$ and $y = 76$

10. **(i)** Arc length of quadrant

$= \frac{1}{4} \times (2 \times \pi \times 10)$

$= 5\pi$ cm

Perimeter of figure

$= 5\pi + (28 - 10) + 3 + 5 + 5 + 5 + (12 - 5 - 3)$
$+ 28 + (12 - 10)$

$= (70 + 5\pi)$ cm

(ii) Area of quadrant

$= \frac{1}{4} \times \pi \times (10)^2$

$= 25\pi \text{ cm}^2$

Area of figure

= area of rectangle – area of quadrant
– area of square

$= (28 \times 12) - 25\pi - (5 \times 5)$

$= 336 - 25\pi - 25$

$= (311 - 25\pi) \text{ cm}^2$

11. **(a)** Size of each interior angle of a 24-sided regular polygon

$= \frac{(24 - 2) \times 180°}{24}$

$= 165°$

(b) An octagon has 8 sides.

Sum of angles in an octagon = $(8 - 2) \times 180°$

$= 1080°$

Let one of the remaining interior angles be $x°$.

$86° + (8 - 1)x° = 1080°$

$86° + 7x° = 1080°$

$7x° = 1080° - 86°$

$= 994°$

$x° = 142°$

∴ The size of one of the remaining interior angles of the octagon is 142°.

(c) A hexagon has 6 sides.

Sum of angles in a hexagon = $(6 - 2) \times 180°$

$= 720°$

3 + 3 + 3 + 3 + 4 + 4 = 20 parts represent 720°

1 part represents 36°.

3 parts represent 36° × 3 = 108°.

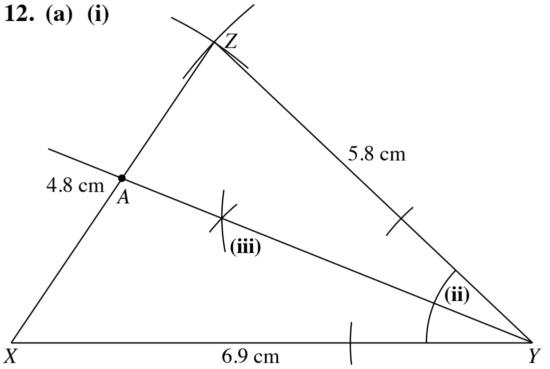
The smallest interior angle gives the largest exterior angle.

∴ Largest exterior angle

$= 180° - 108°$

$= 72°$

12. (a) (i)



(ii) $\angle XYZ = 43°$

(iii) Length of AX = 2.6 cm

(b) (i) $2x° + 72° + x° + 90° = 360°$

$$2x + x + 72 + 90 = 360$$
$$3x = 360 - 72 - 90$$
$$3x = 198$$
$$x = 66$$

(ii) Percentage of students who chose cinema as their favourite form of entertainment

$= \frac{72}{360} \times 100\%$

$= 20\%$

Part II

Section A

1. (a) Percentage of his income on savings

= 100% – 10% – 15% – 12% – 8% – 21%

= 34%

34% represent \$1292.

1% represents \$38.

100% represent 38 × 100 = \$3800

His monthly income is \$3800.

(b) Price of car after first year

= (100 – 15)% of \$56 000

= \$47 600

Price of car after second year

= (100 – 15)% of \$47 600

= \$40 460

Price of car after third year

= (100 – 15)% of \$40 460

= \$34 391

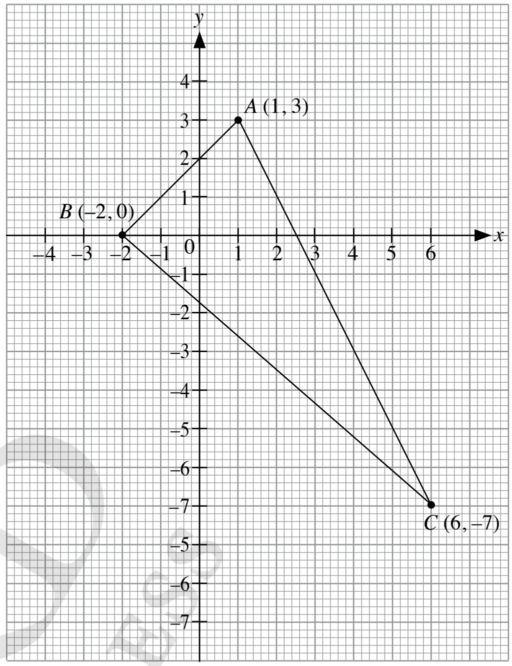
Price of car after fourth year

= (100 – 15)% of \$34 391

= \$29 232.35

= \$29 200 (to the nearest 100 dollars)

2. (i)



(ii) Area of $\triangle ABC$

$= \left[\frac{1}{2} \times (2.5 + 2) \times 3\right] + \left[\frac{1}{2} \times (2.5 + 2) \times 7\right]$

= 6.75 + 15.75

= 22.5 square units

3. (a) When $s = 110$, $v = 36.5$ and $u = 2.56$,

$$110 = \frac{(36.5)^2 - (2.56)^2}{2a}$$
$$2a = \frac{(36.5)^2 - (2.56)^2}{110}$$
$$2a = 12.051\,785\,45$$
$$\therefore a = 6.025\,892\,727$$
$$= 6.03 \text{ (to 3 s.f.)}$$

(b) $2xa - 8pa + 4ya - 6a$

$= 2a(x - 4p + 2y - 3)$

(c) $\frac{4x - 1}{4} - \frac{5 - x}{2} = \frac{5(7 - 2x)}{6} + \frac{11}{12}$

$$12 \times \left(\frac{4x - 1}{4} - \frac{5 - x}{2}\right) = \left(\frac{5(7 - 2x)}{6} + \frac{11}{12}\right) \times 12$$
$$3(4x - 1) - 6(5 - x) = 10(7 - 2x) + 11$$
$$12x - 3 - 30 + 6x = 70 - 20x + 11$$
$$12x + 20x + 6x = 70 + 11 + 3 + 30$$
$$38x = 114$$
$$x = 3$$

4. We observe that $5 = 1 + 2^2$, $14 = 5 + 3^2$ and so on.

(i) She counted $14 + 4^2 = 30$ squares.

(ii) $1 = \frac{1(1+1)(2+1)}{6}$, $5 = \frac{2(2+1)(4+1)}{6}$,

$14 = \frac{3(3+1)(6+1)}{6}$, $30 = \frac{4(4+1)(8+1)}{6}$

$\therefore$ The general formula for the n^{th} term of the sequence is $\frac{n(n+1)(2n+1)}{6}$.

(iii) When $n = 51$, the number of squares is

$= \frac{51(51+1)(2 \times 51+1)}{6}$

$= \frac{51(92)(103)}{6}$

$= 45\ 526$

Section B

5. **(a)** Let 1 part of the length of the sides of the quadrilateral be n cm.

$2n + 3n + 6n + 7n = 108$

$18n = 108$

$n = 6$

The longest side is $7 \times 6 = 42$ cm.

The shortest side is $2 \times 6 = 12$ cm.

Difference between the length of the longest and the shortest sides

$= 42 - 12$

$= 30$ cm

(b) Percentage of bulbs that are not defective

$= 100\% - 6\%$

$= 94\%$

94% represents 611 bulbs.

1% represents 6.5 bulbs.

100% represents $6.5 \times 100 = 650$ bulbs.

He must produce 650 bulbs in order to obtain 611 bulbs which are not defective.

6. **(i)** Ratio of weight of pineapple, sugar and water

$= 6 : 9 : \left(16\frac{2}{3} - 6 - 9\right)$

$= 6 : 9 : 1\frac{2}{3}$

$= 18 : 27 : 5$

(ii) Total weight loss $= 16\frac{2}{3} - 15 = 1\frac{2}{3}$ kg

Ratio of total weight loss and the original weight of mixture

$= 1\frac{2}{3} : 16\frac{2}{3}$

$= 5 : 50$

$= 1 : 10$

(iii) Total cost of producing 15 kg of pineapple jam

= cost of pineapples + cost of sugar + cost of electricity

$= [(0.80) \times 6] + [\$1.20 \times 9] + [2 \times (0.45)]$

$= 4.80 + 10.80 + 0.90$

$= \$16.50$

Cost of each kg of pineapple jam

$= \frac{16.50}{15}$

$= \$1.10$

7. **(i)** Area of cross-section *ABCDEFG*

= area of rectangle *ABCG* – area of semicircle of diameter 40 cm *DEF*

$= (40 \times 60) - \left[\frac{1}{2} \times 3.142 \times \left(\frac{40}{2}\right)^2\right]$

$= 2400 - 628.4$

$= 1771.6$

$= 1770$ cm^2 (to 3 s.f.)

(ii) Volume of slab

= area of cross-section *ABCDEFG* $\times$ length of slab

$= 1771.6 \times 120$

$= 212\ 592$

$= 213\ 000$ cm^3 (to 3 s.f.)

(iii) Total surface area of the slab

$= 2(1771.6) + 2(120 \times 40) + (60 \times 120)$

$+ 2(10 \times 120) + \left[\frac{1}{2} \times 2 \times 3.142 \times \frac{40}{2} \times 120\right]$

$= 3543.2 + 9600 + 7200 + 2400 + 7540.8$

$= 30\ 284$

$= 30\ 300$ cm^2 (to 3 s.f.)

(iv) Convert density to kg/cm^3.

$2300 \text{ kg/m}^3 = \frac{2300 \text{ kg}}{(100 \times 100 \times 100) \text{ cm}^3}$

$= 0.0023$ kg/cm^3

Mass of slab

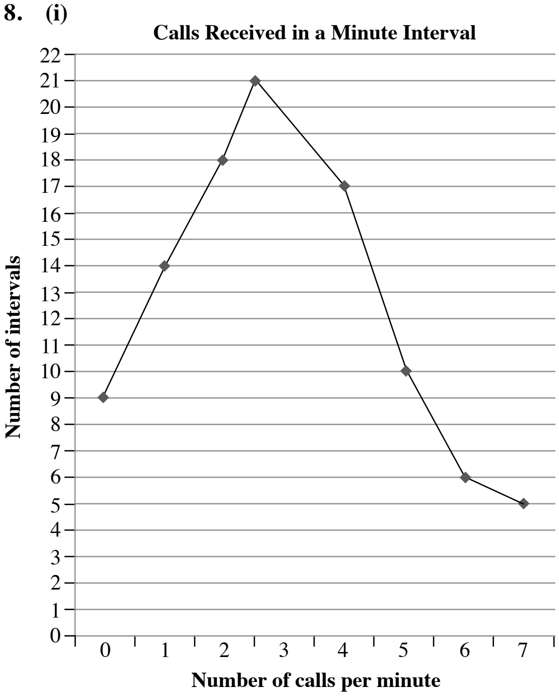
= density $\times$ volume

$= 0.0023 \times 212\ 592$

$= 488.9616$

$= 490$ kg (to 3 s.f.)

8. (i)



(ii) From the line graph, the most common number of calls per minute is 3.

(iii) 18 intervals

(iv) Total number of intervals

$= 9 + 14 + 18 + 21 + 17 + 10 + 6 + 5$

$= 100$

Number of intervals with more than 3 calls per minute

$= 17 + 10 + 6 + 5$

$= 38$

Ratio of the number of intervals with more than 3 calls to total number of intervals

$= 38 : 100$

$= 19 : 50$

(v) Number of intervals with 3 or less calls per minute

$= 100 - 38$

$= 62$

Percentage of intervals with 3 or less calls

per minute $= \frac{62}{100} \times 100\%$

$= 62\%$

(vi) Angle representing 0 calls per minute

$= \frac{9}{100} \times 360° = 32.4°$

Angle representing 1 call per minute

$= \frac{14}{100} \times 360° = 50.4°$

Angle representing 2 calls per minute

$= \frac{18}{100} \times 360° = 64.8°$

Angle representing 3 calls per minute

$= \frac{21}{100} \times 360° = 75.6°$

Angle representing 4 calls per minute

$= \frac{17}{100} \times 360° = 61.2°$

Angle representing 5 calls per minute

$= \frac{10}{100} \times 360° = 36°$

Angle representing 6 calls per minute

$= \frac{6}{100} \times 360° = 21.6°$

Angle representing 7 calls per minute

$= \frac{5}{100} \times 360° = 18°$

(vii) Answer varies. One possible reason: for calls of more than 4, it may be because some callers hang up the phone before even speaking to the operators. Therefore, more calls are recorded.

NOTES